

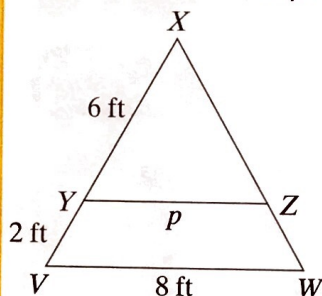
# Transformations and Similarity

## Check Skills You'll Need

### 1. Vocabulary Review

If two polygons are  $\sim$ , then the corresponding angles are congruent and the lengths of the corresponding sides are proportional.

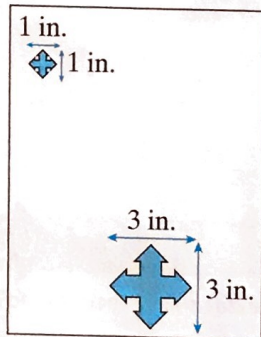
2. In the figure below,  $\triangle VWX \sim \triangle YZX$ . Find the value of  $p$ .



**GO for Help**  
Lesson 7-4

### CONTENT STANDARD

8.G.4



## What You'll Learn

To describe a sequence of transformations that maps one figure onto a figure that is similar; to determine whether two figures are similar by using a sequence of transformations

## Why Learn This?

Smartphones and tablets have a “zoom” feature that makes a figure larger or smaller. A “swipe” feature lets you slide the figure around on the screen. When you use these two features together, the final figure is similar to the original figure.



If two figures are similar, but not congruent, then a dilation, or a dilation and a sequence of transformations, will map one figure onto the other.

## EXAMPLE

### Determining Similarity using Transformations

- 1 You use the zoom and swipe features on your smartphone to enlarge and then move an image of a bee on the screen. Describe the sequence of transformations that maps the original image of the bee onto an image that is similar.



A dilation with a scale factor of 3.5 maps the original image of the bee onto the first zoomed-in image. A translation to the left maps the zoomed-in image onto the final image similar to the original image.

## Quick Check

1. Using a computer, a graphic designer moves a company logo from the top left of a page to the bottom center of the page and then enlarges the logo, as shown at the left. Describe the sequence of transformations that maps the original logo onto the final logo.

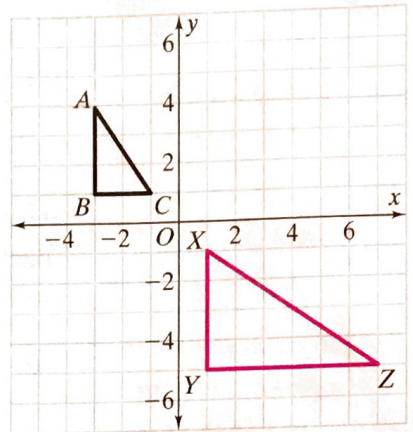


## More Than One Way

Determine whether  $\triangle ABC$  is similar to  $\triangle ZYX$ . Explain your reasoning.

### GO for Help

For help rotating a figure about the origin, go to Lesson 8-3, Example 2.



### Tina's Method

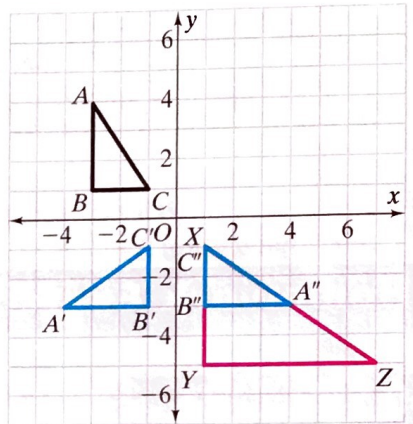
I can use a sequence of transformations to prove that  $\triangle ABC$  is similar to  $\triangle ZYX$ .

Rotate  $\triangle ABC$   $90^\circ$  about the origin to get  $\triangle A'B'C'$ .

