1-1 Variables and Expressions

Common Core State Standards

A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MP 1, MP 3, MP 4, MP 7

Objective To write algebraic expressions

Can the number of states in the United States vary?

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

Consider the population of Florida, the area of Colorado, and the flight time from Philadelphia to San Francisco. Which of these has a value that varies? Explain.

A mathematical **quantity** is anything that can be measured or counted. Some quantities remain constant. Others change, or vary, and are called *variable quantities*.

Lesson Vocabulary
- quantity
- variable
- algebraic expression
- numerical expression

Essential Understanding Algebra uses symbols to represent quantities that are unknown or that vary. You can represent mathematical phrases and real-world relationships using symbols and operations.

A **variable** is a symbol, usually a letter, that represents the value(s) of a variable quantity. An **algebraic expression** is a mathematical phrase that includes one or more variables. A **numerical expression** is a mathematical phrase involving numbers and operation symbols, but no variables.

Problem 1 Writing Expressions With Addition and Subtraction

What is an algebraic expression for the word phrase?

Plan

How can a diagram help you write an algebraic expression?

Models like the ones shown can help you to visualize the relationships described by the word phrases.

	Word Phrase	Model	Expression
A	32 more than a number n	? = n + 32	$n + 32$
B	58 less a number n	58 = n + ?	$58 - n$

Got It? 1. What is an algebraic expression for 18 more than a number n?

Problem 2 Writing Expressions With Multiplication and Division

What is an algebraic expression for the word phrase?

Think

Is there more than one way to write an algebraic expression with multiplication?
Yes. Multiplication can be represented using a dot or parentheses in addition to an ×.

	Word Phrase	Model	Expression
A	8 times a number n	? (8 boxes: n n n n n n n n)	$8 \times n$, $8 \cdot n$, $8n$
B	the quotient of a number n and 5	n (5 boxes: ? ? ? ? ?)	$n \div 5$, $\frac{n}{5}$

Got It? **2.** What is an algebraic expression for each word phrase in parts (a) and (b)?

a. 6 times a number n

b. the quotient of 18 and a number n

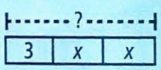
c. Reasoning Do the phrases *6 less a number y* and *6 less than a number y* mean the same thing? Explain.

Problem 3 Writing Expressions With Two Operations

What is an algebraic expression for the word phrase?

Plan

How can I represent the phrases visually?
Draw a diagram. You can represent the phrase in Problem 2, part (A), as shown below.



	Word Phrase	Expression
A	3 more than twice a number x	$3 + 2x$
B	9 less than the quotient of 6 and a number x	$\frac{6}{x} - 9$
C	the product of 4 and the sum of a number x and 7	$4(x + 7)$

Got It? **3.** What is an algebraic expression for each word phrase?

a. 8 less than the product of a number x and 4

b. twice the sum of a number x and 8

c. the quotient of 5 and the sum of 12 and a number x

In Problems 1, 2, and 3, you were given word phrases and wrote algebraic expressions. You can also translate algebraic expressions into word phrases.

Problem 4 Using Words for an Expression

What word phrase can you use to represent the algebraic expression $3x$?

Think

Is there only one way to write the expression in words?
No. The operation performed on 3 and x can be described by different words like "multiply," "times," and "product."

Expression $3x$ or $3 \cdot x$ — A number and a variable side by side indicate a product.

Words three times a number x or the product of 3 and a number x

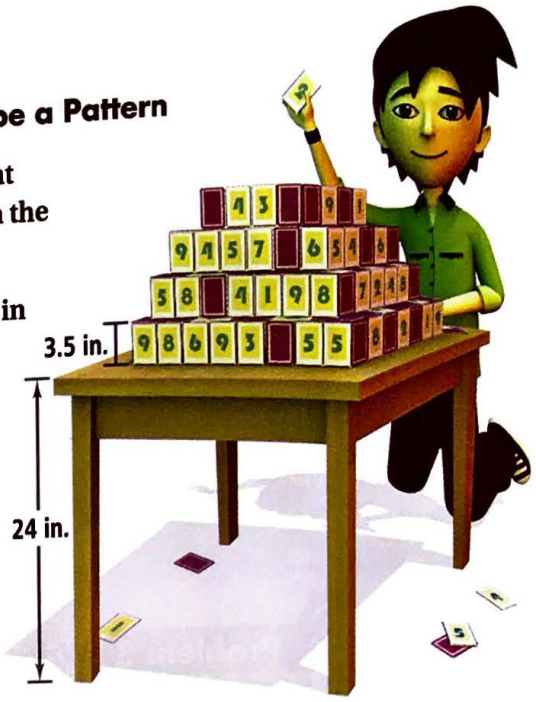
Got It? **4.** What word phrase can you use to represent the algebraic expression?

