2-5 Types of Solutions of Linear Equations

Check Skills You'll Need

1. **Vocabulary Review** A value of the variable for which an equation is true is a ? of the equation.

Solve each equation. Check the solution.

2. $2a + 4 = 4a - 2$
3. $3w - 1 = -2(w + 3)$
4. $3(z - 2) = 5z + 1$

GO for Help Lesson 2-4

What You'll Learn

To identify whether a linear equation in one variable has one, infinitely many, or no solutions

Why Learn This?

Equations with variables on both sides can help you check whether two ways of paying for something, such as swimming lessons, cost the same.

The equations solved in this chapter so far have resulted in a variable equal to a number. Sometimes, solving an equation may result in a number equal to the same number, or a different number. Each of these results indicates how many solutions an equation has. This is summarized in the table below, where x represents the variable and a and b represent different numbers.

CONTENT STANDARDS
8.EE.7, 8.EE.7a, 8.EE.7b

KEY CONCEPTS Types of Solutions

Algebraic Form	Number of Solutions	Description
$a = b$	None	There are *no* values of the variable for which the equation is true.
$x = a$	One	The equation is true for *exactly one* value of the variable.
$a = a$	Infinitely many	The equation is true for *all* values of the variable.

EXAMPLE 1 Identifying Types of Solutions

Tell whether each equation has one solution, infinitely many solutions, or no solution. Justify your answer.

a. $2x - 4 = -x - 1$

$2x + 1x - 4 = -x + 1x - 1$ ← Add $1x$ to each side.

$3x - 4 = -1$ ← Simplify.

$3x - 4 + 4 = -1 + 4$ ← Add 4 to each side.

$3x = 3$ ← Simplify.

$x = 1$ ← Divide both sides by 3.

The result is an equation of the form $x = a$. This equation is true for exactly one value. So, the equation has one solution.

Vocabulary Tip

The possible solutions to an equation that has infinitely many solutions are endless.

b.
$$\begin{aligned} 2x - 4 &= 2(x - 2) \\ 2x - 4 &= 2x - 4 && \leftarrow \text{Use the Distributive Property.} \\ 2x - 4 - 2x &= 2x - 4 - 2x && \leftarrow \text{Subtract } 2x \text{ from each side.} \\ -4 &= -4 && \leftarrow \text{Simplify.} \end{aligned}$$

The result is an equation of the form $a = a$. This equation is true for all values of x. So, the equation has infinitely many solutions.

c.
$$\begin{aligned} 2x - 4 &= 2(x + 1) \\ 2x - 4 &= 2x + 2 && \leftarrow \text{Use the Distributive Property.} \\ 2x - 4 - 2x &= 2x + 2 - 2x && \leftarrow \text{Subtract } 2x \text{ from each side.} \\ -4 &= 2 && \leftarrow \text{Simplify.} \end{aligned}$$

The result is an equation of the form $a = b$. There are no values of x for which the equation is true. So, the equation has no solution.

Quick Check

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For help solving equations with variables on both sides, see Lesson 2-4, Examples 1 and 2.

1. Tell whether each equation has one solution, infinitely many solutions, or no solution. Justify your answer.
   - **a.** $5x + 8 = 5(x + 3)$
   - **b.** $9x = 8 + 5x$
   - **c.** $6x + 12 = 6(x + 2)$
   - **d.** $7x - 11 = 11 - 7x$

EXAMPLE 2 Application: Comparing Costs

Sports You want to take 10 lessons at a swim club. You can pay a membership fee of \$20 plus a fee per lesson. You can also decide not to pay a membership fee. In that case, the fee per lesson is \$3 more. Is there any lesson fee for which these two plans cost the same? Justify your answer.

Words $\text{membership fee} + \frac{10}{\text{lessons}} \cdot \text{lesson fee} = \frac{10}{\text{lessons}} \cdot (\text{lesson fee} + \$3)$

Equation Let f = lesson fee.

$$20 + 10 \cdot f = 10 \cdot (f + 3)$$

$$\begin{aligned} 20 + 10f &= 10(f + 3) \\ 20 + 10f &= 10f + 30 && \leftarrow \text{Use the Distributive Property.} \\ 20 + 10f - 10f &= 10f + 30 - 10f && \leftarrow \text{Subtract } 10f \text{ from each side.} \\ 20 &= 30 && \leftarrow \text{Simplify.} \end{aligned}$$

The result is an equation of the form $a = b$. So the equation has no solution. There is no lesson fee for which these two plans cost the same.

Test Prep Tip

Check the type of solution by substituting different values for the variable in the original equation.

Quick Check

2. Admission to the museum is \$8 for students and \$16 for adults. Yesterday, twice as many students as adults came to the museum. The total admissions paid by students and the total admissions paid by adults were equal. Can you find the number of adults that came to the museum yesterday? Justify your answer.

