8-4 Transformations and Congruence

Check Skills You'll Need

1. **Vocabulary Review** The ? is the number of degrees a figure rotates about a fixed point.

Give the coordinates of the image of each point after being rotated the given number of degrees about the origin.

2. $A(2, 1)$, 90°
3. $B(0, 3)$, 180°
4. $C(-1, 4)$, 270°

GO for Help Lesson 8-3

What You'll Learn

To describe a sequence of transformations that maps one figure onto another; to determine whether two figures are congruent by using a sequence of transformations

Why Learn This?

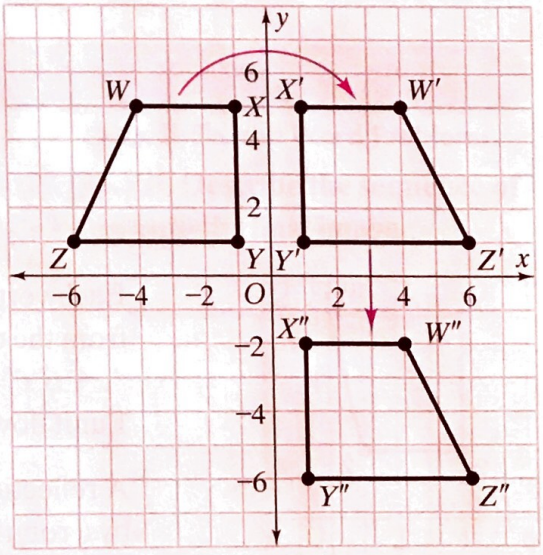
When you walk in the sand, you leave a trail of footprints that are congruent to each other. You can use transformations to map one footprint onto another.

If two figures are congruent, then a transformation, or a sequence of transformations, will map one figure onto the other.

CONTENT STANDARD 8.G.2

EXAMPLE 1 Recognizing a Series of Transformations

The three trapezoids are congruent. Describe the sequence of transformations that maps $WXYZ$ onto $W''X''Y''Z''$.



A reflection over the y-axis maps $WXYZ$ onto $W'X'Y'Z'$.

A translation 7 units down maps $W'X'Y'Z'$ onto $W''X''Y''Z''$.

So, a reflection over the y-axis, followed by a translation 7 units down, maps $WXYZ$ onto $W''X''Y''Z''$.

Quick Check

1. Describe the sequence of transformations that maps $W''X''Y''Z''$ onto $WXYZ$.

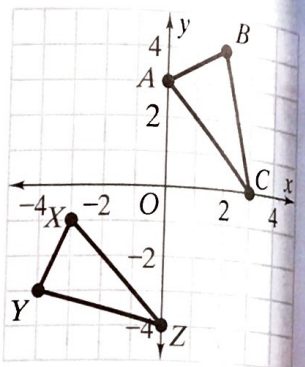
If you can use a sequence of transformations to map one figure onto another, then the two figures are congruent.

EXAMPLE 2 Using Transformations to Determine Congruence

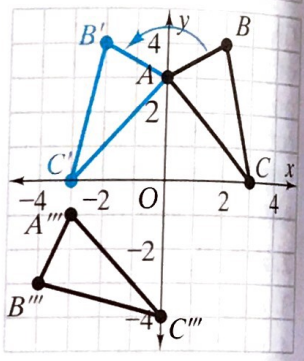
Vocabulary Tip

Two figures have the same *orientation* if you can use a translation, a rotation, or a combination of the two to map one figure onto the other.

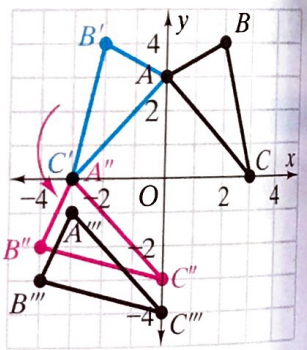
Determine whether the two triangles in the diagram are congruent. If they are, write a congruence statement. If they are not congruent, explain why.



$\triangle ABC$ and $\triangle XYZ$ have opposite orientations and are on opposite sides of the y-axis, so start by reflecting $\triangle ABC$ over the y-axis to get $\triangle A'B'C'$.

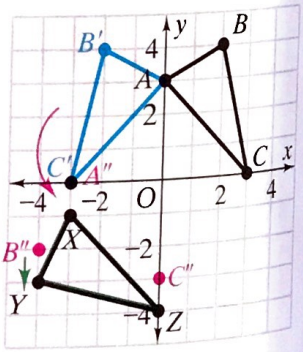


Since $\triangle A'B'C'$ and $\triangle XYZ$ are in different positions, rotate $\triangle A'B'C'$ 90° about the origin to get $\triangle A''B''C''$.



