Name: $\qquad$ Date: $\qquad$

## Student Exploration: Golf Range

Vocabulary: acceleration, air resistance, gravity, hang time, launch angle, projectile motion, trajectory, vector, velocity

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. You are in a contest with your friends to see who can drive a golf ball the farthest. Should you hit a "line drive" (low to the ground) or a shot with a very high angle? Explain.
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$\qquad$
2. Golf drives travel much farther than Major League home runs. Why might this be?
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$\qquad$
$\qquad$

## Gizmo Warm-up

Have you ever hit a hole-in-one? You will have a chance to do that in the Golf Range Gizmo ${ }^{\text {TM }}$, where you will see how a variety of factors affect the path of a golf ball. The movement of objects such as a ball through space is called projectile motion.

1. Press Play $(\downarrow)$. Did the ball go too far, the right distance, or not far enough? $\qquad$

2. Click Reset (అ). Move the $\boldsymbol{v}_{\text {initial }}$ and $\boldsymbol{\theta}$ sliders to adjust the velocity and launch angle until you get a hole-in-one. (With the Gizmo sound on ( $(\sqrt{2})$ ) you will hear "Hole in one!")

What were the velocity and launch angle values? $\qquad$
3. Can you get holes-in-one using other combinations of $\boldsymbol{v}_{\text {initial }}$ and $\boldsymbol{\theta}$ ? If so, give an example.

| Activity A: <br> Maximum <br> distance | Get the Gizmo ready: <br> - Click Reset and check that Atmosphere: Air is <br> selected. <br> - Set $\boldsymbol{v}$ intita |  |
| :--- | :--- | :--- |

## Question: What launch angle will produce the longest drive?

1. Form hypothesis: What launch angle do you think will yield the longest drive? $\qquad$
2. Experiment: Turn on the Show grid checkbox. With the velocity set to $75 \mathrm{~m} / \mathrm{s}$, experiment with a variety of launch angles until you find the one that yields the longest driving distance.
A. What launch angle produced the longest drive? $\qquad$
B. How far did the ball travel? $\qquad$
3. Observe: Click Reset and turn on Show paths. Click Clear paths. Take a swing with the optimum launch angle. The curved path the ball takes through the air is its trajectory.

Look closely at the trajectory. Does it appear symmetrical? $\qquad$
The curve is slightly steeper on the right than on the left as a result of air resistance.
4. Experiment: Click Reset, then select Atmosphere: None. As before, use trial and error until you find the launch angle that produces the longest drive.
A. What launch angle produced the longest drive? $\qquad$
B. How far did the ball travel? $\qquad$
C. Why do you think the ball traveled farther in this situation? $\qquad$
$\qquad$
5. Extend your thinking: The Moon has much less gravity than Earth and has an extremely thin atmosphere. How would these factors affect the trajectory of a golf ball on the Moon?
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$\qquad$
$\qquad$
$\qquad$

| Activity B: | Get the Gizmo ready: |  |
| :--- | :--- | :--- |
| Physics of <br> projectile motion | - Click Clear paths and select Atmosphere: None. <br> - Turn off Show grid and Show paths. <br> - You will need a scientific calculator for this activity. |  |

Introduction: Velocity is an example of a vector quantity because it describes the speed and direction of an object. The velocity of an object through space can be shown by two components: a horizontal component ( $\mathbf{v}_{\mathbf{x}}$ ) and a vertical component ( $\mathbf{v}_{\mathbf{y}}$ ).

