

# 7-6

# Angles and Polygons

## Check Skills You'll Need

1. **Vocabulary Review**  
What is the sum of the measures of a pair of *supplementary angles*?

Find the measure of the supplement of each angle.

2.  $54^\circ$     3.  $90^\circ$   
4.  $108^\circ$    5.  $18^\circ$   
6.  $76^\circ$     7.  $150^\circ$

**GO for Help**  
Lesson 7-1

## What You'll Learn

To find the angle measures of a polygon

**New Vocabulary** interior angle, exterior angle

## Why Learn This?

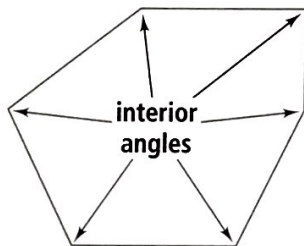
Polygons often appear in art and architecture. In designing tile patterns, it helps to know about the angles of polygons.

Here is a list of common polygons.

Polygon Name	Number of Sides	Polygon Name	Number of Sides
Triangle	3	Octagon	8
Quadrilateral	4	Nonagon	9
Pentagon	5	Decagon	10
Hexagon	6	Dodecagon	12
Heptagon	7		

## CONTENT STANDARD

8.G.5



Two consecutive sides of a polygon form one **interior angle**. The sum of the measures of the interior angles depends on the number of sides.

## KEY CONCEPTS Polygon Angle Sum

For a polygon with  $n$  sides, the sum of the measures of the interior angles is  $(n - 2)180^\circ$ .

## EXAMPLE Sum of the Interior Angle Measures

- 1 What is the sum of the measures of the interior angles of a nonagon?

$$\begin{aligned} (n - 2)180^\circ &= (9 - 2)180^\circ && \leftarrow \text{A nonagon has nine sides. Substitute 9 for } n. \\ &= 1,260^\circ && \leftarrow \text{Simplify.} \end{aligned}$$

The sum of the interior angle measures of a nonagon is  $1,260^\circ$ .

## Quick Check

1. What is the sum of the measures of the interior angles of a heptagon?



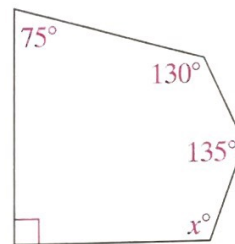
PearsonSuccessNet.com

For: Polygon Angle Sums Activity  
Use: Interactive Textbook, 7-6

You can use the same formula to find angle measures in a polygon.

## EXAMPLE Angle Measures of a Polygon

- 2 **Algebra** Find the unknown angle measure in the pentagon at the right.



**Step 1** Find the sum of the angle measures.

$$\begin{aligned}(n - 2)180^\circ &= (5 - 2)180^\circ && \leftarrow \text{Substitute 5 for } n. \\ &= 540^\circ && \leftarrow \text{Simplify.}\end{aligned}$$

**Step 2** Write an equation. Let  $x$  = the unknown angle measure.

$$540^\circ = 90^\circ + 75^\circ + 130^\circ + 135^\circ + x^\circ \quad \leftarrow \text{Write an equation.}$$

$$540^\circ = 430^\circ + x^\circ \quad \leftarrow \text{Simplify.}$$

$$110^\circ = x^\circ \quad \leftarrow \text{Subtract } 430^\circ \text{ from each side.}$$

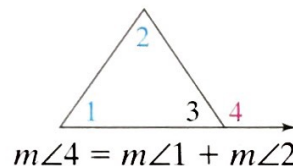
The unknown angle measure is  $110^\circ$ .

### Quick Check

2. A hexagon has five angles with measures of  $142^\circ$ ,  $84^\circ$ ,  $123^\circ$ ,  $130^\circ$ , and  $90^\circ$ . What is the measure of the sixth angle?

An **exterior angle** of a polygon is an angle formed by a side and an extension of an adjacent side.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the interior angles at the other two vertices.