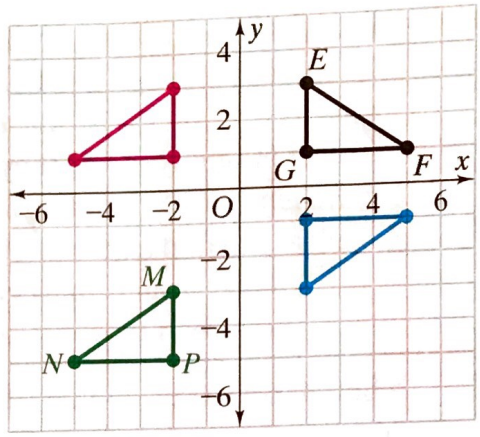
Each vertex of $\triangle XYZ$ is 1 unit down from the corresponding vertex of $\triangle A''B''C''$. So translating $\triangle A''B''C''$ 1 unit down will map it onto $\triangle XYZ$.

A reflection over the y-axis, followed by a rotation of 90° about the origin, followed by a translation 1 unit down maps $\triangle ABC$ onto $\triangle XYZ$. So, $\triangle ABC \cong \triangle XYZ$.



Quick Check

2. Determine whether $\triangle EFG$ is congruent to $\triangle MNP$. If the triangles are congruent, tell what sequence of transformations will map $\triangle EFG$ onto $\triangle MNP$. Then write a congruence statement. If they are not congruent, explain why.

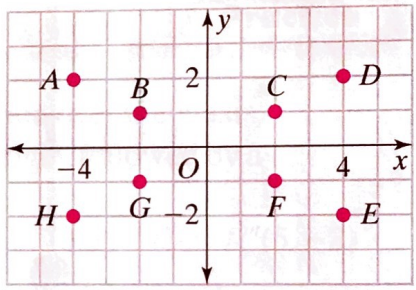


Check Your Understanding

1. **Reasoning** What three transformations can you use to prove that two figures are congruent? Explain your reasoning.

Use the graph at the right. Match each point with its image after the given sequence of transformations.

2. A, reflection over y-axis, 90° rotation about origin, translation 3 units down
3. B, reflection over x-axis, 180° rotation about origin, translation 2 units down
4. C, reflection over y-axis, 270° rotation about origin, translation 3 units right



Homework Exercises

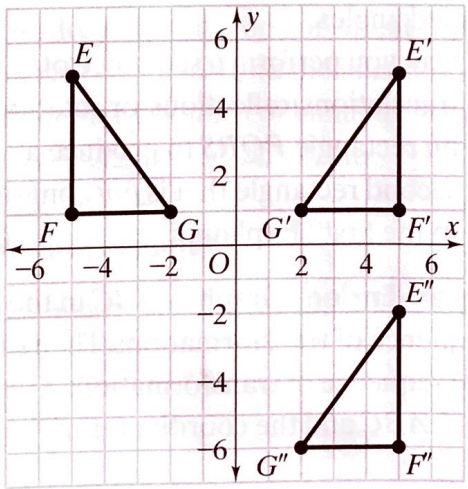
For more exercises, see Extra Skills and Word Problems.

GO for Help

Exercise	See Examples
5–6	1
7–8	2

The three figures in each diagram are congruent. Describe the sequence of transformations that maps the original figure onto the final image.

5.



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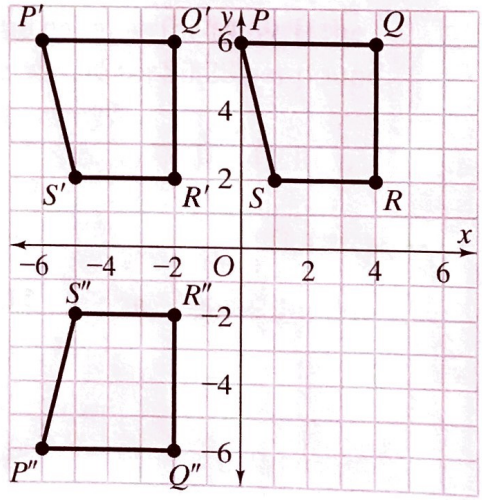
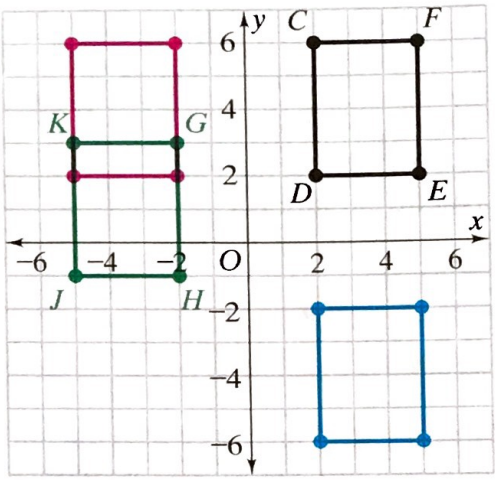


Determine whether the black figure is congruent to the green figure. If the figures are congruent, tell what sequence of transformations will map the black figure onto the green figure. Then write a congruence statement. If they are not congruent, explain why.

7.