a. $x + 8.1$ **b.** $10x + 9$ **c.** $\frac{n}{3}$ **d.** $5x - 1$

You can use words or an algebraic expression to write a mathematical rule that describes a real-life pattern.

Problem 5 Writing a Rule to Describe a Pattern

Hobbies The table below shows how the height above the floor of a house of cards depends on the number of levels.



A **What is a rule for the height? Give the rule in words and as an algebraic expression.**

House of Cards

Number of Levels	Height (in.)
2	$(3.5 \cdot 2) + 24$
3	$(3.5 \cdot 3) + 24$
4	$(3.5 \cdot 4) + 24$
n	?

Know
Numerical expressions for the height given several different numbers of levels

Need
A rule for finding the height given a house with n levels

Plan
Look for a pattern in the table. Describe the pattern in words. Then use the words to write an algebraic expression.

Rule in Words

Multiply the number of levels by 3.5 and add 24.

Rule as an Algebraic Expression

The variable n represents the number of levels in the house of cards.

$3.5n + 24$ ← This expression lets you find the height for n levels.

B **A group of students built another house of cards that had 10 levels. Each card was 4 inches tall, and the height from the floor to the top of the house of cards was 70 inches. How tall would the house of cards be if they built an 11th level?**

Since each card was 4 inches tall, adding 1 more level would increase the total height of the house of cards by 4 inches.

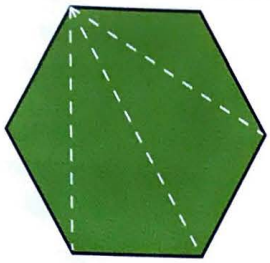
The house of cards would be 70 + 4, or 74 inches tall if the 11th level were added.

C **Another group of students built a third house of cards with n levels. Each card was 5 inches tall, and the height from the floor to the top of the house of cards was $34 + 5n$ inches. How tall would the house of cards be if the group added 1 more level of cards?**

Since each card was 5 inches tall, adding 1 more level would increase the total height of the house of cards by 5 inches.

The house of cards would be $34 + 5n + 5$ in. tall if the next level were added.

Got It? 5. Suppose you draw a segment from any one vertex of a regular polygon to the other vertices. A sample for a regular hexagon is shown below. Use the table to find a pattern. What is a rule for the number of nonoverlapping triangles formed? Give the rule in words and as an algebraic expression.



Triangles in Polygons

Number of Sides of Polygon	Number of Triangles
4	4 − 2
5	5 − 2
6	6 − 2
n	■

Lesson Check

Do you know HOW?

1. Is each expression *algebraic* or *numerical*?

   a. $7 \div 2$ b. $4m + 6$ c. $2(5 - 4)$

2. What is an algebraic expression for each phrase?

   a. the product of 9 and a number t

   b. the difference of a number x and $\frac{1}{2}$

   c. the sum of a number m and 7.1

   d. the quotient of 207 and a number n

Use words to describe each algebraic expression.

3. $6c$
4. $x - 1$
5. $\frac{t}{2}$
6. $3t - 4$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. **Vocabulary** Explain the difference between numerical expressions and algebraic expressions.

8. **Reasoning** Use the table to decide whether $49n + 0.75$ or $49 + 0.75n$ represents the total cost to rent a truck that you drive n miles.

Truck Rental Fees

Number of Miles	Cost
1	\$49 + (\$.75 × 1)
2	\$49 + (\$.75 × 2)
3	\$49 + (\$.75 × 3)
n	■

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Write an algebraic expression for each word phrase. See Problems 1–3.

