

## 6-4

## Exponents and Division

 **Check Skills You'll Need**

- 1. Vocabulary Review**  
Is the expression  $x^5$  an exponent or a power?

Write each expression using exponents.

2.  $7 \cdot 7 \cdot 7 \cdot 7$

3.  $4 \cdot 4 \cdot 4$

4.  $5 \cdot 5$

5.  $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

**What You'll Learn**

To divide powers with the same base and to simplify expressions with negative exponents

**Why Learn This?**

Nanorobots are microscopic machines that may soon be used to fight illness inside the human body. When working with very small numbers, such as the length of a nanorobot, you often divide expressions with exponents.



You can divide powers with the same base by writing out all the factors.

$$\frac{7^5}{7^3} = \frac{\underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{7}} \cdot 7 \cdot 7}{\underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{7}}} = \frac{7 \cdot 7}{1} = 7^2$$

Notice that  $5 - 3 = 2$ . This example suggests the following rule.

 **CONTENT STANDARD**

8.EE.1

**KEY CONCEPTS** **Dividing Powers With the Same Base**

To divide nonzero numbers or variables with the same nonzero base, subtract the exponents.

**Arithmetic**

$$\frac{8^5}{8^3} = 8^{(5-3)} = 8^2$$

**Algebra**

$$\frac{a^m}{a^n} = a^{(m-n)}, \text{ where } a \neq 0$$

**EXAMPLE** **Dividing Powers**

- 1** Write  $\frac{m^{12}}{m^5}$  using a single exponent.

$$\begin{aligned} \frac{m^{12}}{m^5} &= m^{(12-5)} && \leftarrow \text{Subtract exponents with the same base.} \\ &= m^7 && \leftarrow \text{Simplify.} \end{aligned}$$

 **Quick Check**

- 1.** Write  $\frac{w^8}{w^5}$  using a single exponent.

What does the exponent 0 mean? Consider finding the quotient  $\frac{3^5}{3^5}$ .

If you subtract exponents,  $\frac{3^5}{3^5} = 3^{(5-5)} = 3^0$ .

If you write factors,  $\frac{3^5}{3^5} = \frac{\cancel{3}^1 \cdot \cancel{3}^1 \cdot \cancel{3}^1 \cdot \cancel{3}^1 \cdot \cancel{3}^1}{\cancel{3}_1 \cdot \cancel{3}_1 \cdot \cancel{3}_1 \cdot \cancel{3}_1 \cdot \cancel{3}_1}$   
 $= \frac{1}{1} = 1$ .

Notice that  $\frac{3^5}{3^5} = 3^0$  and  $\frac{3^5}{3^5} = 1$ . This suggests the following rule.

### KEY CONCEPTS Zero as an Exponent

For any nonzero number  $a$ ,  $a^0 = 1$ .

**Example**  $9^0 = 1$

### EXAMPLE Expressions With a Zero Exponent

2 Simplify each expression.

a.  $(-8)^0$

b.  $3m^0$

$(-8)^0 = 1$

← Simplify. →

$3m^0 = 3 \cdot 1 = 3$

### Quick Check

2. Simplify each expression.

a.  $(-9)^0$

b.  $(2r)^0$

c.  $2r^0$

To understand negative exponents, consider finding the quotient  $\frac{6^2}{6^5}$ .

If you subtract exponents,  $\frac{6^2}{6^5} = 6^{(2-5)} = 6^{-3}$ .

If you write factors,  $\frac{6^2}{6^5} = \frac{\cancel{6}^1 \cdot \cancel{6}^1}{\cancel{6}_1 \cdot \cancel{6}_1 \cdot 6 \cdot 6 \cdot 6}$   
 $= \frac{1}{6 \cdot 6 \cdot 6} = \frac{1}{6^3}$ .

Notice that  $\frac{6^2}{6^5} = 6^{-3}$  and  $\frac{6^2}{6^5} = \frac{1}{6^3}$ . This suggests the following rule.

### KEY CONCEPTS Negative Exponents

For any nonzero number  $a$  and integer  $n$ ,  $a^{-n} = \frac{1}{a^n}$ .

**Example**  $8^{-5} = \frac{1}{8^5}$

### Vocabulary Tip

Read  $3^0$  as "3 to the zero power."

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To simplify an expression with negative exponents, you can first write the expression with a positive exponent.

### EXAMPLE Expressions With Negative Exponents

3 Simplify each expression.

a.  $3^{-2}$

$$3^{-2} = \frac{1}{3^2} \leftarrow \text{Use a positive exponent.} \rightarrow (y)^{-6} = \frac{1}{y^6}$$

$$= \frac{1}{9} \leftarrow \text{Simplify.}$$

b.  $(y)^{-6}$

### ✓ Quick Check

3. Simplify each expression.

a.  $3^{-1}$

b.  $w^{-4}$

c.  $(-2)^{-3}$

### ● More Than One Way

Simplify the expression  $4^3 \cdot 4^{-5}$ .

#### Tina's Method

I can rewrite the expression with positive exponents.

$$4^3 \cdot 4^{-5} = 4^3 \cdot \frac{1}{4^5} \leftarrow \text{Use a positive exponent.}$$

$$= \frac{4^3}{4^5} \leftarrow \text{Multiply fractions.}$$

$$= 4^{(3-5)} \leftarrow \text{Subtract exponents with the same base.}$$

$$= 4^{-2} \leftarrow \text{Simplify.}$$

$$= \frac{1}{4^2} \leftarrow \text{Use a positive exponent.}$$

$$= \frac{1}{16} \leftarrow \text{Simplify.}$$

So the expression is equal to  $\frac{1}{16}$ .



