1-3 Real Numbers and the Number Line

Common Core State Standards

Prepares for N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational . . .

MP 1, MP 3, MP 6

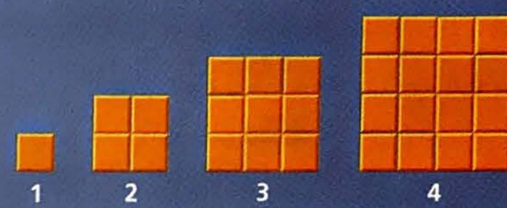
Objectives To classify, graph, and compare real numbers
To find and estimate square roots

This problem involves a special group of numbers.

MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

If the pattern continues, which will be the first figure to contain more than 200 square units? Explain your reasoning.



The diagrams in the Solve It model what happens when you multiply a number by itself to form a product. When you do this, the original number is called a *square root* of the product.

Lesson Vocabulary
- square root
- radicand
- radical
- perfect square
- set
- element of a set
- subset
- rational numbers
- natural numbers
- whole numbers
- integers
- irrational numbers
- real numbers
- inequality

take note

Key Concept Square Root

Algebra A number a is a **square root** of a number b if $a^2 = b$.

Example $7^2 = 49$, so 7 is a square root of 49.

Essential Understanding You can use the definition above to find the exact square roots of some nonnegative numbers. You can approximate the square roots of other nonnegative numbers.

The radical symbol $\sqrt{}$ indicates a nonnegative square root, also called a *principal square root.* The expression under the radical symbol is called the **radicand.**

$$\text{radical symbol} \rightarrow \sqrt{a} \leftarrow \text{radicand}$$

Together, the radical symbol and radicand form a **radical.** You will learn about negative square roots in Lesson 1-6.

Problem 1 Simplifying Square Root Expressions

Think
How can you find a square root?
Find a number that you can multiply by itself to get a product that is equal to the radicand.

What is the simplified form of each expression?

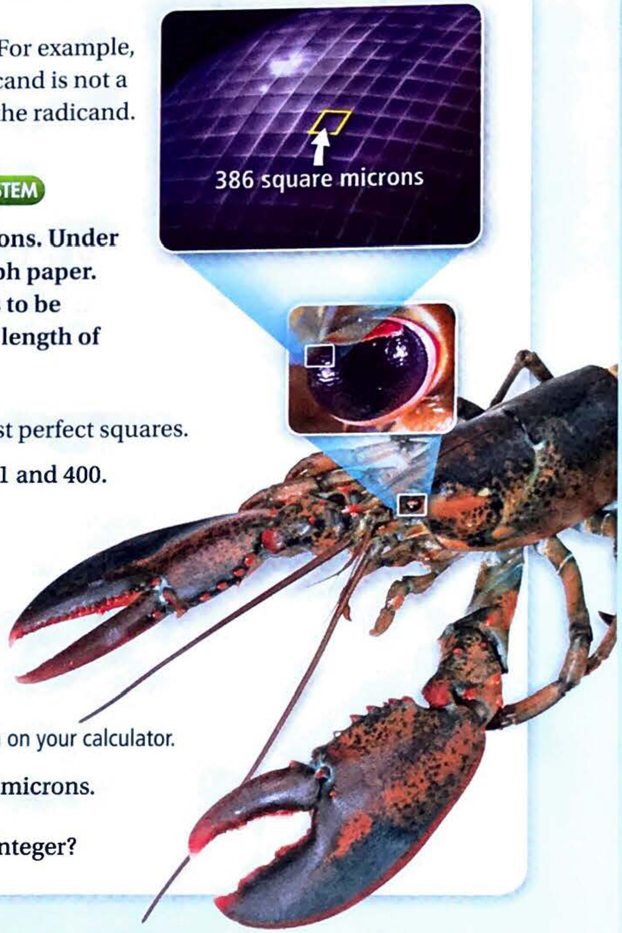
A $\sqrt{81} = 9$ $\quad 9^2 = 81$, so 9 is a square root of 81.

B $\sqrt{\frac{9}{16}} = \frac{3}{4}$ $\quad \left(\frac{3}{4}\right)^2 = \frac{9}{16}$, so $\frac{3}{4}$ is a square root of $\frac{9}{16}$.

Got It? **1.** What is the simplified form of each expression?

a. $\sqrt{64}$ **b.** $\sqrt{25}$ **c.** $\sqrt{\frac{1}{36}}$ **d.** $\sqrt{\frac{81}{121}}$

The square of an integer is called a **perfect square.** For example, 49 is a perfect square because $7^2 = 49$. When a radicand is not a perfect square, you can estimate the square root of the radicand.