An ancient Roman mosaic made of tiles

## EXAMPLE Finding the Measure of an Exterior Angle

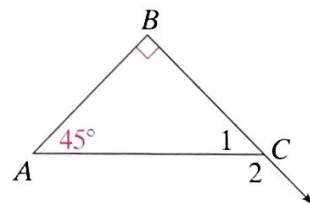
- 3 **Art** The diagram shows a portion of the design of a mosaic.  $\angle 2$  is an exterior angle of  $\triangle ABC$ . What is  $m\angle 2$ ?

$$m\angle 2 = m\angle A + m\angle B \quad \leftarrow \text{Exterior angle of triangle}$$

$$= 45^\circ + 90^\circ \quad \leftarrow \text{Substitute.}$$

$$= 135^\circ \quad \leftarrow \text{Simplify.}$$

$\angle 2$  measures  $135^\circ$ .



**Check** By the angle sum of a triangle,  $m\angle 1 = 180^\circ - 45^\circ - 90^\circ = 45^\circ$ .

$\angle 1$  and  $\angle 2$  are supplementary, so  $m\angle 2 = 180^\circ - 45^\circ = 135^\circ$ . ✓

### Quick Check

3. In  $\triangle RST$ ,  $m\angle R = 63^\circ$  and  $m\angle S = 84^\circ$ . What is the measure of the exterior angle at vertex  $T$ ?

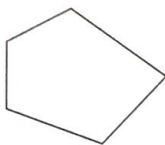


# Check Your Understanding

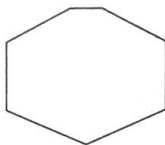
1. **Vocabulary** How does an interior angle of a polygon differ from an exterior angle?

Classify each polygon by the number of its sides.

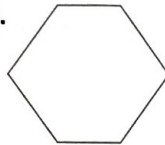
2.



3.



4.



5.



6. **Error Analysis** Jason knows the sum of the angle measures of a hexagon is  $720^\circ$ . To find the sum of the angle measures of a dodecagon, he multiplies  $720^\circ$  by 2 since  $12 = 6 \cdot 2$ . Miranda multiplies  $180^\circ$  by 10. Who is correct? Explain.

## Homework Exercises

For more exercises, see Extra Skills and Word Problems.

### GO for Help

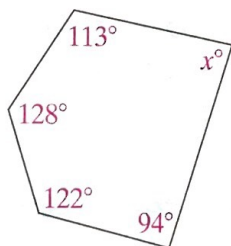
For Exercises	See Examples
7–12	1
13–15	2
16–19	3

Find the sum of the measures of the interior angles of each polygon.

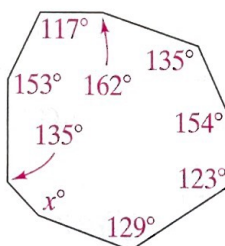
7. pentagon      8. octagon      9. hexagon  
10. decagon      11. triangle      12. dodecagon

**Algebra** Find the missing angle measure in each figure.

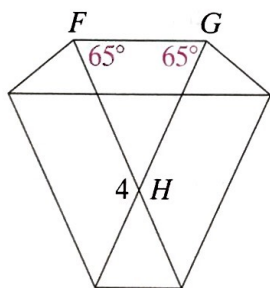
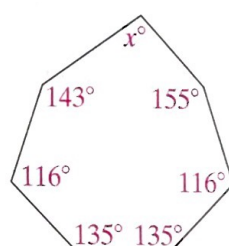
13.



14.



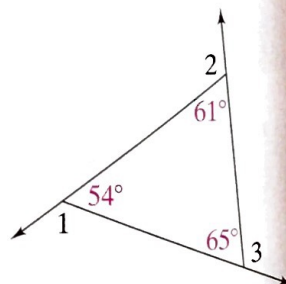
15.



Find the measure of each exterior angle of the triangle below.