#### Eric's Method

To multiply numbers with the same base, I can add the exponents.

$$4^3 \cdot 4^{-5} = 4^{(3+(-5))} \leftarrow \text{Add the exponents.}$$

$$= 4^{-2} \leftarrow \text{Simplify.}$$

$$= \frac{1}{4^2} \leftarrow \text{Use a positive exponent.}$$

$$= \frac{1}{16} \leftarrow \text{Simplify.}$$

So the expression is equal to  $\frac{1}{16}$ .



#### Choose a Method

Simplify the expression  $5^2 \cdot 5^{-5}$ .

# Check Your Understanding

- Reasoning** Is  $(-1)^0$  a positive or a negative number? Explain.
- Mental Math** Find the value of  $\frac{123^5}{123^4}$ .

Write out the factors of each expression. Then simplify using a single exponent. Exercise 3 has been started for you.

$$3. \frac{2^6}{2^5} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \qquad 4. \frac{3^4}{3^2} \qquad 5. \frac{8^5}{8^2}$$

# Homework Exercises

For more exercises, see **Extra Skills and Word Problems**.

## GO for Help

For Exercises	See Examples
6-13	1
14-22	2-3

Write each expression using a single exponent.

$$6. \frac{a^5}{a^3} \qquad 7. \frac{x^9}{x^5} \qquad 8. \frac{c^7}{c^2} \qquad 9. \frac{(-1)^5}{(-1)^4}$$

$$10. \frac{23^{12}}{23^8} \qquad 11. \frac{135^{10}}{135^1} \qquad 12. \frac{(-7)^{99}}{(-7)^{98}} \qquad 13. \frac{(-9)^{32}}{(-9)^{15}}$$

- Zoology** The slowest mammal on Earth is the sloth. A sloth might move along a tree at a maximum rate of just  $3^{-1}$  m each minute. At this rate, how long will it take a sloth to climb a tree that is 33 m tall?

Simplify each expression.

$$15. 4^0 \qquad 16. (-3)^0 \qquad 17. u^0 \qquad 18. (3t)^0$$

$$19. 10^{-2} \qquad 20. b^{-6} \qquad 21. x^{-4} \qquad 22. 7^{-1}$$

## GPS

- Guided Problem Solving** Snow is falling at an average rate of  $2^{-2}$  cm each hour. At this rate, how long will it take  $2^2$  cm of snow to fall?

- Make a Plan** Use the formula  $\text{time} = \frac{\text{distance}}{\text{rate}}$ .
- Carry Out the Plan** Divide powers with the same base. Simplify.

- Earth Science** Earth's crust is divided into large pieces called tectonic plates. The Pacific tectonic plate is moving northwest at a rate of about  $4^{-2}$  m each year. At this rate, how long will it take the plate to move  $4^6$  m (about 2.5 miles)?

Complete each equation.

$$25. \frac{4^{\square}}{4^2} = 4^{10} \qquad 26. \frac{x^6}{x^{\square}} = x^4 \qquad 27. \frac{14x^5}{7x^3} = 2x^{\square} \qquad 28. \frac{1}{c^7} = c^{\square}$$

Use  $w = -1$  and  $x = 2$ . Simplify each expression.

29.  $(w + x)^{-4}$

30.  $x^w$

31.  $-2^w + 2x$

32.  $(2x)^w + 1$

33. **Error Analysis** A student wrote that  $-3^0 = 1$ . What error did the student make?

 **Writing in Math** Is each statement *true* or *false*? Explain your reasoning.

34.  $4^0 = 4^{-1}$

35.  $8^{-1} = (-8)^1$

36.  $2^1 \cdot 2^{-1} = 2^0$

37.  $(-2)^{-1} = 2$

38. **Astronomy** The Moon is moving away from Earth at a rate of more than  $3^{-5}$  m each year. At this rate, how long will it take the Moon to move 34 m away from Earth?

39. **Challenge** You can divide a polynomial by a monomial by dividing each term of the numerator by the denominator.

**Sample** 
$$\frac{6x^4 + 10x^3}{2x^2} = \frac{6x^4}{2x^2} + \frac{10x^3}{2x^2}$$

$$= 3x^2 + 5x$$

a.  $\frac{6n^5 - 12n^2}{3n^2}$

b.  $\frac{4m^9 + 6m^6 + 2m^3}{2m^3}$



## Test Prep and Mixed Review

**Practice**

### Gridded Response

40. A garden snail is moving at a rate of  $4^{-3}$  m/s. At this rate, how many seconds will it take the snail to move across a garden that is  $4^2$  m wide?

41. A quadratic function is represented by the ordered pairs in the table below. What is the value of  $y$  when  $x = 12$ ?

$x$	1	3	5	7
$y$	-2	7	23	47

42. Two groups of students order burritos and tacos. One order of 3 burritos and 4 tacos costs \$11.33, while the other order of 9 burritos and 5 tacos costs \$23.56. Write and solve a linear system to find the cost of one burrito.

### GO for Help

For Exercises

See Lesson

43-46

1-2

**Find the square roots of each number.**

43. 121

44. 625

45.  $\frac{1}{144}$

46.  $\frac{1}{81}$