Reflect  $\triangle A'B'C'$  over the y-axis to get  $\triangle A''B''C''$ .

Dilate  $\triangle A''B''C''$  by the scale factor 2 with center  $C''$  to get  $\triangle ZYX$ .

I can map  $\triangle ABC$  onto  $\triangle ZYX$  using transformations with a dilation, so the triangles are similar.



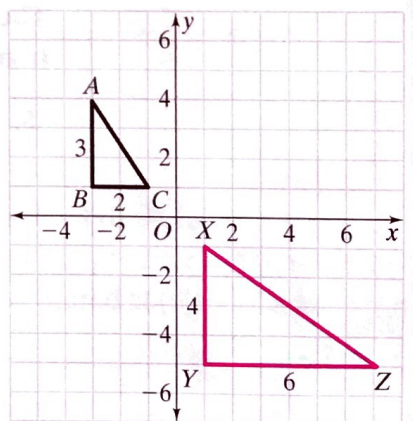
### Roberto's Method

I can use the SAS Similarity Theorem to prove that the two triangles are similar.

$\angle B \cong \angle Y$  because they are right angles.

$$\frac{AB}{BC} = \frac{3}{2} \text{ and } \frac{ZY}{YX} = \frac{6}{4} = \frac{3}{2}.$$

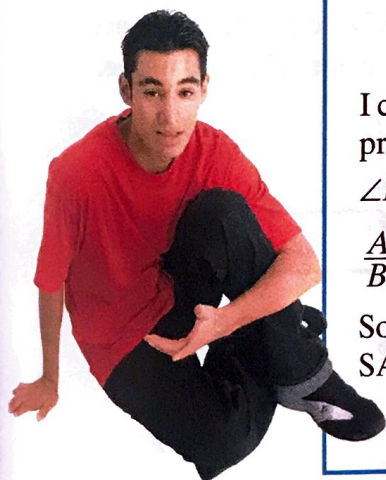
So,  $\triangle ABC$  is similar to  $\triangle ZYX$  by the SAS Similarity Theorem.



### Choose a Method

Quadrilateral  $QRST$  has vertices  $Q(1, 1)$ ,  $R(3, 1)$ ,  $S(4, 3)$ , and  $T(2, 3)$ .

Quadrilateral  $JKLM$  has vertices  $J(-2, 2)$ ,  $K(-6, 2)$ ,  $L(-8, 6)$ , and  $M(-4, 6)$ . Determine whether Quadrilateral  $QRST$  is similar to Quadrilateral  $JKLM$ . Explain your reasoning.



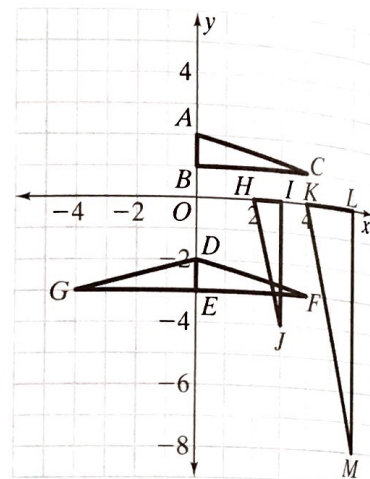


# Check Your Understanding

- Reasoning** What type of transformation can change the size of a figure? Explain your reasoning.

Use the graph at the right. Identify each mapping as a translation, reflection, rotation, or dilation.

- $\triangle ABC \rightarrow \triangle DEF$
- $\triangle DEF \rightarrow \triangle DEG$
- $\triangle DEG \rightarrow \triangle HIJ$
- $\triangle HIJ \rightarrow \triangle KLM$
- What sequence of transformations maps  $\triangle ABC$  onto  $\triangle KLM$ ?



