

5-2

Solving Systems by Substitution

Check Skills You'll Need

1. Vocabulary Review

How can you use the *Distributive Property* to solve for x in this equation?

$$2x + 3(x + 2) = 16$$

2. Substitute $2x - 1$ for y in each equation. Solve for x .

$$y + 2x = 3$$

$$x - 2y = 8$$

$$2x + 3y = -15$$



What You'll Learn

To solve a system of linear equations by substitution

New Vocabulary substitution method

Why Learn This?

You can use a system of equations to compare the costs of two phone plans. To solve a system, you can solve one of the equations for one of the variables. This is called the **substitution method**.

EXAMPLE Using Substitution

- 1 Solve the system by substitution. $y = 4x - 1$
 $2x + 2y = 3$

Step 1 Because $y = 4x - 1$, substitute $4x - 1$ for y in $2x + 2y = 3$.

$$2x + 2y = 3$$

← Write the second equation.

$$2x + 2(4x - 1) = 3$$

← Substitute $4x - 1$ for y .

$$2x + 8x - 2 = 3$$

← Use the Distributive Property.

$$10x - 2 = 3$$

← Simplify.

$$10x = 5$$

← Add 2 to each side.

$$x = \frac{1}{2}$$

← Divide each side by 10.

Step 2 Substitute $\frac{1}{2}$ for x in either equation and solve for y .

$$y = 4x - 1$$

← Write either equation.

$$y = 4\left(\frac{1}{2}\right) - 1$$

← Substitute $\frac{1}{2}$ for x .

$$y = 1$$

← Simplify.

The solution of the system is $\left(\frac{1}{2}, 1\right)$.

Check Replace x with $\frac{1}{2}$ and y with 1 in each equation.

$$y = 4x - 1$$

$$2x + 2y = 3$$

$$1 \stackrel{?}{=} 4\left(\frac{1}{2}\right) - 1$$

$$2\left(\frac{1}{2}\right) + 2(1) \stackrel{?}{=} 3$$

$$1 = 1 \quad \checkmark$$

$$3 = 3 \quad \checkmark$$

Quick Check

1. Solve the system by substitution.

$$y = x + 1$$

Check your answer.

$$2x + y = -2$$



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EXAMPLE

Solving for a Variable and Using Substitution

2 Solve the system by substitution.

$$\begin{aligned} -9x + 5y &= -1 \\ -3x + y &= 4 \end{aligned}$$

Step 1 Solve one of the equations for one of the variables.

$$\begin{aligned} -3x + y &= 4 \\ y &= 3x + 4 \end{aligned}$$

← Write the second equation.
← Add 3x to each side.

Step 2 Substitute $3x + 4$ for y in the other equation and solve for x .

$$\begin{aligned} -9x + 5y &= -1 \\ -9x + 5(3x + 4) &= -1 \\ -9x + 15x + 20 &= -1 \\ 6x &= -21 \\ x &= -3.5 \end{aligned}$$

← Write the first equation.
← Substitute $3x + 4$ for y .
← Use the Distributive Property.
← Simplify. Subtract 20 from each side.
← Divide each side by 6.

Step 3 Substitute -3.5 for x in either equation and solve for y .

$$\begin{aligned} -3x + y &= 4 \\ -3(-3.5) + y &= 4 \\ 10.5 + y &= 4 \\ y &= -6.5 \end{aligned}$$

← Write either equation.
← Substitute -3.5 for x .
← Simplify.
← Subtract 10.5 from each side.

The solution of the system is $(-3.5, -6.5)$.

 **Quick Check**

2. Solve the system by substitution.
Check your answer.

$$\begin{aligned} -2x + y &= 3 \\ 3x - 2y &= 0 \end{aligned}$$

EXAMPLE

Application: Phone Plans

3 Nia's phone plan costs \$42 per month plus \$.20 per text. Rick's phone plan costs \$45 per month plus \$.10 per text. For what number of texts will their plans cost the same per month? What will the monthly cost be?

Step 1 Write a system of equations. Let x = the number of texts and y = the cost per month.

$$\begin{aligned} y &= 42 + 0.2x && \leftarrow \text{Cost of Nia's plan} \\ y &= 45 + 0.1x && \leftarrow \text{Cost of Rick's plan} \end{aligned}$$

Step 2 $42 + 0.2x = 45 + 0.1x$ ← Substitute $42 + 0.2x$ for y in the second equation.

$$\begin{aligned} 0.1x &= 3 \\ x &= 30 \end{aligned}$$

← Subtraction Property of Equality.
← Divide each side by 0.1.

Step 3 $y = 42 + 0.2(30)$ ← Substitute 30 for x in either equation.
 $y = 48$ ← Simplify.

For 30 texts per month, both plans will cost \$48.

Test Prep Tip 

Check your solution by substituting 30 for x into the other equation to see if the monthly charge is the same.

$$\begin{aligned} y &= 45 + 0.1x \\ y &= 45 + 0.1(30) \\ y &= 45 + 3 \\ y &= 48 \checkmark \end{aligned}$$

✓ Quick Check

3. Juan's phone plan costs \$40 per month plus \$.40 per text. For what number of texts will Juan's and Nia's plans cost the same per month? What will the monthly cost be?

● More Than One Way

Solve the linear system.

$$-x + y = 5$$

$$\frac{1}{2}x + y = 8$$

Jasmine's Method

I can solve the system by substitution. First, I can solve the equation $-x + y = 5$ for y to get $y = x + 5$.

$$\frac{1}{2}x + y = 8$$

← Write the second equation.

$$\frac{1}{2}x + (x + 5) = 8$$

← Substitute $x + 5$ for y .

$$\frac{3}{2}x + 5 = 8$$

← Simplify.

$$\frac{3}{2}x = 3$$

← Subtract 5 from each side.

$$x = 2$$

← Divide each side by $\frac{3}{2}$.

