3-5 Nonlinear Functions

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8.F.3

What You'll Learn

To identify nonlinear functions

New Vocabulary nonlinear function, quadratic function, parabola

Why Learn This?

Think of a juggler's ball being tossed up in the air and falling back down. As the ball rises, its speed decreases. Then it falls, and the speed increases. The relationship between speed and time is not always the same.

In cases like this, **nonlinear functions** are needed to describe how variables are related. Nonlinear functions are functions whose graphs are not straight lines. One example of a nonlinear function is a **quadratic function** in which the greatest exponent of a variable is 2.

The graph of a quadratic function is a U-shaped curve called a **parabola.** The curve may open upward or downward. When you throw a ball into the air, the path it follows is a parabola.
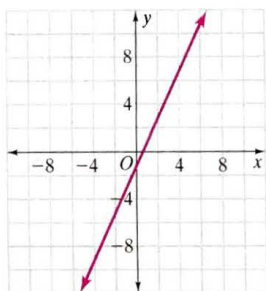
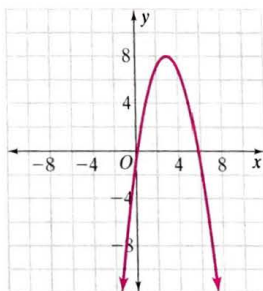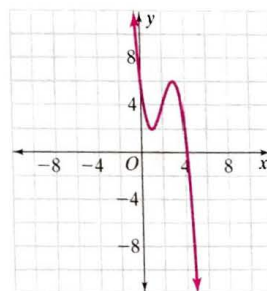### EXAMPLE Nonlinear Functions From Graphs

1 Which functions appear to be nonlinear?

Function 1   **Function 2**   **Function 3**

The graph of function 1 is a straight line and the graph of functions 2 and 3 are not straight lines. Functions 2 and 3 are nonlinear.

✓ Quick Check

1. Sketch the function that passes through the points in the table. Does the function appear to be nonlinear?

x	2	4	6	8
y	4	8	12	16

You can identify nonlinear functions from equations and tables.

Equation: If a function's greatest exponent is 2 or greater, or if it has a variable for an exponent, the function is nonlinear.

Table: If the ratios between the changes in variables in a table are *not* the same, then the function is nonlinear.

> **EXAMPLE** **Nonlinear Functions from Tables and Equations**

2 Determine which function is nonlinear.

a. $y = 4x^3 - 7$

$$y = 8(x + 1) - 12$$
$$y = 8x + 8 - 12$$
$$y = 8x - 4$$

$y = 4x^3 - 7$: greatest exponent is 3 $y = 8x^1 - 4$: greatest exponent is 1

The nonlinear function is $y = 4x^3 - 7$.

b.

x	2	4	6	8
y	10	7	4	1

(+2, +2, +2 across x; −3, −3, −3 across y)

x	1	2	3	4
y	1	8	27	64

(+1, +1, +1 across x; +7, +19, +37 across y)

The ratio between the changes in variables is $-\frac{2}{3}$. The ratios are the same.

The ratios between the changes in variables are $\frac{1}{7}, \frac{1}{19}$, and $\frac{1}{31}$. The ratios are *not* the same.

The second table represents a nonlinear function.

✓ Quick Check

2. Which of the two given functions is nonlinear?

a. $y = 17 - 4^x$
$y = 4 + 2(x + 7)$

b.

x	3	6	9	12
y	3	5	7	10

x	6	7	8	9
y	−2	1	4	7

Sometimes you can determine if a function is nonlinear by its description.

> **EXAMPLE** **Nonlinear Functions from Descriptions**

3 Decide if the function described is linear or nonlinear.

A single cell divides to form two cells after 1 minute. All cells formed after this also divide in two every minute. A function relates time in minutes to the total number of cells.

As time increases by 1 minute, the total number of cells double which is not a constant rate of change. The function is nonlinear.

✓ Quick Check

3. John mows lawns. He charges his customers $15 per hour. A function relates the number of hours John works and the amount he earns. Is the function nonlinear? Explain.

1. **Vocabulary** Explain how a nonlinear function differs from a linear function.

2. Sketch a linear function.

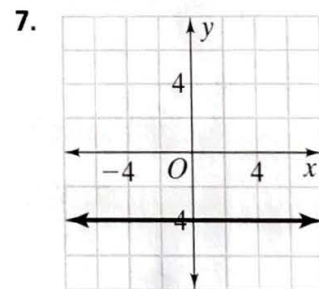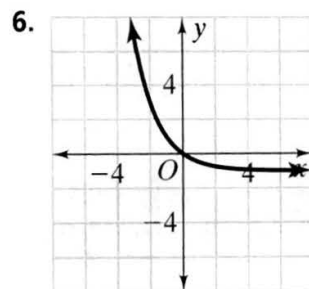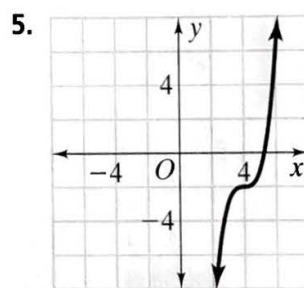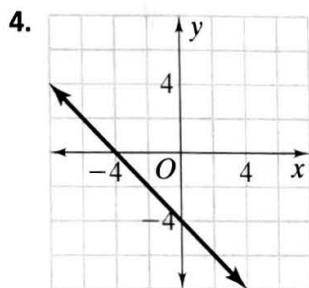3. Make a table of values that represent a nonlinear function.