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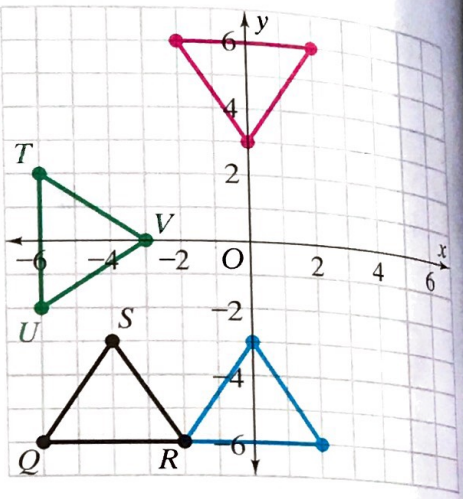


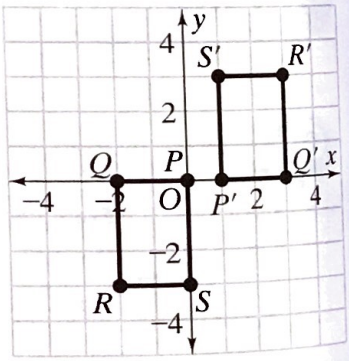
GPS

9. Guided Problem Solving Draw $\triangle EFG$ with vertices at $E(-2,3)$, $F(0,4)$, and $G(0,0)$ and $\triangle JKL$ with vertices at $J(3,-2)$, $K(4,-4)$, and $L(0,-4)$.

Describe two sequences of transformations that you can use to map $\triangle EFG$ onto $\triangle JKL$.

- To map $\triangle EFG$ onto $\triangle JKL$, should you start by translating, reflecting, or rotating $\triangle EFG$?
- What transformation should you perform second?
- Does the order in which you perform the sequence of transformations matter?

10. Rectangle $PQRS$ is transformed to rectangle $P'Q'R'S'$ as shown on the graph.



a. Describe a sequence of transformations to map rectangle $PQRS$ to rectangle $P'Q'R'S'$.

b. Identify all congruent line segments and angles.

c. Can you perform a sequence of translations, reflections, or rotations on rectangle $PQRS$ to produce a second rectangle that is *not* congruent to the first? Explain.

11. Open-Ended Graph $\triangle ABC$ in the coordinate plane. Describe a sequence of transformations. Then draw the image of $\triangle ABC$ after the sequence of transformations. Give the coordinates of the vertices of $\triangle ABC$ and the coordinates of the vertices of its image.

12. **Geometry** A *glide reflection* is a transformation that is made up of a translation and a reflection across a line that is parallel to the direction of the translation.
    a. Graph $\triangle DEF$ with vertices $D(1, -1)$, $E(7, -3)$, and $F(2, -7)$.
    b. Graph the image of $\triangle DEF$ under a glide reflection where the translation is $(x, y) \rightarrow (x, y + 8)$ and the reflection line is the y-axis. Give the coordinates of the vertices of the image.
    c. Are the two figures congruent? Explain your reasoning.

13. **Writing in Math** Suppose you reflect a point over the y-axis and then translate its image 5 units down. Describe what you would do to the coordinates of the original point to find the coordinates of the final image.

14. **Challenge** Graph $\triangle MNP$ with vertices $M(-2, 2)$, $N(-4, 2)$, and $P(-4, 5)$. Reflect $\triangle MNP$ across the y-axis. Then reflect its image across the x-axis. How can you get the same final image using a single rotation? Explain.

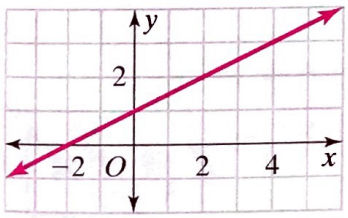
Test Prep and Mixed Review

Practice

Multiple Choice

15. What is the image of $T(1, 3)$ after a reflection over the y-axis, followed by a rotation of 180° about the origin, followed by a translation 4 units left?
    (A) $T'''(-5, 3)$ (B) $T'''(-3, -3)$ (C) $T'''(3, 3)$ (D) $T'''(5, -3)$

16. The graph of $y = \frac{1}{2}x + 1$ is shown on the coordinate grid at the right. Which table of ordered pairs contains only points on this line?



(F)

x	y
−4	1
2	2
3	2.5

(G)

x	y
−2	0
1	1.5
4	3

(H)

x	y
0	−2
1	0
2	2

(J)

x	y
−3	−1.5
0	2
5	3.5

17. Audrey bought a box of cereal and some bananas for $4.69. If the cereal cost $3.99 and the bananas were on sale for $0.28 per pound, how many pounds of bananas did Audrey buy?
    (A) 0.42 lb (B) 2.2 lb (C) 2.5 lb (D) 4.2 lb

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Exercise	See Lesson
18–20	2-1

Solve each equation.

18. $2b + 6 = 20$ 19. $3k - 2 = 24$ 20. $-5n + 4 = 29$