1-6 Converse of the Pythagorean Theorem

Check Skills You'll Need

1. **Vocabulary Review** In a right triangle, the length of a ? cannot be longer than the length of the ?.

Find the missing leg length b given one leg length a and the length of the hypotenuse c. Round to the nearest tenth.

2. $a = 6, c = 10$
3. $a = 9, c = 15$

GO for Help Lesson 1-5

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8.G.6

What You'll Learn

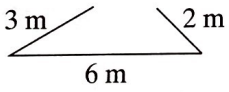
To solve problems using the Triangle Inequality Theorem and the Converse of the Pythagorean Theorem

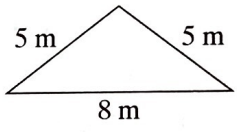
New Vocabulary Triangle Inequality Theorem, Converse of the Pythagorean Theorem

Why Learn This?

You can use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle.

Notice that the segments on the left below do *not* form a triangle. The segments on the right below do form a triangle. The lengths of three segments must be related in a special way in order to form a triangle.





KEY CONCEPTS Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

EXAMPLE Using the Triangle Inequality Theorem

1 Is it possible to construct a triangle with the given side lengths? Explain.

a. 6 cm, 5 cm, 4 cm

$4 + 5 > 6$ ✔
$5 + 6 > 4$ ✔
$4 + 6 > 5$ ✔

Yes. The sum of any two lengths is greater than the third length.

b. 7 mi, 15 mi, 6 mi

$6 + 15 > 7$ ✔
$15 + 7 > 6$ ✔
$7 + 6 > 15$ ✘

No. The sum of 7 and 6 is *not* greater than 15.

Quick Check

1. Is it possible to construct a triangle with the given side lengths? Explain.
   a. 6 mi, 10 mi, 20 mi
   b. 1.5 m, 2.5 m, 3.5 m

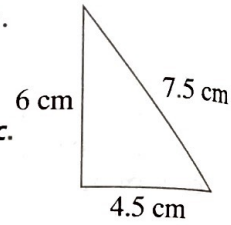
If the equation $a^2 + b^2 = c^2$ is true for the lengths of the sides of a triangle, then the triangle is a right triangle. This method is called the **Converse of the Pythagorean Theorem**. You can use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle.

EXAMPLE 2 Identifying a Right Triangle

Vocabulary Tip

When using the Converse of the Pythagorean Theorem, substitute the greatest side length for *c*.

Determine whether the triangle is a right triangle. Explain.



$a^2 + b^2 \stackrel{?}{=} c^2$ ← **Use the Pythagorean Theorem.**

$6^2 + 4.5^2 \stackrel{?}{=} 7.5^2$ ← **Substitute 6 for *a*, 4.5 for *b*, and 7.5 for *c*.**

$36 + 20.25 \stackrel{?}{=} 56.25$ ← **Simplify. Use a calculator.**

$56.25 = 56.25$

The equation is true, so the triangle is a right triangle.

Quick Check

2. Determine whether the given lengths can be side lengths of a right triangle. Explain.

a. 10 in., 24 in., 26 in.

b. 8 cm, 9 cm, 12 cm

EXAMPLE 3 Application: Surveying

A land surveyor determines that a triangular plot of land has boundary lines that are 400 yd, 600 yd, and 720 yd in length. The 400-yd boundary line runs east-west. Determine whether either of the other two boundary lines runs north-south.

If either of the other two boundary lines runs north-south, then the boundary lines must form a right triangle. So use the Converse of the Pythagorean Theorem.

$a^2 + b^2 \stackrel{?}{=} c^2$ ← **Use the Pythagorean Theorem.**

$400^2 + 600^2 \stackrel{?}{=} 720^2$ ← **Substitute 400 for *a*, 600 for *b*, and 720 for *c*.**

$160{,}000 + 360{,}000 \stackrel{?}{=} 518{,}400$ ← **Simplify.**

$520{,}000 \neq 518{,}400$

The equation is not true, so the boundary lines do not form a right triangle. Neither of the other two boundary lines runs north-south.

Quick Check

3. A triangular field has boundary lines that are 40 yd, 75 yd, and 85 yd long. Determine whether the boundary lines form a right triangle. Explain.

Check Your Understanding

1. **Vocabulary** How is the Triangle Inequality Theorem different from the Converse of the Pythagorean Theorem?

Three lengths are given. Use the inequality that follows to determine if it is possible to construct a triangle with the lengths. Explain.

