

7-5

Proving Triangles Similar

 Check Skills You'll Need

- Vocabulary Review**
If two triangles are *similar*, then their corresponding angles are ? and their corresponding side lengths are ?.
- Is a square with a side length of 6 in. similar to a square with a side length of 10 in.? Explain.

GO for Help
Lesson 7-4

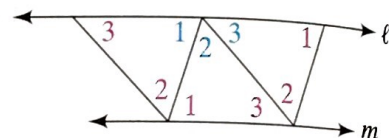
What You'll Learn

To determine measures of the angles of triangles and use them to help prove that triangles are similar

Why Learn This?

Architects and engineers often make use of triangles when designing buildings and other structures.

Three copies of the same triangle have been arranged as shown. Lines ℓ and m are parallel because their alternate interior angles are congruent.



The angles labeled in blue show that $\angle 1$, $\angle 2$, and $\angle 3$ form a straight angle along line ℓ , which means that the sum of their measures is 180° . Therefore, the sum of the measures of the angles of each of the triangles is 180° .

© CONTENT STANDARD
8.G.5

KEY CONCEPTS Angle Sum of a Triangle

The sum of the measures of the angles of any triangle is 180° .

EXAMPLE Finding an Angle Measure

- Multiple Choice** $\triangle RST$ forms part of the front of a cabin as shown at the left. What is the measure of $\angle S$?

(A) 55° (B) 70° (C) 125° (D) 110°

$$m\angle R + m\angle S + m\angle T = 180^\circ$$

← Angle sum of a triangle

$$55^\circ + m\angle S + 55^\circ = 180^\circ$$

← Substitute.

$$m\angle S + 110^\circ = 180^\circ$$

← Simplify.

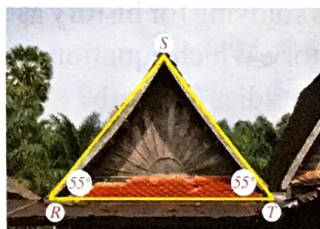
$$m\angle S + 110^\circ - 110^\circ = 180^\circ - 110^\circ$$

← Subtract 110° from each side.

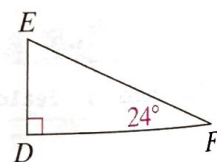
$$m\angle S = 70^\circ$$

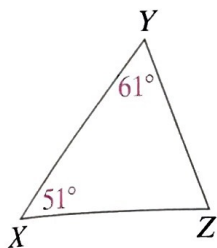
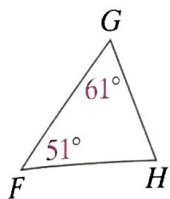
← Simplify.

$\angle S$ measures 70° . The correct answer is choice B.


 Quick Check

1. What is the measure of $\angle E$ in $\triangle DEF$?





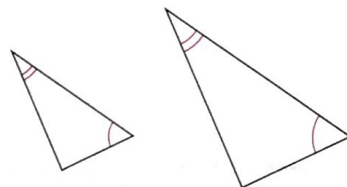
If all pairs of corresponding angles of two triangles are congruent, then the triangles have the same shape and the triangles are similar. In the diagram at the left, $\angle F \cong \angle X$ and $\angle G \cong \angle Y$.

What about the third pair of corresponding angles? By using the angle sum of a triangle, you can determine that $m\angle H = 68^\circ$ and $m\angle Z = 68^\circ$. Because all three pairs of corresponding angles are congruent, $\triangle FGH \sim \triangle XYZ$.

This example shows that if two pairs of corresponding angles of two triangles are congruent, then the third pair of corresponding angles must also be congruent and the triangles are similar.

KEY CONCEPTS Angle-Angle (AA) Similarity

If two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.



EXAMPLE Similar Triangles

- 2 Show that the pair of triangles is similar.