9. 4 more than p
10. y minus 12
11. the quotient of n and 8
12. the product of 15 and c
13. a number t divided by 82
14. the sum of 13 and twice a number h
15. 6.7 more than the product of 5 and n
16. 9.85 less than the product of 37 and t

Write a word phrase for each algebraic expression. See Problem 4.

17. $q + 5$
18. $\frac{y}{5}$
19. $12x$
20. $49 + m$
21. $9n + 1$
22. $\frac{z}{8} - 9$
23. $15 - \frac{1.5}{d}$
24. $2(5 - n)$

Write a rule in words and as an algebraic expression to model the relationship in each table. See Problem 5.

25. Sightseeing While on vacation, you rent a bicycle. You pay \$9 for each hour you use it. It costs \$5 to rent a helmet while you use the bicycle.

Bike Rental

Number of Hours	Rental Cost
1	$(\$9 \times 1) + \$5$
2	$(\$9 \times 2) + \$5$
3	$(\$9 \times 3) + \$5$
n	■

26. Sales At a shoe store, a salesperson earns a weekly salary of \$150. A salesperson is also paid \$2.00 for each pair of shoes he or she sells during the week.

Shoe Sales

Pairs of Shoes Sold	Total Earned
5	$\$150 + (\$2 \times 5)$
10	$\$150 + (\$2 \times 10)$
15	$\$150 + (\$2 \times 15)$
n	■

B Apply

Write an algebraic expression for each word phrase.

27. 8 minus the product of 9 and r

28. the sum of 15 and x, plus 7

29. 4 less than three sevenths of y

30. the quotient of 12 and the product of 5 and t

31. Error Analysis A student writes the word phrase "the quotient of n and 5" to describe the expression $\frac{5}{n}$. Describe and correct the student's error.

32. Think About a Plan The table at the right shows the number of bagels a shop gives you per "baker's dozen." Write an algebraic expression that gives the rule for finding the number of bagels in any number b of baker's dozens.

- What is the pattern of increase in the number of bagels?
- What operation can you perform on b to find the number of bagels?

Bagels

Baker's Dozens	Number of Bagels
1	13
2	26
3	39
b	■

33. Tickets You and some friends are going to a museum. Each ticket costs \$4.50.

a. If n is the number of tickets purchased, write an expression that gives the total cost of buying n tickets.

b. Suppose the total cost for n tickets is \$36. What is the total cost if one more ticket is purchased?

34. Volunteering Serena and Tyler are wrapping gift boxes at the same pace. Serena starts first, as shown in the diagram. Write an algebraic expression that represents the number of boxes Tyler will have wrapped when Serena has wrapped x boxes.

35. Multiple Choice Which expression gives the value in dollars of d dimes?

(A) $0.10d$ (B) $0.10 + d$ (C) $\frac{0.10}{d}$ (D) $10d$

Open-Ended **Describe a real-world situation that each expression might model. Tell what each variable represents.**

36. $5t$ **37.** $b + 3$ **38.** $\frac{40}{h}$

Challenge

39. Reasoning You write $(5 - 2) \div n$ to represent the phrase *2 less than 5 divided by a number n*. Your friend writes $(5 \div n) - 2$. Are these both reasonable interpretations? Can verbal descriptions lack precision? Explain.

Write two different expressions that could both represent the given diagram.

40.

x	1	1	1	1
x	1	1	1	1
x	1	1	1	1

41.

x	1	1	1	1
x	1	1		

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 7

Look back at the figures on page 3 showing the pattern of the tiles of the walkway.

Complete the table that shows the relationship between n, the number of names on each side of the walk, and the number of inscribed tiles.

Walkway Tiles

n	Number of Inscribed Tiles
1	2
a. ?	4
3	b. ?
4	c. ?

d. Write a rule in words and as an algebraic expression to model the relationship shown in the table.

e. How many plain tiles are in the walk when there are 3 names on each side? Write an expression for the number of plain tiles when there are n names on each side of the walk.

f. If $n = 8$, how many plain and inscribed tiles will there be in the walkway? Explain.