1-2 Order of Operations and Evaluating Expressions

Common Core State Standards

A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MP 1, MP 3, MP 4, MP 6, MP 8

Objectives To simplify expressions involving exponents
To use the order of operations to evaluate expressions

What is your plan for making a good choice?

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

You've won! For a door prize, you get to choose between the two options shown. Which is the better prize? Why?

PRIZE 1

You get $60 immediately.

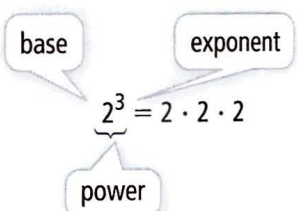
PRIZE 2

You get $1 the first day. Then, each day for the next five days, you get twice the previous day's amount.

Essential Understanding You can use *powers* to shorten how you represent repeated multiplication, such as $2 \times 2 \times 2 \times 2 \times 2 \times 2$.

Lesson Vocabulary
- power
- exponent
- base
- simplify
- evaluate

A **power** has two parts, a *base* and an *exponent*. The **exponent** tells you how many times to use the **base** as a factor. You read the power 2^3 as "two to the third power" or "two cubed." You read 5^2 as "five to the second power" or "five squared."



You **simplify** a numerical expression when you replace it with its single numerical value. For example, the simplest form of $2 \cdot 8$ is 16. To simplify a power, you replace it with its simplest name.

Problem 1 Simplifying Powers

What is the simplified form of the expression?

Think

What does the exponent indicate?
It shows the number of times you use the base as a factor.

A 10^7

$10^7 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

$= 10{,}000{,}000$

B $(0.2)^5$

$(0.2)^5 = 0.2 \cdot 0.2 \cdot 0.2 \cdot 0.2 \cdot 0.2$

$= 0.00032$

Got It? **1.** What is the simplified form of each expression?

a. 3^4 **b.** $\left(\frac{2}{3}\right)^3$ **c.** $(0.5)^3$

Essential Understanding When simplifying an expression, you need to perform operations in the correct order.

You might think about simplifying the expression $2 + 3 \times 5$ in two ways:

Add first.

$2 + 3 \times 5 = 5 \times 5 = 25$ ✗

Multiply first.

$2 + 3 \times 5 = 2 + 15 = 17$ ✔

Both results may seem sensible, but only the second result is considered correct. This is because the second way uses the order of operations that mathematicians have agreed to follow. Always use the following order of operations:

take note

Key Concept Order of Operations

1. Perform any operation(s) inside grouping symbols, such as parentheses ( ) and brackets [ ]. A fraction bar also acts as a grouping symbol.
2. Simplify powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Problem 2 Simplifying a Numerical Expression

What is the simplified form of each expression?

Think

How do you simplify an expression that contains a fraction? You start by simplifying the numerator and denominator. Then you divide the numerator by the denominator.

A $(6 - 2)^3 \div 2$

$(6 - 2)^3 \div 2 = 4^3 \div 2$ Subtract inside parentheses.

$= 64 \div 2$ Simplify the power.

$= 32$ Divide.

B $\frac{2^4 - 1}{5}$

$\frac{2^4 - 1}{5} = \frac{16 - 1}{5}$ Simplify the power.

$= \frac{15}{5}$ Subtract.

$= 3$ Divide.

Got It? 2. What is the simplified form of each expression?

a. $5 \cdot 7 - 4^2 \div 2$

b. $12 - 25 \div 5$

c. $\frac{4 + 3^4}{7 - 2}$

d. **Reasoning** How does a fraction bar act as a grouping symbol? Explain.

When two or more variables, or a number and variables, are written together, treat them as if they were within parentheses. So $4xy$ is equivalent to $(4xy)$, and $xy^2 = (xy^2)$. You **evaluate** an algebraic expression by replacing each variable with a given number. Then simplify the expression using the order of operations.

Problem 3 Evaluating Algebraic Expressions

What is the value of the expression for $x = 5$ and $y = 2$?

Plan

How is this Problem like ones you've seen before?
You begin by substituting numbers for the variables. After substituting, you have numerical expressions just like the ones in Problem 2.

A $x^2 + x - 12 \div y^2$

$x^2 + x - 12 \div y^2 = 5^2 + 5 - 12 \div 2^2$ Substitute 5 for x and 2 for y.

$= 25 + 5 - 12 \div 4$ Simplify powers.

$= 25 + 5 - 3$ Divide.

$= 27$ Add and subtract from left to right.

B $(xy)^2 \div (xy)$

$(xy)^2 \div xy = (5 \cdot 2)^2 \div (5 \cdot 2)$ Substitute 5 for x and 2 for y.

$= 10^2 \div 10$ Multiply inside parentheses.

$= 100 \div 10$ Simplify the power.