Now I can substitute 2 for x in either equation and solve for y .

$$-x + y = 5$$

← Write the first equation.

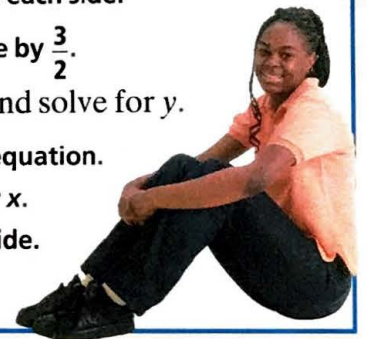
$$-(2) + y = 5$$

← Substitute 2 for x .

$$y = 7$$

← Add 2 to each side.

The solution of the system is $(2, 7)$.

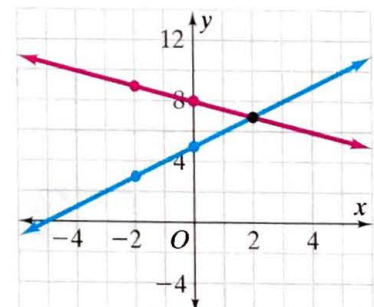


Kevin's Method

I can solve the system by graphing. I can make a table for each equation in the system. Then I can graph the lines.

$-x + y = 5$	
x	y
-5	0
0	5
5	10

$\frac{1}{2}x + y = 8$	
x	y
-2	9
0	8
2	7



The solution of the system is $(2, 7)$.

Solve the linear system.

Explain why you chose the method you used.

$$-x + y = -2$$

$$\frac{1}{3}x + y = 2$$



Check Your Understanding

1. Which of the following systems of equations would be most efficiently solved by using the substitution method? Explain.

System A: $6x + 3y = -3$	System B: $4x + 3y = 11$
$4x + 2y = 2$	$2x - y = 3$

2. Suppose you want to solve the system at the right using substitution. What should your first step be?

$$\begin{aligned} x - 4y &= 2 \\ 3x + 5y &= 40 \end{aligned}$$

3. Explain why you might choose to solve a system of equations using substitution rather than by graphing.

Homework Exercises

For more exercises, see **Extra Skills and Word Problems**.

Solve each system of equations using substitution. Check your answer.

GO for Help	
For Exercises	See Examples
4–6	1
7–9	2
10, 11	3

- | | | |
|----------------------------------|------------------------------------|---|
| 4. $y = -x + 8$
$-x + 3y = 0$ | 5. $2x - 2y = 2$
$y = -5x + 14$ | 6. $2y - x = 3$
$y = 2x - 6$ |
| 7. $-x + y = 4$
$-2x - y = 2$ | 8. $x - 2y = 2$
$3x + 4y = 3$ | 9. $2y - \frac{8}{3}x = 6$
$y + \frac{2}{3}x = -3$ |

10. The cost of membership in a discount grocery club includes a monthly charge and a one-time joining fee. The total cost of membership for 6 months is \$80, and the total cost of membership for 1 year is \$110. Write and solve a system of equations to find the monthly charge and the joining fee.
11. Trey's online music club charges a monthly fee of \$20 plus \$.80 per song downloaded. Deb's online music club charges a monthly fee of \$21 plus \$.60 per song downloaded. For what number of songs downloaded will the monthly charge be the same for both clubs?




12. **Guided Problem Solving** Natasha is going to the local spring festival. The table shows the ticket options. Which ticket option should she choose? Justify your answer.

Ticket Option	Admission Price	Price per Ride
A	\$5	\$.30
B	\$3	\$.80

- **Understand the Problem** Write a system of equations that models the situation.
- **Make a Plan** How can you find the answer using your system?

13. A group of friends wants to play paintball. The pricing options are shown in the table. Which option should the friends choose? Justify your answer.

Pricing Option	Equipment Rental	Price per Game
1	\$25	\$7
2	\$40	\$4

14. **Writing in Math**  What would your first step be in solving the system below? Explain your reasoning?

$$y = 2x - 4$$

$$4x - 2y = 0$$

15. **Reasoning** A community theater group put on a play last month. Twenty more adults attended than children. Adult tickets cost \$8 each and children's tickets cost \$3 each. The theater group collected \$1,260. How many adult tickets were sold?

16. A grocery store makes a 20-pound mixture of almonds and cashew nuts. The store charges \$4 per pound for almonds and \$5.50 per pound for cashews. The total value of the mixture is \$92. How many pounds of each type of nut are in the mixture?



17. **Challenge** Use substitution to solve the system of equations. Write the solution in the form (x, y, z) .

$$x + y + z = -54$$

$$x = -6y$$

$$z = 14y$$

Test Prep and Mixed Review

Practice

Multiple Choice

18. Which of the following is the solution of the system of equations?

$$y = 3x - 8$$

$$y = 4 - x$$

- (A) $(3, 1)$ (B) $(1, 3)$ (C) $(-3, 1)$ (D) $(3, -1)$

19. What is the decimal expansion of $\frac{1}{6}$?

- (F) 1.6 (G) 0.6 (H) $0.1\bar{6}$ (J) $0.01\bar{6}$

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For Exercises	See Lesson
20–22	4-1

Find the slope of the line that passes through each pair of points.

20. $(3, 7)$ and $(2, 2)$ 21. $(3, 0)$ and $(1, -6)$ 22. $(-2, -1)$ and $(-5, -2)$