Check Your Understanding

1. **Vocabulary** When any value can be substituted for the variable in an equation to create a true equation, the equation has __?__ solution(s).

Match each equation to the correct number of solutions.

2. $4x + 8 = 4(x + 4)$
3. $5x = 9 + 2x$
4. $x + 9 = 7x + 9 - 6x$

A. one
B. infinitely many
C. none

5. **Reasoning** What number could you substitute for a in the equation $6x + 9 = a(2x + 3)$ to create an equation with infinitely many solutions? Explain.

Homework Exercises

For more exercises, see Extra Skills and Word Problems.

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For Exercises	See Example
6–15	1
16	2

Show whether each equation has one solution, infinitely many solutions, or no solution. Justify your answer.

6. $7x = 3x - 12$
7. $3x + 3 = 3(x + 1)$
8. $22y = 11(3 + y)$
9. $-3t + 1 = t + 9 - 4t$
10. $16z - 24 = 8(2z - 3)$
11. $-5w = 7 - 4w + 8$
12. $4(-x - 1.6) = -4x + 6.4$
13. $1 + c + 1.4 = c + 2.4$
14. $\frac{5}{3}s = \frac{15}{3} + \frac{3}{2}s - \frac{1}{4}$
15. $-\frac{2}{9}\left(-n + \frac{3}{9}\right) = \frac{2}{9}n - \frac{5}{9}$

16. A recreation center offers a membership for \$75. Members may take classes for \$10 each. Nonmembers must pay \$15 for these classes. How many classes could a nonmember take and pay the same total amount as a member? Justify your answer.

GPS

17. **Guided Problem Solving** Six more than a number equals two times the sum of one-half the number plus three. Is this statement true for only one number, for all numbers, or for no numbers? Explain your reasoning.
    - Write an equation to represent the statement.
    - Simplify the equation until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results.

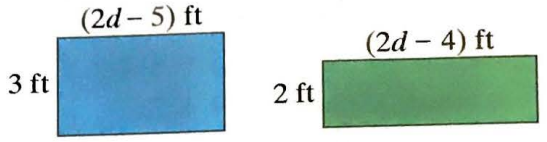
18. Two more than a number equals three times the sum of one third of the number plus six. Is this statement true for only one number, for all numbers, or for no numbers? Explain your reasoning.

19. **Open Ended** The equation $20y + 4 = 4(3y + 1)$ has exactly one solution. Change one number in the original equation to create a new equation that has infinitely many solutions. Then change one number in the new equation to create another equation that has no solution.

20. **Geometry** Greg is buying fabric from a store. He has the choice of buying fabric that is 2 feet wide or 3 feet wide. The diagrams show how much fabric of each type he can buy for d dollars.

a. For what value(s) of d is the perimeter of both choices the same?

b. For what value(s) of d is the area of both choices the same?



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21. **Writing in Math** In the process of simplifying an equation, Marco eliminated all the variables. How many solutions could the original equation have? Explain your reasoning.

22. **Open Ended** Write three equations with the same variable on each side. One equation should have one solution, one equation should have infinitely many solutions, and one equation should have no solutions. Justify your work.

23. **Challenge** Find the number of solutions of the equation $6(0.8 + 2z) - 3.2 = 4(3z - 0.3) + 2.8$. Justify your answer.

Test Prep and Mixed Review

Practice

Multiple Choice

24. Which equation has exactly one solution?

Ⓐ $3x + 7 = 2x + 14 + x$ Ⓒ $3(x + 7) = 4x + 21 - x$

Ⓑ $3(x + 7) = 2(x + 7) + x$ Ⓓ $3x + 7 = 5x + 7 - 3x$

25. For a party, you buy 2 dozen cupcakes and 3 quarts of ice cream. Your friend buys 1 dozen cupcakes and 2 quarts of ice cream. Let d = cost of a dozen cupcakes and q = cost of a quart of ice cream. Which expression represents the total cost of the cupcakes and ice cream?

Ⓕ $2d + 2q$ Ⓗ $3d + 5q$

Ⓖ $2d + 3q$ Ⓙ $5d + 3q$

26. You hike 3 mi west of your campsite. Then you hike 2 mi south. To the nearest tenth of a mile, how far are you from your campsite?

Ⓐ 5.0 mi Ⓒ 3.2 mi

Ⓑ 3.6 mi Ⓓ 2.2 mi

27. A diagonal path through a rectangular garden is 12 yd long. The length of the garden is 11 yd. To the nearest tenth, what is the width of the garden?

Ⓕ 1.0 yd Ⓗ 16.3 yd

Ⓖ 4.8 yd Ⓙ 23.0 yd

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For Exercises	See Lesson
28–30	1-1

Write each fraction as a decimal.

28. $\frac{13}{16}$ 29. $\frac{3}{11}$ 30. $\frac{5}{9}$