2 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Equivalence
You can represent an equation in many ways. Equivalent representations have the same solution as the original equation.

Solving Equations (Lessons 2-1, 2-2, 2-3, 2-4)
Equivalent equations have the same solution(s). To solve a given equation, form a series of simpler equivalent equations that isolate the variable.

2 Solving Equations and Inequalities
You can use properties of numbers and equality to transform equations into equivalent, simpler equations and find solutions.

Solving Equations (Lessons 2-1, 2-2, 2-3, 2-4)
Use equations to model real-world situations and find unknown quantities.

Literal Equations and Formulas (Lesson 2-5)
Formulas represent reliable real-world relationships. Use them to solve problems.

3 Proportionality
In a proportional relationship, the ratios of two quantities are equal. You can use this relationship to describe similar figures, scale models, and rates.

Rates, Proportions, and Similar Figures (Lessons 2-6, 2-7, 2-8)
Use rates to model ideas like growth, speed, and unit prices. Use proportions to interpret scale drawings.

Percents (Lessons 2-9, 2-10)
Formulas represent reliable real-world relationships. Use them to solve problems.

Chapter Vocabulary

- Addition Property of Equality (p. 81)
- conversion factor (p. 117)
- cross products (p. 125)
- Cross Products Property (p. 125)
- Division Property of Equality (p. 83)
- equivalent equations (p. 81)
- formula (p. 110)
- identity (p. 104)
- inverse operations (p. 82)
- isolate (p. 82)
- literal equation (p. 109)
- Multiplication Property of Equality (p. 83)
- percent error (p. 146)
- percent change (p. 144)
- percent decrease (p. 144)
- percent increase (p. 144)
- proportion (p. 124)
- rate (p. 116)
- ratio (p. 116)
- relative error (p. 146)
- scale (p. 132)
- scale drawing (p. 132)
- scale model (p. 132)
- similar figures (p. 130)
- Subtraction Property of Equality (p. 81)
- unit analysis (p. 117)
- unit rate (p. 116)

Choose the correct term to complete each sentence.

1. Addition and subtraction are examples of __?__ because they undo each other.
2. An equation that is true for every value of the variable is a(n) __?__.
3. A ratio of two equivalent measures given in different units is a(n) __?__.
4. On a map, information such as "1 in. : 5 mi" is the __?__ of the map.
5. In the proportion $\frac{a}{b} = \frac{c}{d}$, ad and bc are the __?__.

2-1 and 2-2 Solving One- and Two-Step Equations

Quick Review

To solve an equation, get the variable by itself on one side of the equation. You can use **properties of equality** and **inverse operations** to isolate the variable. For example, use multiplication to undo its inverse, division.

Example

What is the solution of $\frac{y}{2} + 5 = 8$?

$\frac{y}{2} + 5 - 5 = 8 - 5$	Subtract to undo addition.
$\frac{y}{2} = 3$	Simplify.
$2 \cdot \frac{y}{2} = 3 \cdot 2$	Multiply to undo division.
$y = 6$	Simplify.

Exercises

Solve each equation. Check your answer.

6. $x + 5 = -2$

7. $a - 2.5 = 4.5$

8. $3b = 42$

9. $\frac{n}{5} = 13$

10. $7x - 2 = 22.5$

11. $\frac{y}{4} - 3 = -4$

12. $8 + 3m = -7$

13. $-\frac{3d}{4} + 5 = 11$

14. Dining Five friends equally split a restaurant bill that comes to \$32.50. How much does each pay?

15. Reasoning Justify each step in solving $4x - 3 = 9$.

$4x - 3 + 3 = 9 + 3$	?
$4x = 12$	?
$\frac{4x}{4} = \frac{12}{4}$	?
$x = 3$	?

2-3 Solving Multi-Step Equations

Quick Review

To solve some equations, you may need to combine like terms or use the Distributive Property to clear fractions or decimals.

Example

What is the solution of $12 = 2x + \frac{4}{3} - \frac{2x}{3}$?

$3 \cdot 12 = 3\left(2x + \frac{4}{3} - \frac{2x}{3}\right)$	Multiply by 3.
$36 = 6x + 4 - 2x$	Simplify.
$36 = 4x + 4$	Combine like terms.
$36 - 4 = 4x + 4 - 4$	Subtract 4.
$32 = 4x$	Combine like terms.
$\frac{32}{4} = \frac{4x}{4}$	Divide each side by 4.
$8 = x$	Simplify.