$= 10$ Divide.

Got It? **3.** What is the value of each expression when $a = 3$ and $b = 4$ in parts (a)-(b)?

a. $3b - a^2$ **b.** $2b^2 - 7a$

Problem 4 Evaluating a Real-World Expression

Banking **What is an expression for the spending money you have left after depositing $\frac{2}{5}$ of your wages in savings? Evaluate the expression for weekly wages of \$40, \$50, \$75, and \$100.**

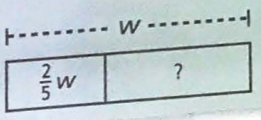
Know
- Savings equals $\frac{2}{5}$ of wages.
- Various weekly wages

Need
- Expression for spending money
- Amount of spending money for various weekly wages

Plan
Write an algebraic expression and evaluate it for each amount of weekly wages. Use a table to organize your results.

Think

How can a model help you write the expression?
This model shows that spending money equals your wages w minus the amount you save: $\frac{2}{5}w$.



Relate spending money equals wages minus $\frac{2}{5}$ of wages

Define Let w = your wages.

Write $w - \frac{2}{5} \cdot w$

The expression $w - \frac{2}{5} \cdot w$ represents the amount of money you have left after depositing $\frac{2}{5}$ of your wages in savings.

Spending Money

Wages (w)	$w - \frac{2}{5}w$	Total Spending Money (\$)
40	$40 - \frac{2}{5}(40)$	24
50	$50 - \frac{2}{5}(50)$	30
75	$75 - \frac{2}{5}(75)$	45
100	$100 - \frac{2}{5}(100)$	60

Got It? 4. The shipping cost for an order at an online store is $\frac{1}{10}$ the cost of the items you order. What is an expression for the total cost of a given order? What are the total costs for orders of \$43, \$79, \$95, and \$103?

Lesson Check

Do you know HOW?

What is the simplified form of each expression?

1. 5^2
2. 2^3
3. $\left(\frac{3}{4}\right)^2$

Evaluate each expression for $x = 3$ and $y = 4$.

4. $x^2 + 2(x + y)$
5. $(xy)^3$
6. $4x^2 - 3xy$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. **Vocabulary** Identify the exponent and the base in 4^3.
8. **Error Analysis** A student simplifies an expression as shown below. Find the error and simplify the expression correctly.

$$23 - 8 \cdot 2 + 3^2 = 23 - 8 \cdot 2 + 9$$
$$= 15 \cdot 2 + 9$$
$$= 30 + 9$$
$$= 39 \quad \times$$

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Simplify each expression. See Problems 1 and 2.

9. 3^5
10. 4^3
11. 2^4
12. 10^8
13. $\left(\frac{2}{3}\right)^3$
14. $\left(\frac{1}{2}\right)^4$
15. $(0.4)^6$
16. 7^4
17. $20 - 2 \cdot 3^2$
18. $6 + 4 \div 2 + 3$
19. $(6^2 - 3^3) \div 2$
20. $5 \cdot 2^2 \div 2 + 8$
21. $80 - (4 - 1)^3$
22. $52 + 8^2 - 3(4 - 2)^3$
23. $\frac{6^4 \div 3^2}{9}$
24. $\frac{2 \cdot 7 + 4}{9 \div 3}$

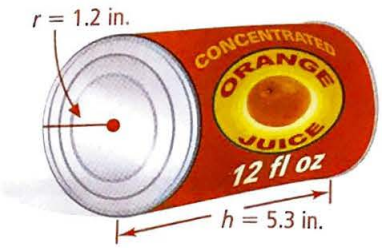
Evaluate each expression for $s = 4$ and $t = 8$. See Problem 3.

25. $(s + t)^3$
26. $s^4 + t^2 + s \div 2$
27. $(st)^2 \div (st^2)$
28. $3st^2 \div (st) + 6$
29. $(t - s)^5$
30. $(2s)^2t$
31. $2st^2 - s^2$
32. $2s^2 - t^3 \div 16$
33. $\frac{(3s)^3t + t}{s}$

34. Write an expression for the amount of change you will get when you pay for a purchase p with a \$20 bill. Make a table to find the amounts of change you will get for purchases of \$11.59, \$17.50, \$19.00, and \$20.00. See Problem 4.
35. An object's momentum is defined as the product of its mass m and velocity v. Write an expression for the momentum of an object. Make a table to find the momentums of a vehicle with a mass of 1000 kg moving at a velocity of 15 m/s, 20 m/s, and 25 m/s.

B Apply

36. Geometry The expression $\pi r^2 h$ represents the volume of a cylinder with radius r and height h.



a. What is the volume, to the nearest tenth of a cubic inch, of the juice can at the right? Use 3.14 for π.

b. Reasoning About how many cubic inches, to the nearest tenth of a cubic inch, does a fluid ounce of juice fill?