## Homework Exercises

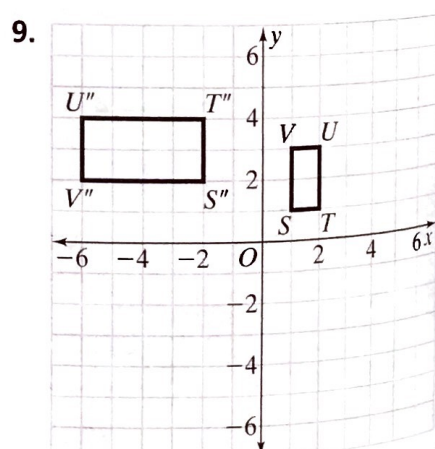
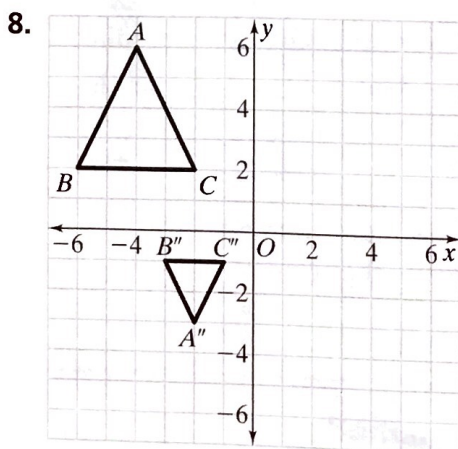
For more exercises, see **Extra Skills and Word Problems**.

### GO for Help

Exercise	See Examples
6–8	1
9–10	More Than One Way

- You use a word-processing program to create invitations for a party. You decrease the font size from 32 to 16 points. Then you move the words from the left side to the center of the invitation. Describe the sequence of transformations that maps the original words onto their final size and placement.

The two figures in each diagram are similar. Describe the sequence of two transformations that maps the original figure onto the final image.





**Test Prep Tip**

When describing transformations on the coordinate plane, remember to specify the direction of a translation, the center of a rotation or a dilation, and the scale of a dilation.

**GPS**

Determine whether the two figures are similar. If they are similar, describe a sequence of transformations that can be used to map one figure onto the other figure. If they are not similar, explain why.

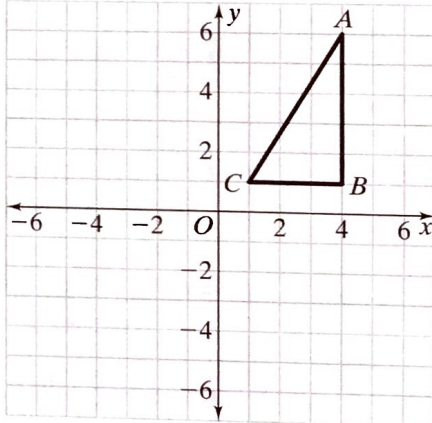
- 10. Quadrilateral  $ABCD$  with vertices  $A(-1, 1)$ ,  $B(-4, 1)$ ,  $C(-4, 3)$ , and  $D(-2, 3)$ ; quadrilateral  $JKLM$  with vertices  $J(6, 1)$ ,  $K(0, 1)$ ,  $L(0, 5)$ , and  $M(4, 5)$
- 11.  $\triangle DEF$  with vertices  $D(0, 0)$ ,  $E(-3, 0)$ , and  $F(-3, 6)$ ;  $\triangle PQR$  with vertices  $P(0, 0)$ ,  $Q(0, 2)$ , and  $R(4, 2)$

- 12. **Guided Problem Solving** Point  $P''(-12, -9)$  is the image of point  $P$  after the following sequence of transformations: a reflection over the  $x$ -axis, followed by a dilation with a scale factor of 3 centered at  $(0, 0)$ . What are the coordinates of point  $P$ ?

- What do you need to do in order to find the coordinates of  $P'$ ?
- What are the coordinates of  $P'$ ?
- How does knowing the coordinates of  $P'$  help you find the coordinates of  $P$ ?