Exercises

Solve each equation. Check your answer.

16. $7(s - 5) = 42$

17. $3a + 2 - 5a = -14$

18. $-4b - 5 + 2b = 10$

19. $3.4t + 0.08 = 11$

20. $10 = \frac{c}{3} - 4 + \frac{c}{6}$

21. $\frac{2x}{7} + \frac{4}{5} = 5$

Write an equation to model each situation. Then solve the equation.

22. Earnings You work for 4 h on Saturday and 8 h on Sunday. You also receive a \$50 bonus. You earn \$164. How much did you earn per hour?

23. Entertainment Online concert tickets cost \$37 each, plus a service charge of \$8.50 per ticket. The Web site also charges a transaction fee of \$14.99 for the purchase. You paid \$242.49. How many tickets did you buy?

2-4 Solving Equations With Variables on Both Sides

Quick Review

When an equation has variables on both sides, you can use properties of equality to isolate the variable on one side. An equation has no solution if no value of the variable makes it true. An equation is an **identity** if every value of the variable makes it true.

Example

What is the solution of $3x - 7 = 5x + 19$?

$3x - 7 - 3x = 5x + 19 - 3x$	Subtract 3x.
$-7 = 2x + 19$	Simplify.
$-7 - 19 = 2x + 19 - 19$	Subtract 19.
$-26 = 2x$	Simplify.
$\frac{-26}{2} = \frac{2x}{2}$	Divide each side by 2.
$-13 = x$	Simplify.

Exercises

Solve each equation. If the equation is an identity, write *identity*. If it has no solution, write *no solution*.

24. $\frac{2}{3}x + 4 = \frac{3}{5}x - 2$

25. $6 - 0.25f = f - 3$

26. $3(h - 4) = -\frac{1}{2}(24 - 6h)$

27. $5n = 20(4 + 0.25n)$

28. **Architecture** Two buildings have the same total height. One building has 8 floors with height h. The other building has a ground floor of 16 ft and 6 other floors with height h. Write and solve an equation to find the height h of these floors.

29. **Travel** A train makes a trip at 65 mi/h. A plane traveling 130 mi/h makes the same trip in 3 fewer hours. Write and solve an equation to find the distance of the trip.

2-5 Literal Equations and Formulas

Quick Review

A **literal equation** is an equation that involves two or more variables. A **formula** is an equation that states a relationship among quantities. You can use properties of equality to solve a literal equation for one variable in terms of others.

Example

What is the width of a rectangle with area 91 ft^2 and length 7 ft?

$A = \ell w$	Write the appropriate formula.
$\frac{A}{\ell} = w$	Divide each side by ℓ.
$\frac{91}{7} = w$	Substitute 91 for A and 7 for ℓ.
$13 = w$	Simplify.

The width of the rectangle is 13 ft.

Exercises

Solve each equation for x.

30. $ax + bx = -c$

31. $\frac{x + r}{t} + 1 = 0$

32. $m - 3x = 2x + p$

33. $\frac{x}{p} + \frac{x}{q} = s$

Solve each problem. Round to the nearest tenth, if necessary. Use 3.14 for π.

34. What is the width of a rectangle with length 5.5 cm and area 220 cm^2?

35. What is the radius of a circle with circumference 94.2 mm?

36. A triangle has height 15 in. and area 120 in.^{2}. What is the length of its base?

2-6 Ratios, Rates, and Conversions

Quick Review

A ratio between numbers measured in different units is called a **rate.** A **conversion factor** is a ratio of two equivalent measures in different units such as $\frac{1\text{ h}}{60\text{ min}}$, and is always equal to 1. To convert from one unit to another, multiply the original unit by a conversion factor that has the original units in the denominator and the desired units in the numerator.

Example

A painting is 17.5 in. wide. What is its width in centimeters? Recall that 1 in. = 2.54 cm.

$$17.5\,\cancel{\text{in.}} \cdot \frac{2.54\text{ cm}}{1\,\cancel{\text{in.}}} = 44.45\text{ cm}$$

The painting is 44.45 cm wide.