16.  $\angle 1$   
17.  $\angle 2$   
18.  $\angle 3$

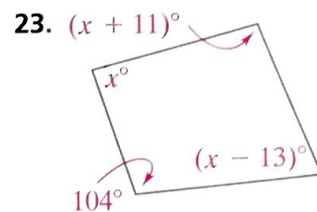
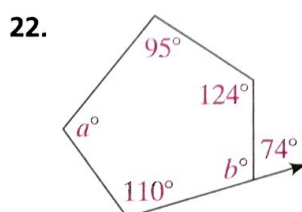
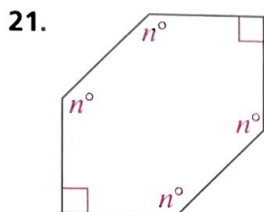
19. The diagram at the left shows the plans for a kite.  $\angle 4$  is an exterior angle of  $\triangle FGH$ . What is  $m\angle 4$ ?



### GPS

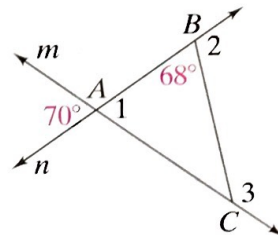
20. **Guided Problem Solving** The measure of each interior angle of a polygon is  $157.5^\circ$ . How many sides  $n$  does the polygon have?
- What two expressions can you write for the sum of the measures of the interior angles of the polygon?
  - What is the solution for  $n$  when you set the two expressions equal to each other?

**Algebra** Find the missing angle measures in each figure.



In the diagram, line  $m$  intersects line  $n$ . Find the measure of each angle, and justify your answers.

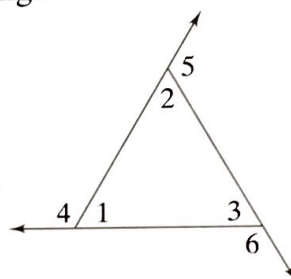
24.  $\angle 1$                       25.  $\angle 2$                       26.  $\angle 3$



27. **Writing in Math** Could the sum of the measures of the interior angles of a polygon be equal to  $810^\circ$ ? Explain why or why not.

28. The measures of six angles of a heptagon are  $145^\circ$ ,  $115^\circ$ ,  $152^\circ$ ,  $87^\circ$ ,  $90^\circ$ , and  $150^\circ$ . Find the measure of the seventh angle.

29. **Baseball** In the home plate at the left,  $\angle 1 \cong \angle 2$ . Find  $m\angle 1$ .



30. **Challenge** Use the triangle at the right and what you have learned about the exterior angles of triangles to show that the sum of the measures of the exterior angles of a triangle is  $360^\circ$ .

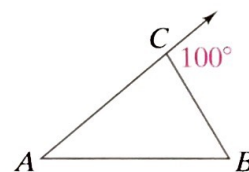


**Test Prep and Mixed Review**

**Practice**

**Multiple Choice**

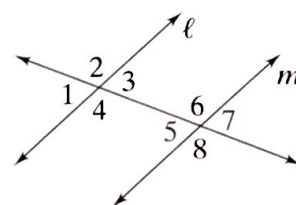
31. The following statements are true about  $\triangle ABC$ .
- $m\angle A$  is less than  $m\angle B$ .
  - $m\angle B$  is less than  $m\angle C$ .
  - An exterior angle at vertex  $C$  measures  $100^\circ$ .



Which are possible measures of  $\angle A$  and  $\angle B$ ?

- (A)  $m\angle A = 10^\circ$ ,  $m\angle B = 90^\circ$       (C)  $m\angle A = 40^\circ$ ,  $m\angle B = 60^\circ$   
 (B)  $m\angle A = 20^\circ$ ,  $m\angle B = 80^\circ$       (D)  $m\angle A = 50^\circ$ ,  $m\angle B = 50^\circ$
32. The area of a square is 275 square feet. Which is closest to the side length of the square?  
 (F) 15.8 ft      (G) 16.1 ft      (H) 16.4 ft      (J) 16.6 ft

In the diagram, line  $\ell \parallel m$ , and  $m\angle 5 = 63^\circ$ . Find the measure of each angle.



33.  $\angle 1$                       34.  $\angle 2$                       35.  $\angle 3$   
 36.  $\angle 4$                       37.  $\angle 6$                       38.  $\angle 7$

**GO for Help**

For Exercises	See Lesson
33–38	7-2