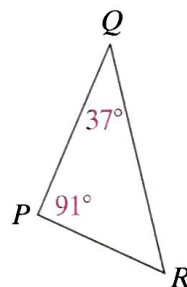
Step 1 Use the angle sum of a triangle to find $m\angle R$.

$$91^\circ + 37^\circ + m\angle R = 180^\circ$$

$$128^\circ + m\angle R = 180^\circ$$

$$128^\circ - 128^\circ + m\angle R = 180^\circ - 128^\circ$$

$$m\angle R = 52^\circ$$

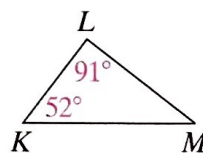


Step 2 Use AA similarity.

$$\angle P \cong \angle L \quad \leftarrow \text{Each measures } 91^\circ.$$

$$\angle R \cong \angle K \quad \leftarrow \text{Each measures } 52^\circ.$$

$\triangle PQR \sim \triangle LMK$ by AA similarity.



GO for Help

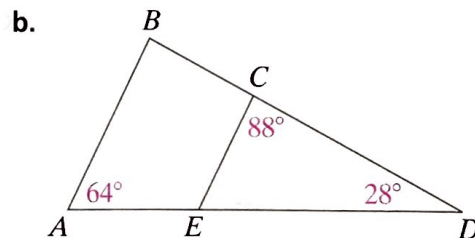
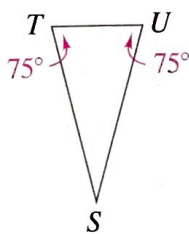
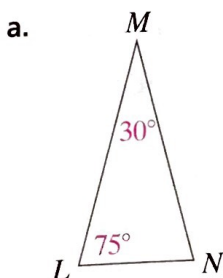
For help solving equations, go to Lesson 2-1, Example 1.

Test Prep Tip

There is often more than one way to show that two triangles are similar. For example, you could have started by finding $m\angle M$ instead of $m\angle R$.

Quick Check

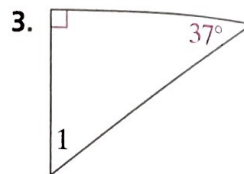
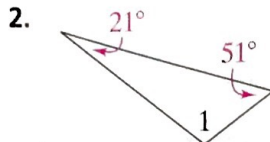
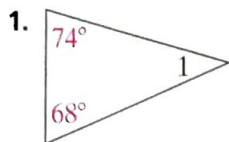
2. Show that each pair of triangles is similar.





Check Your Understanding

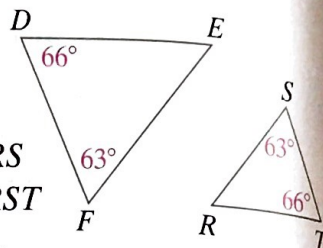
For each triangle, find $m\angle 1$.



Use the triangles below for Exercises 4 and 5.

4. List the congruent corresponding angles of the two triangles.

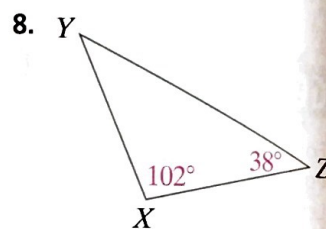
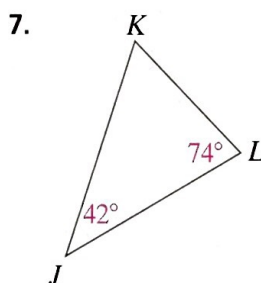
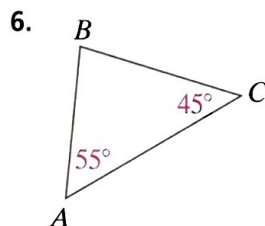
5. **Error Analysis** Daisy writes $\triangle DEF \sim \triangle TRS$ by AA similarity. Erika writes $\triangle DEF \sim \triangle RST$ by AA similarity. Who is incorrect? Explain.



Homework Exercises

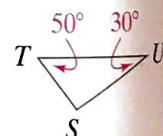
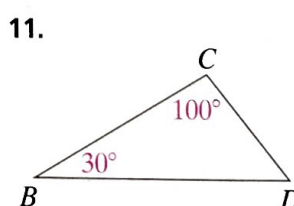
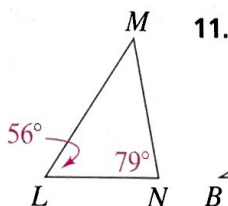
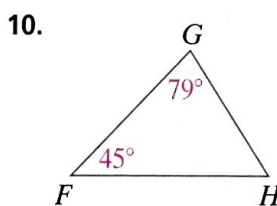
For more exercises, see Extra Skills and Word Problems.