## Question: How does the velocity of an object change as it flies through space?

1. Observe: Click Reset. Turn on Show velocity vectors, set $\boldsymbol{v}_{\text {initial }}$ to $50 \mathrm{~m} / \mathrm{s}$, and set $\boldsymbol{\theta}$ to 45.0 degrees. Click Play, and focus on the blue and red arrows that represent the vertical and horizontal components of the golf ball's velocity.
A. As the ball flies through the air, what do you notice about the blue (vertical) arrow?
B. As the ball flies through the air, what do you notice about the red (horizontal) arrow?
C. Try other velocities and launch angles. Do these results hold up? $\qquad$
2. Calculate: You can use trigonometry to find the initial horizontal and vertical components of the golf ball's velocity. Take out your calculator now. Click Reset, and turn off Show velocity vectors. Set $\boldsymbol{v}_{\text {initial }}$ to $50.0 \mathrm{~m} / \mathrm{s}$ and $\boldsymbol{\theta}$ to 60.0 degrees.
A. To calculate $\mathbf{v}_{\mathbf{x}}$, multiply $v_{\text {initial }}$ by the cosine of the angle: $\mathbf{v}_{\mathbf{x}}=v_{\text {initial }} \cdot \cos (\theta)$ : $\qquad$
B. To calculate $\mathbf{v}_{\mathbf{y}}$, multiply $v_{\text {initial }}$ by the sine of the angle: $\mathbf{v}_{\mathbf{y}}=v_{\text {initial }} \bullet \sin (\theta)$ : $\qquad$
C. Turn on Show velocity vectors. Were you correct? $\qquad$
3. Analyze: An object flying through the air is said to be in free fall. As you observed, the horizontal component of velocity ( $\mathbf{v}_{\mathbf{x}}$ ) does not change as the object moves, but the vertical component ( $\mathbf{v}_{\mathbf{y}}$ ) decreases over time. (Note: Air resistance is not included in this model.)
A. What force causes $\mathbf{v}_{\mathbf{y}}$ to change as the golf ball travels? $\qquad$
B. Why doesn't $\mathbf{v}_{\mathbf{x}}$ change as the object travels? (Hint: Are there any horizontal forces acting on the ball?) $\qquad$

## (Activity B continued on next page)

## Activity B (continued from previous page)

4. Set up Gizmo: Acceleration is a change in velocity. As the ball moves through its trajectory, it undergoes a downward acceleration due to the force of gravity. To calculate the acceleration of a falling object, divide the velocity change by the time interval.

$$
a=\left(v_{\text {current }}-v_{\text {initial }}\right) / t
$$

Set $\boldsymbol{v}_{\text {initial }}$ to $75.0 \mathrm{~m} / \mathrm{s}$ and $\boldsymbol{\theta}$ to 60.0 degrees. Record the initial vertical velocity of the golf ball in the first row of the table below. Include all units.

| Time (s) | $\mathbf{v}_{\mathbf{y}}(\mathbf{m} / \mathbf{s})$ |
| :---: | :---: |
| 0.00 s |  |
|  |  |

5. Gather data: Click Play, and then click Pause (II) at some point before the ball reaches its peak height. Record the time and $\mathbf{v}_{\mathbf{y}}$ in the table.
6. Calculate: Compute the velocity difference by subtracting the initial velocity from the current velocity (your answer should be a negative number). Then divide this number by the time to find the acceleration. (Units of acceleration are meters per second per second, or $\mathrm{m} / \mathrm{s}^{2}$.)

Velocity difference: $\qquad$ Time: $\qquad$ Acceleration: $\qquad$
7. Compare: Turn on Advanced features and observe the value of $g$, a measure of gravitational acceleration. Gravitational acceleration is the negative of $g$ : $a=-g$.

Is the value of $g$ equal to the negative of the acceleration you measured? $\qquad$
8. Experiment: Click Reset. Try launching the ball with different values of $g$. How does the value of $g$ affect the flight of the ball?
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$\qquad$
9. Extend your thinking: One of the problems aeronautical engineers face is building rockets that generate enough thrust to escape Earth's gravitational field. How would this problem be affected if the rocket was launched from the Moon? From a massive planet such as Jupiter?
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$\qquad$

|  | Get the Gizmo ready: |  |
| :--- | :--- | :--- |
| Activity C: | - Set $\boldsymbol{v}_{\text {initial }}$ to $75 \mathrm{~m} / \mathrm{s}, \boldsymbol{\theta}$ to 50.0 degrees, and $\boldsymbol{g}$ to |  |
|  | $9.8 \mathrm{~m} / \mathrm{s}^{2}$. (Be sure Advanced features is on.) |  |
| Hang time | Check that Atmosphere: None is selected. |  |

## Question: How can you calculate the distance a golf ball travels?

1. Think about it: Suppose you know a golf ball's horizontal velocity $\left(\mathbf{v}_{\mathbf{x}}\right)$ and the time it had traveled through the air $(t)$. How could you calculate how far the ball traveled?
2. Observe: What is the initial value of $\mathbf{v}_{\mathbf{y}}$ ? $\qquad$
3. Calculate: The vertical velocity of a projectile is found with the equation: $\mathbf{v}_{\mathbf{y}}=\mathbf{v}_{\mathbf{y} \text { (initial) }}-g t$.
A. What will be the value of $\mathbf{v}_{\mathbf{y}}$ when the ball is at the top of its trajectory? $\qquad$
B. Using the equation above, solve for $t$ when $\mathbf{v}_{\mathbf{y}}=0.0 \mathrm{~m} / \mathrm{s}, \mathbf{v}_{y}($ initial $)=57.45 \mathrm{~m} / \mathrm{s}$, and $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$. $\qquad$
C. Now use the same method to determine how long it will take the ball to fall from its maximum height to the ground: $\qquad$
D. Based on your answers to $B$ and $C$, how long will the ball spend in the air? $\qquad$ This is the hang time of the ball.
4. Check: Now press Play and observe the total time the ball spends in the air.

