## 3.3

## Proportional Relationship

## Check Skills You'll Need

## 1. Vocabulary Review

What is the input variable in the function $y=2 x+1$ ?

Use the function $d=3 t$ to complete the function table.
2.

| $t$ | $d$ |
| :---: | :---: |
| 1 | $\square$ |
| 2 | $\square$ |
| 3 | $\square$ |
| 4 | $\square$ |

## What You'll Learn

To determine if relationships are proportional
New Vocabulary proportional relationship

## Why Learn This?

Proportional relationships are used for many things like cooking, chemical mixtures, medicines, and shopping.

A proportional relationship is a relationship between inputs and outputs in which the ratio of inputs and outputs is always the same. If a relationship is in a table,
 write the ratio of each input to its corresponding output.
If the ratios are the same, then the relationship is proportional.

## EXAMPLES Proportional Relationships in Tables

(1) Distance-Time Relationships Determine if the relationship is proportional.

| Mario's Car Trip |  |
| :---: | :---: |
| Time $t(\mathrm{hr})$ | Distance $d(\mathrm{mi})$ |
| 1 | 50 |
| 2 | 100 |
| 3 | 150 |
| 4 | 240 |

$$
\begin{aligned}
& 1 / 50 \\
& 2 / 100=1 / 50 \\
& 3 / 150=1 / 50 \\
& 4 / 240=1 / 60
\end{aligned} \quad \leftarrow \quad \begin{aligned}
& \text { Write the ratio of each input } \\
& \text { to its corresponding output. } \\
& \text { Then simplify. }
\end{aligned}
$$

The ratios are not all the same so the relationship is not proportional.
(2) Comparison Shopping Determine if the relationship is proportional. Tomato Prices

| 2 pounds for $\$ 3$ | $2 / \$ 3$ | The number of pounds is the <br> 4 pounds for $\$ 6$ |
| :--- | :--- | :--- |
| 6 pounds for $\$ 9$ | $4 / \$ 6=2 / \$ 3$ | $6 / \$ 9=2 / \$ 3$ | | input and the cost is the |
| :--- |
| output. |

The ratios are the same so the relationship is proportional.

## COuick Check

1. The ratios of all the inputs to the outputs in a table are $\frac{1}{4}$. Is the relationship proportional?
2. Pizza slices are selling as follows: 1 for $\$ 2,2$ for $\$ 3$, or 4 for $\$ 5$. Is this relationship proportional? Explain.

You can graph a function with a proportional relationship by making an input-output table.

## EXAMPLE Input-Output Tables and Graphs

(3) Jewelry Jenna earns $\$ 12.00$ per hour making bracelets. She uses the function $e=12 h$ to track her earnings where e represents earnings and $h$ represents number of hours. Make an input-output table and graph your results. Does the function have a proportional relationship? Explain.
Step 1 Make an input-output table. Choose values for $h$ and solve for $e$.

| Number of Hours h | Earnings e (in dollars) |
| :---: | :---: |
| 1 | 12 |
| 2 | 24 |
| 3 | 36 |
| 4 | 48 |

This can be shown to be a proportional relationship by checking that the ratios are the same and are equal to $\frac{1}{12}$.
Step 2 Sketch a graph. Label the axes using $h$ and $e$ and with titles. Plot the points and draw a line through them.

Jenna's Earnings


## Quick Check

3. The function $t=4 m$ gives the temperature $t$ in degrees Celsius after $m$ minutes of a liquid during a science experiment. Determine if the function has a proportional relationship.
4. Vocabulary How can you determine if a relationship in a table is a proportional relationship?
5. The ordered pairs $(2,4)$ and $(5, x)$ are a part of the same proportional relationship. Find the value of $x$.
6. Reasoning A baker sells 3 rolls for $\$ 5$ and 6 rolls for $\$ 8$. Is the relationship between the price of selling 3 rolls and 6 rolls proportional? Explain.
7. Make an input-output table for the function $y=4 x$ and graph the function.

