1-3 Cube Roots

Check Skills You'll Need

1. **Vocabulary Review** The ? of a number is a number that, when multiplied by itself, is equal to the given number.

Find the positive and negative square roots of each number.

2. 1
3. $\frac{1}{25}$
4. 81
5. $\frac{1}{144}$

GO for Help Lesson 1-2

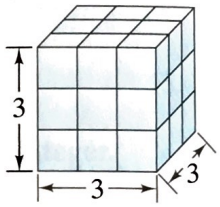
What You'll Learn

To find cube roots and to solve cube root equations

New Vocabulary perfect cube, cube root

Why Learn This?

The large cube at the right is made up of smaller unit cubes. You can use this type of model to help you understand perfect cubes and cube roots.



A cube number is a power with an exponent of 3. A number that is the cube of a whole number is a **perfect cube**. For example, $3 \cdot 3 \cdot 3$, or 3^3, is 27. So 27 is a perfect cube.

The **cube root** of a number is a number that, when used as a factor three times, is equal to the given number. Since 3^3, or $3 \cdot 3 \cdot 3$, is 27, 3 is the cube root of 27.

Perfect Cubes

n	n^3
0	0
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1,000

EXAMPLE Finding Cube Roots of Perfect Cubes

1. Find the cube root of each number.

a. 8 $\quad 2 \cdot 2 \cdot 2 = 8$ ← The cube root of 8 is 2.

b. -125 $\quad -5 \cdot -5 \cdot -5 = -125$ ← The cube root of -125 is -5.

c. $\frac{1}{64}$ $\quad \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64}$ ← The cube root of $\frac{1}{64}$ is $\frac{1}{4}$.

Quick Check

1. Find the cube root of each number.

a. 216 $\quad$ b. -1 $\quad$ c. $\frac{1}{27}$

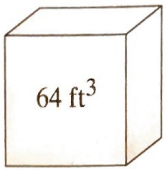
The inverse of cubing a number is finding its cube root. The symbol $\sqrt[3]{\ }$ means the cube root of a number. For example, $\sqrt[3]{1{,}000}$ means the cube root of 1,000, or 10.

EXAMPLE 2 Finding the Side Length of a Cube

Vocabulary Tip

Volume is the number of unit cubes needed to fill a solid. Volume is measured in cubic units.

Measurement A cube-shaped packing box has a volume of 64 cubic feet. What is the side length of the box?



The formula for the volume V of a cube is $V = s^3$, where s is the length of one side of the cube.

Method 1 Solve an Equation

$V = s^3$ ← Volume formula

$64 = s^3$ ← Substitute 64 for V.

$\sqrt[3]{64} = \sqrt[3]{s^3}$ ← Find the cube root of each side.

3 [$\sqrt[x]{}$] 64 [=] 4 ← Use a calculator.

$4 = s$

Method 2 Mental Math

The volume of the box is 64 cubic feet. Since $4^3 = 64$, $\sqrt[3]{64} = 4$.

The side length of the box is 4 feet.

Quick Check

2. A different cube-shaped packing box has a volume of 125 cubic feet. What is the side length of the box?

To find the cube root of a fraction, find the cube root of the numerator and the cube root of the denominator.

EXAMPLE 3 Solving a Cube Root Equation

Solve $x^3 = \frac{8}{343}$.

$x^3 = \frac{8}{343}$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{8}{343}}$ ← Find the cube root of each side.

$= \frac{\sqrt[3]{8}}{\sqrt[3]{343}}$ ← Find the cube root of the numerator. Find the cube root of the denominator.

$x = \frac{2}{7}$ ← Simplify. *Think:* $8 = 2 \cdot 2 \cdot 2$ and $343 = 7 \cdot 7 \cdot 7$.

Quick Check

3. Solve $x^3 = \frac{27}{216}$.

Check Your Understanding

1. **Reasoning** The volume of a cube is 512 cubic inches. What is the length of its side? Explain your reasoning.

Find the cube root of each number.

2. 729
3. 0
4. -8
5. $\frac{1}{1{,}000}$

Homework Exercises

For more exercises, see Extra Skills and Word Problems.

GO for Help

For Exercises	See Examples
6–11	1
12–19	2, 3

Find the cube root of each number.

6. 343
7. 1
8. -27
9. $\frac{1}{125}$
10. -216
11. $\frac{8}{343}$

Solve each equation by finding the value of x.

12. $x^3 = 512$
13. $x^3 = 125$
14. $x^3 = -1{,}000$
15. $x^3 = \frac{27}{125}$
16. $x^3 = \frac{512}{729}$
17. $x^3 = \frac{343}{1{,}000}$

18. A cube-shaped gift box has a volume of 8 cubic inches. What is the side length of the box?

19. A cube-shaped planter holds $\frac{27}{512}$ cubic feet of potting soil. What is the side length of the planter?

GPS

20. **Guided Problem Solving** Find the cube root of 0.008.
    - Can you rename the decimal as a fraction?
    - Write your answer as a decimal. Check by cubing your answer.

21. Find the cube root of 0.216.

22. **Writing in Math** Is 0.3 the cube root of 0.27? Explain.

23. **a.** Copy and complete the table below.

x	1	2	3	4	5	6	7	8	9	10
x^2										
x^3										

b. Are any of the numbers both a perfect square and a perfect cube?

c. When x is a perfect square, what is true about the cube of x?

Find the value of each expression.

24. $(\sqrt[3]{64})^3$ **25.** $\sqrt[3]{9^3}$ **26.** $(\sqrt[3]{n})^3$

Draw and label a cube to model each volume.

27. 1 cubic inch **28.** 8 cubic feet **29.** 125 cubic meters

30. Number Sense For what values of n does $\sqrt[3]{n}$ have the following values?

a. positive

b. 0

c. negative

31. Error Analysis A student evaluated the expression $\sqrt[3]{\frac{8}{64}}$ and got $\frac{1}{8}$. What error did the student make?

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For help using a calculator to find a cube root, see Example 2.

32. Use a calculator to estimate $\sqrt[3]{350}$ to the nearest hundredth.

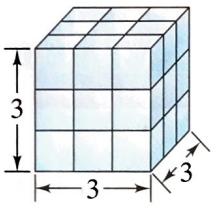
33. Challenge Estimate the value of the expression $\sqrt[3]{100}$ to the nearest integer. Explain your reasoning.

Test Prep and Mixed Review

Practice

Multiple Choice

34. The model represents $\sqrt[3]{27} = 3$. Which model can be used to represent $\sqrt[3]{1{,}000}$?



Ⓐ $100 \times 10 \times 1$ rectangular prism

Ⓑ $25 \times 10 \times 4$ rectangular prism

Ⓒ $20 \times 10 \times 5$ rectangular prism

Ⓓ $10 \times 10 \times 10$ rectangular prism

35. Luisa bought 50 tiles on sale. Each one measured 1 square foot. What is the largest square floor she can tile?

Ⓕ 1 ft × 1 ft

Ⓖ 7 ft × 7 ft

Ⓗ 8 ft × 8 ft

Ⓙ 50 ft × 50 ft

36. Which of the following inequalities is NOT true?

Ⓐ $\sqrt{64} > \sqrt[3]{64}$

Ⓑ $\sqrt{64} \geq \sqrt[3]{64}$

Ⓒ $\sqrt{0} < \sqrt[3]{0}$

Ⓓ $\sqrt{1} \leq \sqrt[3]{1}$

GO for Help

For Exercises	See Lesson
37–39	1-2

Estimate the value of each expression to the nearest tenth.

37. $\sqrt{18}$ **38.** $\sqrt{30}$ **39.** $\sqrt{85}$