2. 3 ft, 6 ft, 8 ft
$3 + 6 \stackrel{?}{>} 8$

3. 7 in., 8 in., 15 in.
$7 + 8 \stackrel{?}{>} 15$

4. 0.5 m, 2.5 m, 4 m
$0.5 + 2.5 \stackrel{?}{>} 4$

The lengths of the sides of a triangle are given. Use the equation that follows to determine whether the triangle is a right triangle. Explain.

5. 6 yd, 8 yd, 10 yd
$6^2 + 8^2 \stackrel{?}{=} 10^2$

6. 16 cm, 63 cm, 65 cm
$16^2 + 63^2 \stackrel{?}{=} 65^2$

7. 8 m, 24 m, 25 m
$8^2 + 24^2 \stackrel{?}{=} 25^2$

8. The sides of a triangular weather flag are 49.6 in., 49.6 in., and 25 in. in length. Is the flag in the shape of a right triangle? Explain.

Homework Exercises

For more exercises, see Extra Skills and Word Problems.

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For Exercises	See Examples
9–14	1
15–20	2
21–22	3

Is it possible to construct a triangle with the given side lengths? Explain.

9. 2 ft, 4 ft, 7 ft

10. 1 in., 5 in., 6 in.

11. 15 m, 25 m, 35 m

12. 3.8 ft, 5.2 ft, 8.5 ft

13. $\frac{2}{3}$ in., $4\frac{2}{3}$ in., 5 in.

14. 22 m, 44 m, 66 m

Determine whether the given lengths can be side lengths of a right triangle. Explain.

15. 12 m, 16 m, 20 m

16. 8 in., 31.5 in., 32.5 in.

17. 10 yd, 12 yd, 16 yd

18. 5 cm, 13 cm, 14 cm

19. 8 ft, 15 ft, 17 ft

20. 7 mi, 24 mi, 26 mi

21. The three streets shown at the right intersect to form a triangle. Do the streets form a right triangle? Explain your reasoning.

22. The front view of an A-frame house is a triangle. The lengths of its sides are 39 ft, 39 ft, and 28 ft. Is the front view of the house in the shape of a right triangle? Explain.

GPS **23. Guided Problem Solving** The lengths of two sides of a triangle are 5 in. and 7 in. What is the range of possible lengths of the third side x of the triangle?

- How can you use the Triangle Inequality Theorem to find the least possible length for the third side?
- How can you use the Triangle Inequality Theorem to find the greatest possible length for the third side?

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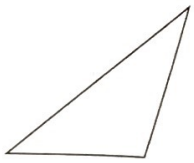
24. Writing in Math Two sides of a right triangle measure 5 in. and 12 in. Why is this not enough information to find the length of the third side of the triangle?

25. Measurement Copy the triangle at the right. Use a ruler to test the Triangle Inequality Theorem.



26. Number Sense How do you know that a triangle with side lengths $\sqrt{1}$, $\sqrt{2}$, and $\sqrt{3}$ is a right triangle? Explain.

27. Challenge You can use the squares of the lengths of the sides of a triangle to find whether the triangle is acute or obtuse.

If $a^2 + b^2 < c^2$, then the triangle is obtuse.
If $a^2 + b^2 > c^2$, then the triangle is acute.

In both cases, c represents the length of the longest side of the triangle. The lengths of the sides of a triangle are 5 m, 6 m, and 7 m. Is the triangle *acute*, *right*, or *obtuse*? Explain.

Vocabulary Tip

An *obtuse triangle* has one obtuse angle. An *acute triangle* has three acute angles.

Test Prep and Mixed Review

Practice

Multiple Choice

28. Which set of lengths could NOT represent the side lengths of a right triangle?

(A) 18 yd, 24 yd, 30 yd
(B) 13 yd, 14 yd, 15 yd
(C) 7 yd, 24 yd, 25 yd
(D) 15 yd, 36 yd, 39 yd

29. A 12-ft ladder leans against the side of a house. The ratio of the length of the ladder to the space between the bottom of the ladder and the house is 4 : 1. To the nearest tenth of a foot, what is the distance from the top of the ladder to the ground?

(F) 15.0 ft (G) 12.4 ft (H) 11.6 ft (J) 9.0 ft

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For Exercises	See Lesson
30–32	1-5

The lengths of two sides of a right triangle are given; a and b represent the lengths of the legs, and c represents the length of the hypotenuse. Find the missing side length.

30. $a = 20, b = 21$ **31.** $a = 13, c = 85$ **32.** $b = 14, c = 50$