- 13. A translation 6 units down followed by a dilation with scale factor  $\frac{1}{2}$  maps  $\triangle ABC$  onto  $\triangle A''B''C''$ . If  $\overline{AB} = 8$  units, what is the length of  $\overline{A''B''}$ ?

- 14. **Open Ended** Copy  $\triangle ABC$ .
  - a. Describe a sequence of two different transformations in which the final image is *congruent* to  $\triangle ABC$ .
  - b. Describe a sequence of two different transformations in which the final image is *similar*, but not congruent, to  $\triangle ABC$ .
  - c. How are the sequences of transformations you described in parts (a) and (b) different?



- 15. **Writing in Math** Does a  $90^\circ$  rotation about  $(0, 0)$ , followed by a dilation with scale factor 2 centered at  $(0, 0)$  produce the same final image as a dilation with scale factor 2 centered at  $(0, 0)$  followed by a  $90^\circ$  rotation about  $(0, 0)$ ? Explain.

- 16. **Challenge** Is a dilation with a scale factor of 2, followed by a dilation with a scale factor of 3, the same as a single dilation with a scale factor of 5? Explain your reasoning.

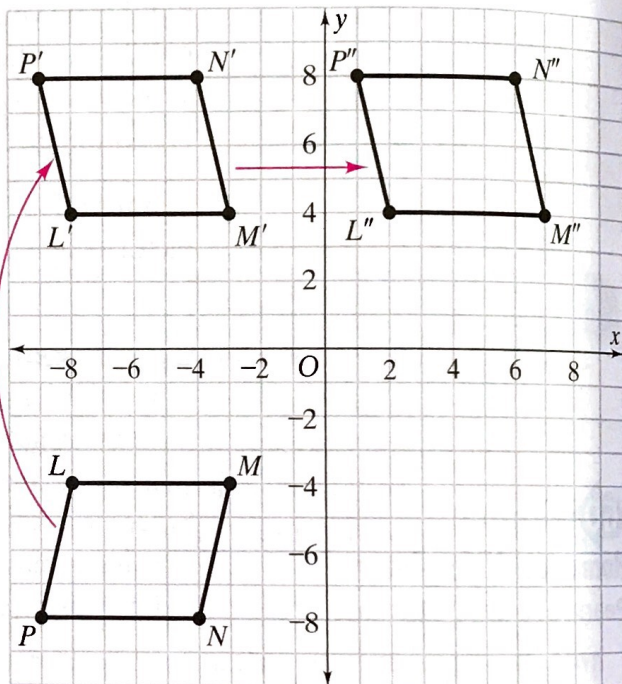


Multiple Choice

17. Quadrilateral  $ABCD$  is similar to quadrilateral  $WXYZ$ .  $ABCD$  has vertices  $A(2, 1)$ ,  $B(6, 1)$ ,  $C(6, 4)$ , and  $D(2, 4)$ .  $WXYZ$  has vertices  $W(4, 2)$ ,  $X(12, 2)$ ,  $Y(12, 8)$ , and  $Z(4, 8)$ . What is the scale factor of the dilation that maps  $ABCD$  onto  $WXYZ$ ?

- (A)  $\frac{1}{2}$
- (B)  $\frac{1}{4}$
- (C) 2
- (D) 4

18. The three parallelograms are congruent. Describe the sequence of transformations that maps  $LMNP$  onto  $L''M''N''P''$ .



19. For every 3 runs that Margo's team scored during a softball game, Addison's team scored 4 runs. What additional information is needed to find the number of runs Margo's team scored?

- (F) Addison's team's score
- (G) The score after the first inning.
- (H) The number of players on Margo's team
- (J) The number of players on both teams

**GO for Help**

Exercise	See Lesson
20-24	7-1

Find the measure of the complement and supplement of each angle.

20.  $37^\circ$       21.  $74^\circ$       22.  $83.2^\circ$       23.  $47.1^\circ$       24.  $5.8^\circ$