Determine the unknown angle measure in each triangle.



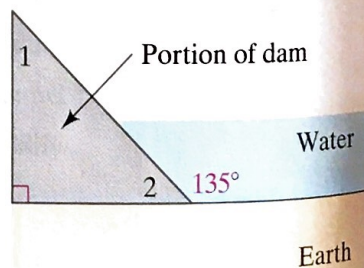
9. A triangular flag has two angles that each measure 74° . What is the measure of the third angle of the flag?

Show that each pair of triangles is similar.

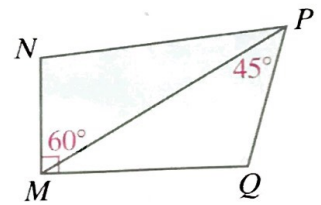


12. **Guided Problem Solving** The diagram shows a cross-section of a portion of a dam. What is the measure of $\angle 1$?

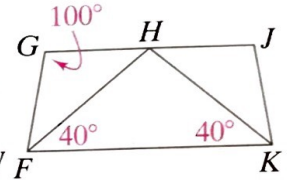
- What is the relationship between $\angle 2$ and the 135° angle?
- How can you find the measure of $\angle 2$?
- Once you know the measure of $\angle 2$, how can you find the measure of $\angle 1$?



13. City workers are laying out the paths in a new park, as shown in the diagram. Do the workers have enough information to determine $m\angle Q$? If so, explain how to find its measure. If not, explain why not.

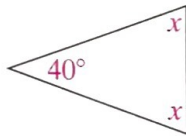


14. **Engineering** The diagram shows a portion of a bridge support. If the support was built correctly, $\triangle FGH$ should be similar to $\triangle FHK$. Given that $\overline{GJ} \parallel \overline{FK}$, was the support built correctly? Justify your answer.

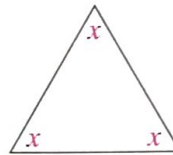


Algebra For each triangle, determine the value of x .

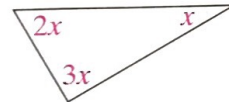
15.



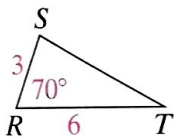
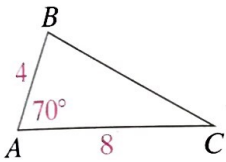
16.



17.



18. **Writing in Math** Is the following statement *true* or *false*? A triangle can have two angles that measure 100° . Explain.
19. **Challenge** Another way to show that two triangles are similar is to show that one pair of corresponding angles is congruent and that the corresponding sides that make up those angles are in proportion. This method is called *side-angle-side (SAS) similarity*. Use this method to show that the triangles at the left are similar.



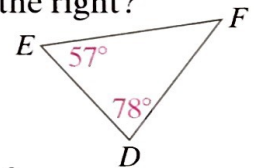
Test Prep and Mixed Review

Practice

Multiple Choice

20. What is the measure of $\angle F$ in the triangle shown at the right?

- (A) 33° (B) 45° (C) 57° (D) 135°



21. What is the solution of the system $\begin{cases} 2x + y = 5 \\ -2x + 4y = 10 \end{cases}$?
- (F) $(-5, 0)$ (G) $(1, 3)$ (H) $(2, 1)$ (J) $(4, -3)$

22. What is the function rule for the linear function shown in the table?

- (A) $y = -2x - 3$ (B) $y = x - 1$ (C) $y = 2x + 1$ (D) $y = 3x + 3$

x	-2	-1	0	1
y	-3	-1	1	3

Algebra Solve each equation.

23. $-2x + 6 = 28$ 24. $1.4n - 0.3 = 8.1$
 25. $\frac{t}{8} - 7 = 3$ 26. $\frac{c}{4} + 8 = 20$

GO for Help

Exercises	See Lesson
23-26	2-1