Simplify each expression.

37. $2[(8-4)^5 \div 8]$

38. $3[(4-2)^5 - 20]$

39. $10 - (2^3 + 4) \div 3 - 1$

40. $\dfrac{22 + 1^3 + (3^4 - 7^2)}{2^3}$

41. $3[42 - 2(10^2 - 9^2)]$

42. $\dfrac{2[8 + (67 - 2^6)^3]}{9}$

43. Think About a Plan The snack bar at your school has added sushi to its menu. The ingredients for one roll include sushi rice, seaweed sheets, cucumbers, cream cheese, and 3 oz of smoked salmon. One roll can be cut into 8 servings. Write an expression for the amount of salmon needed to make s servings of sushi. How much salmon is needed to make 16 servings? 24 servings? 80 servings? 100 servings?

- What operations are needed in your calculations?
- Use a table to help you organize your results. What will you use for the column headings in your table?

44. Salary You earn \$10 for each hour you work at a canoe rental shop. Write an expression for your salary for working the number of hours h. Make a table to find how much you earn for working 10 h, 20 h, 30 h, and 40 h.

Evaluate each expression for the given values of the variables.

45. $3(s-t)^2;\ s=4,\ t=1$

46. $2x - y^2;\ x=7,\ y=3.5$

47. $3m^2 - n;\ m=2,\ n=6$

48. $(2a+2b)^2;\ a=3,\ b=4$

49. $2p^2 + (2q)^2;\ p=4,\ q=3$

50. $(4c - d + 0.2)^2 - 10c;\ c=3.1,\ d=4.6$

51. $\dfrac{3g+6}{h};\ g=5,\ h=7$

52. $\dfrac{2w+3v}{v^2};\ v=6,\ w=1$

53. Writing Consider the expression $(1+5)^2 - (18 \div 3)$. Can you perform the operations in different orders and still get the correct answer? Explain.

54. A student wrote the expressions shown and claimed they were equal for all values of x and y.

$(x+y)^2$
x^2+y^2

a. Evaluate each expression for $x=1$ and $y=0$.

b. Evaluate each expression for $x=1$ and $y=2$.

c. Open-Ended Choose another pair of values for x and y. Evaluate each expression for those values.

d. Writing Is the student's claim correct? Justify your answer.

55. Find the value of $14 + 5 \cdot 3 - 3^2$. Then change two operation signs so that the value of the expression is 8.

Challenge Use grouping symbols to make each equation true.

56. $9 + 3 - 2 + 4 = 6$

57. $16 - 4 \div 2 + 3 = 9$

58. $4^2 - 5 \cdot 2 + 1 = 1$

59. $3 \cdot 4 + 5 - 6 + 7 = 28$

60. a. Geometry A cone has a slant height ℓ of 11 cm and a radius r of 3 cm. Use the expression $\pi r(\ell + r)$ to find the surface area of the cone. Use 3.14 for π. Round to the nearest tenth of a square centimeter.

b. Reasoning Does the surface area of the cone double if the radius doubles? If the slant height doubles? Explain.

Standardized Test Prep

SAT/ACT

61. What is the simplified form of $4 + 10 \div 4 + 6$?

(A) 1.4 (B) 9.5 (C) 12.5 (D) 24

62. What is the value of $(2a)^2b - 2c^2$ for $a = 2$, $b = 4$, and $c = 3$?

(F) 14 (G) 28 (H) 32 (I) 46

63. A shirt is on sale for \$25 at the local department store. The sales tax equals $\frac{1}{25}$ of the shirt's price. What is the total cost of the shirt including sales tax?

(A) \$17 (B) \$26 (C) \$27 (D) \$33

64. You can find the distance in feet that an object falls in t seconds using the expression $16t^2$. If you drop a ball from a tall building, how far does the ball fall in 3 s?

(F) 16 ft (G) 48 ft (H) 96 ft (I) 144 ft

Mixed Review

Write an algebraic expression for each word phrase. See Lesson 1-1.

65. 4 more than p

66. 5 minus the product of y and 3

67. the quotient of m and 10

68. 3 times the difference of 7 and d

Tell whether each number is *prime* or *composite*. See p. 798.

69. 17 **70.** 33 **71.** 43 **72.** 91

Get Ready! To prepare for Lesson 1-3, do Exercises 73–80.

Write each fraction as a decimal and each decimal as a fraction. See p. 802.

73. $\frac{3}{5}$ **74.** $\frac{7}{8}$ **75.** $\frac{2}{3}$ **76.** $\frac{4}{7}$

77. 0.7 **78.** 0.07 **79.** 4.25 **80.** 0.425