Exercises

Convert the given amount to the given unit.

37. $6\frac{1}{2}$ ft; in.

38. 4 lb 7 oz; oz

39. 135 s; min

40. 2.25 mi; yd

41. Production A bread slicer runs 20 h per day for 30 days and slices 144,000 loaves of bread. How many loaves per hour are sliced?

42. Pets A gerbil eats about $\frac{1}{4}$ oz of food per day. About how many pounds of food can a gerbil eat in a year?

43. Sports If a baseball travels at 90 mi/h, how many seconds does it take to travel 60 ft?

2-7 and 2-8 Solving Proportions and Using Similar Figures

Quick Review

The **cross products** of a proportion are equal.

If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

If two figures are **similar,** then corresponding angles are congruent and corresponding side lengths are in proportion. You can use proportions to find missing side lengths in similar figures and for indirect measurement.

Example

A tree casts a shadow 10 m long. At the same time, a signpost next to the tree casts a shadow 4 m long. The signpost is 2.5 m tall. How tall is the tree?

$\frac{x}{10} = \frac{2.5}{4}$	Write a proportion.
$4x = 10(2.5)$	Cross Products Property
$4x = 25$	Simplify.
$x = 6.25$	Divide each side by 4.

Exercises

Solve each proportion.

44. $\frac{3}{7} = \frac{9}{x}$

45. $\frac{-8}{10} = \frac{y}{5}$

46. $\frac{6}{15} = \frac{a}{4}$

47. $\frac{3}{-7} = \frac{-9}{t}$

48. $\frac{b+3}{7} = \frac{b-3}{6}$

49. $\frac{5}{2c-3} = \frac{3}{7c+4}$

50. Models An airplane has a wingspan of 25 ft and a length of 20 ft. You are designing a model of the airplane with a wingspan of 15 in. What will the length of your model be?

51. Projections You project a drawing 7 in. wide and $4\frac{1}{2}$ in. tall onto a wall. The projected image is 27 in. tall. How wide is the projected image?

2-9 Percents

Quick Review

A percent is a ratio that compares a number to 100. If you write a percent as a fraction, you can use a proportion to solve a percent problem.

Example

What percent of 84 is 105?

$\frac{105}{84} = \frac{p}{100}$	Write the percent proportion.
$100(105) = 84p$	Cross Products Property
$10{,}500 = 84p$	Simplify.
$125 = p$	Divide each side by 84.

105 is 125% of 84.

Exercises

52. What percent of 37 is 111?

53. What is 72% of 150?

54. 60% of what number is 102?

55. Gardening A gardener expects that 75% of the seeds she plants will produce plants. She wants 45 plants. How many seeds should she plant?

56. Fundraising A charity sent out 700 fundraising letters and received 210 contributions in response. What was the percent of response?

57. Surveys In a survey, 60% of students prefer bagels to donuts. If 120 students were surveyed, how many students prefer bagels?

2-10 Change Expressed as a Percent

Quick Review

Percent change $p\%$ is the ratio of the amount of change to the original amount.

$$p\% = \frac{\text{amount of increase or decrease}}{\text{original amount}}$$

You can use the percent change formula to express changes as percents.

Example

A bookstore buys a book for $16 and marks it up to $28. What is the markup expressed as a percent change?

$$\text{percent change} = \frac{\text{new amount} - \text{original amount}}{\text{original amount}}$$

$= \frac{28 - 16}{16}$	Substitute.
$= \frac{12}{16}$	Simplify.
$= 0.75$ or 75%	Write the result as a percent.

The price of the book increased by 75%.

Exercises

Tell whether each percent change is an increase or decrease. Then find the percent change. Round to the nearest percent.

58. original amount: 27
new amount: 30

59. original amount: 250
new amount: 200

60. original amount: 873
new amount: 781

61. original amount: 4.7
new amount: 6.2

62. Demographics In 1970, the U.S. population was about 205 million people. In 2007, it was about 301 million. What was the percent increase?

63. Astronomy The time from sunrise to sunset on the shortest day of the year in Jacksonville, Florida, is about 10 h 11 min. On the longest day, the time is 14 h 7 min. What is the percent increase?

64. Weather This morning the temperature was 38°F. This afternoon it is 57°F. Did the temperature increase by 50%? Explain.