## 60 for Help

For Exercises See Examples

| $5-8$ | 1 |
| :---: | :---: |
| $9-12$ | 2 |
| 13 | 3 |

## Determine if the relationship is proportional.

5. 

| $x$ | $y$ |
| ---: | ---: |
| -2 | -10 |
| -4 | -20 |
| 6 | 30 |
| 8 | 40 |

7. 

| $\boldsymbol{m}$ | $\boldsymbol{n}$ |
| :---: | :---: |
| -12 | 6 |
| -14 | 7 |
| -18 | 8 |
| -20 | 9 |

9. 


11.

6.

| $\boldsymbol{d}$ | $\boldsymbol{r}$ |
| :---: | :---: |
| 30 | 40 |
| 45 | 60 |
| 60 | 90 |
| 75 | 100 |

8. 

| $\boldsymbol{h}$ | $\boldsymbol{t}$ |
| :---: | :---: |
| 150 | 50 |
| 270 | 90 |
| 360 | 120 |
| 600 | 200 |

10. 

## Bike Rentals

1 hr for \$8
2 hr for $\$ 16$
3 hr for $\$ 24$
4 hr for $\$ 32$
12.

Ski Equipment Rentals
4 hr for $\$ 10$
6 hr for \$15
12 hr for $\$ 30$
1 day for \$60
13. Landscaping Tamika earns $\$ 15.00$ per hour working for a landsca company plus $\$ 10$ for lunch. She uses the function $e=15 h+10$ to track her daily earnings where $e$ represents earnings and $h$ represer number of hours. Make an input-output table, graph your results, determine if the function has a proportional relationship. Explain
14. Guided Problem Solving The graph shows the relationship between time and distance of a train from the Central Train Station Reynaldo is traveling to his hometown 360 miles away from the station on the train. Based on the graph, estimate how long it will take the train to arrive at Reynaldo's hometown.

- See if the graph for train A passes through a distance that is a factor of 360 .
- Use the corresponding time to estimate your answer.

Kudzu


Basketball

15. Data Analysis The graph at the right shows the relationship between time and total snowfall for a December blizzard. Based on the graph, estimate how long it will take for the amount of snow to total 18 inches.
16. Reasoning A scientist measures the growth rate of Kudzu at 4.5 in . every 12 hr . Sketch the graph at the top left on your paper. Use the scientist's information to complete the graph.

Blizzard Snowfall

17. Challenge Jake burns calories when cross-country skiing at the rate of 11 calories per minute. The graph at the left shows the calories he burns while playing basketball. Which activity burns calories at a faster rate? Explain.

## rest Prep and mixed Review

## Multiple Choice

| 60) for Help |  |
| :---: | :---: |
| For Exercises | See Lesson |
| $20-22$ | $1-7$ |

18. Which relationship is proportional?
(A)

| $\boldsymbol{d}$ | $\boldsymbol{t}$ |
| ---: | ---: |
| 8 | 4 |
| 12 | 6 |
| 20 | 10 |
| 24 | 11 |
| 26 | 13 |


Pizza Slices
1 for $\$ 1.50$
2 for $\$ 2.25$
3 for $\$ 3.75$
4 for $\$ 4.00$
(D)

| $x$ | $y$ |
| :---: | :---: |
| -20 | -15 |
| -60 | -45 |
| -120 | -90 |
| -240 | -180 |
| -400 | -300 |

19. Which is the solution to the equation $x^{3}=-64$ ?
(F) $x=-8$
(G) $x=-4$
(H) $x=4$
$x=8$

Find the distance between the points. Round your answer to the nearest tenth.
20. $A(0,0)$ and $B(3,4)$
21. $A(-2,1)$ and $B(2,6)$
22. $A(4,-3)$ and $B(7,6)$

## Proportional Relationships in Graphs

You can determine if relationships are proportional from looking at graphs of the relationships,

## ACTIVITY

Copy and complete the table of ordered pairs for each graph.

Function A


Function C


| $x$ | -6 | -2 | 2 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ |  |  |  |  |

Function B


| $x$ | 8 | 6 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ |  |  |  |  |

Function D


| $\boldsymbol{x}$ | -3 | -2 | -1 | 1 |
| :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ |  |  |  |  |

1. Which functions have proportional relationships? Explain.
2. Look at the graphs of the functions that have proportional relationships. What do they have in common?
3. Look at the graphs of the functions that do not have proportional relationships. What do they have in common?
4. How can you determine if a function is proportional by looking at its graph?