Were your calculations correct? $\qquad$
5. Evaluate: Click Reset. If the ball has a horizontal velocity $\left(\mathbf{v}_{\mathbf{x}}\right)$ and a hang time ( $t$ ), you can find the horizontal distance the ball travels using $d=\mathbf{v}_{\mathbf{x}} \cdot t$ (distance $=$ velocity $\times$ time $)$.
A. What is the horizontal velocity of the golf ball? $\qquad$
B. What is the hang time of the ball? $\qquad$
C. How far will the ball travel before it hits the ground? $\qquad$
D. Turn on Show grid and click Play. About how far did the ball travel? $\qquad$

## (Activity C continued on next page)

## Activity C (continued from previous page)

6. Calculate: Click Reset. Set $\boldsymbol{v}_{\text {initial }}$ to $50 \mathrm{~m} / \mathrm{s}$ and $\boldsymbol{\theta}$ to 40 degrees. Use what you have learned to calculate $\mathbf{v}_{\mathbf{x}}, \mathbf{v}_{\mathbf{y}}$, the hang time of the ball, and the horizontal distance the ball will travel.
$\mathbf{v}_{\mathrm{x}}$ $\qquad$ $v_{y}$ $\qquad$ Hang time $\qquad$ Distance $\qquad$
7. Test: Check your answers using the Gizmo. Were your calculations correct? $\qquad$
8. Apply: Complete the following table, first calculating the answers and then verifying them with the Gizmo. Include all units.

| $\boldsymbol{v}_{\text {initial }}$ | $\boldsymbol{\theta}$ | $\mathbf{v}_{\mathbf{x}}(\mathbf{m} / \mathbf{s})$ | $\mathbf{v}_{\mathbf{y}(\text { initial) }}(\mathbf{m} / \mathbf{s})$ | Hang time (s) | Distance (m) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $60 \mathrm{~m} / \mathrm{s}$ | $30^{\circ}$ |  |  |  |  |
| $30 \mathrm{~m} / \mathrm{s}$ | $45^{\circ}$ |  |  |  |  |
| $80 \mathrm{~m} / \mathrm{s}$ | $60^{\circ}$ |  |  |  |  |
| $50 \mathrm{~m} / \mathrm{s}$ | $75^{\circ}$ |  |  |  |  |

9. Challenge yourself: A classic problem in projectile motion is how far a projectile will travel if launched from a cliff. To solve this problem, you need to use the free-fall equation: $h=g t^{2} / 2$.

Click Reset. Check that the selected atmosphere is None. With the Advanced features checkbox turned on, set the height of the person ( $\boldsymbol{h}_{\text {person }}$ ) to 200.0 m . Set $\boldsymbol{v}_{\text {initial }}$ to $50.0 \mathrm{~m} / \mathrm{s}$, $\boldsymbol{\theta}$ to 0.0 degrees, and $\boldsymbol{g}$ to $9.8 \mathrm{~m} / \mathrm{s}^{2}$.
A. Solve the free-fall equation $\left(h=g t^{2} / 2\right)$ for $t$. $\qquad$
B. Calculate the time it will take the ball to fall to the ground from a height of 200 meters and acceleration $(g)$ of $9.81 \mathrm{~m} / \mathrm{s}^{2}$. $\qquad$
C. Based on the time and the horizontal velocity, how far will the ball travel horizontally?
D. Press Play. What were the actual hang time and distance? $\qquad$
10. Advanced challenge: Click Reset. Change $\boldsymbol{\theta}$ to $30^{\circ}$. Calculate the hang time and distance traveled. (Hint: Use $\mathbf{v}_{\mathbf{y}}=\mathbf{v}_{\mathbf{y} \text { (initial) }}-g t$ for the time to apex, $h=h_{\text {initial }}+g t^{2} / 2$ for the height of the apex, and $h=g t^{2} / 2$ for the time from apex to ground.)

Predicted hang time: $\qquad$ Predicted distance traveled: $\qquad$
Check your answers: Actual hang time: $\qquad$ Actual distance: $\qquad$