Homework Exercises

GO for Help

For Exercises	See Examples
4–7	1
8–15	2

Identify the function as linear or nonlinear.

4.

5.

6.

7.

Which of the two given functions is nonlinear?

8. $y = x^2 + 2x + 12$
   $y = x$

9. $y = 8x - 9$
   $y = 5x^4 - 7x^3 + 13$

10. $y = 7.75 + 3(x - 2)$
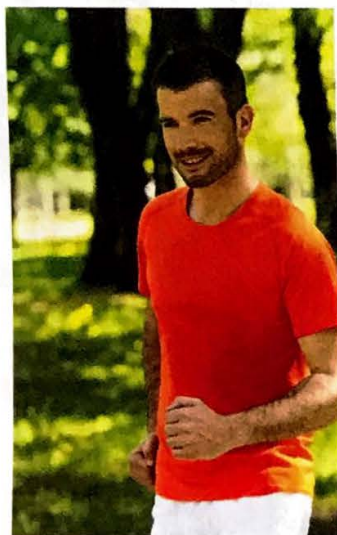 $y = 15x^5 - 4$

11. $y = 2^x$
   $y = 2x + 5 + 5x$

12. $y = \frac{1}{2}x - 2 - 2x$
   $y = \frac{3}{4}x^4 - 1$

13. $y = -0.5 + x - x^2$
   $y = 9.2(x - 2)$

Determine if the function described is linear or nonlinear. Explain.

14. Diego is jogging at a rate of 5 mi/h. A function relates how far Diego jogs to his rate of speed.

15. Ricardo is at the playground and is swinging on a swing set. Jasmine records Ricardo's height above the ground as it relates to time.

16. Guided Problem Solving Copy and complete the table. Identify the function relating area and perimeter as linear or nonlinear.

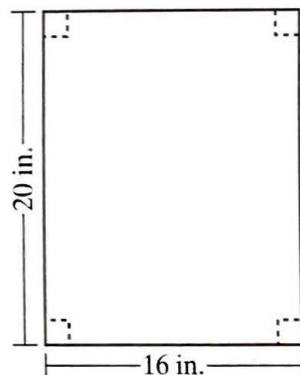| Side Length of Square (cm)	2	3	4	5	6
Perimeter (cm)					
Area (cm²)					

- Find the ratio of the change in perimeter to the change in area.

17. Geometry Copy and complete the table. Identify the function relating area and perimeter as linear or nonlinear.

Length of Rectangle (cm)	1	2	3	4	5
Width of Rectangle (cm)	2	3	4	5	6
Perimeter (cm)					
Area (cm²)					

18. Writing in Math Describe a function relating y to x so that for $x = 0$ to $x = 5$, its graph is linear and for $x = 5$ to $x = 10$, its graph is nonlinear. Describe what both x and y represent.

19. Challenge A box is to be formed by cutting squares from the corners of the rectangle and folding up the sides. Find the volume of the box for a variety of different sized squares. About what size square will give the box the greatest volume? Is the relationship between the square's side length and the volume of the box linear or nonlinear?
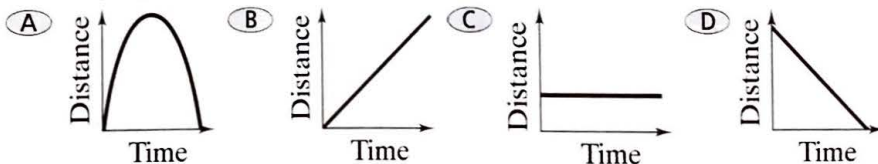
20 in.

16 in.

Test Prep and Mixed Review

Practice

Multiple Choice

20. Which graph is nonlinear?

A [Distance vs Time graph]   B [Distance vs Time graph]   C [Distance vs Time graph]   D [Distance vs Time graph]

21. On a coordinate grid, Lola's house is located at $(1, 2)$. The library is located at $(1, 10)$ and Lola's school is located at $(7, 2)$. Each unit on the grid represents 1 mi. How far is it from the library to Lola's school?

    F 100 mi   G 10 mi   H 8 mi   J 6 mi

Use the function $y = 2x + 5$. Find each output for the given input.

22. $x = 2$ **23.** $x = -2$ **24.** $x = 13$ **25.** $x = 27$