Problem 2 Estimating a Square Root STEM

Plan
How can you get started?
The square root of the area of a square is equal to its side length. So, find $\sqrt{386}$.

Biology **Lobster eyes are made of tiny square regions. Under a microscope, the surface of the eye looks like graph paper. A scientist measures the area of one of the squares to be 386 square microns. What is the approximate side length of the square to the nearest micron?**

Method 1 Estimate $\sqrt{386}$ by finding the two closest perfect squares.

The perfect squares closest to 386 are **361 and 400.**

$19^2 = 361$

$20^2 = 400 \longleftarrow 386$

Since 386 is closer to 400, $\sqrt{386} \approx 20$, and the side length is about 20 microns.

Method 2 Estimate $\sqrt{386}$ using a calculator.

$\sqrt{386} \approx 19.6$ $\quad$ Use the square root function on your calculator.

The side length of the square is about 20 microns.

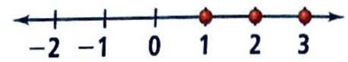
Got It? **2.** What is the value of $\sqrt{34}$ to the nearest integer?

Essential Understanding Numbers can be classified by their characteristics. Some types of numbers can be represented on the number line.

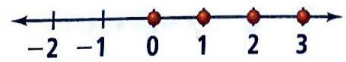
You can classify numbers using *sets*. A **set** is a well-defined collection of objects. Each object is called an **element of the set.** A **subset** of a set consists of elements from the given set. You can list the elements of a set within braces { }.

A **rational number** is any number that you can write in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. A rational number in decimal form is either a terminating decimal such as 5.45 or a repeating decimal such as 0.41666 . . . , which you can write as $0.41\overline{6}$. Each graph below shows a subset of the rational numbers on a number line.

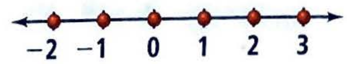
Natural numbers $\{1, 2, 3, \ldots\}$



Whole numbers $\{0, 1, 2, 3, \ldots\}$



Integers $\{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$



An **irrational number** cannot be represented as the quotient of two integers. In decimal form, irrational numbers do not terminate or repeat. Here are some examples.

$$0.1010010001\ldots \qquad \pi = 3.14159265\ldots$$

Some square roots are rational numbers and some are irrational numbers. If a whole number is not a perfect square, its square root is irrational.

Rational $\sqrt{4} = 2$ $\quad \sqrt{25} = 5$

Irrational $\sqrt{3} = 1.73205080\ldots$ $\quad \sqrt{10} = 3.16227766\ldots$

Rational numbers and irrational numbers form the set of **real numbers.**

Think

What clues can you use to classify real numbers?
Look for negative signs, fractions, decimals that do or do not terminate or repeat, and radicands that are not perfect squares.

Problem 3 Classifying Real Numbers

To which subsets of the real numbers does each number belong?

A **15** natural numbers, whole numbers, integers, rational numbers

B -1.4583 rational numbers (since -1.4583 is a terminating decimal)

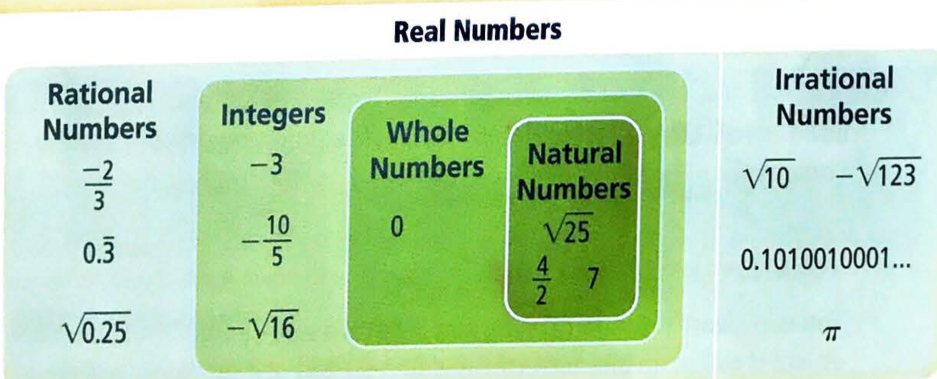
C $\sqrt{57}$ irrational numbers (since 57 is not a perfect square)

Got It? **3.** To which subsets of the real numbers does each number belong?

a. $\sqrt{9}$ **b.** $\frac{3}{10}$ **c.** -0.45 **d.** $\sqrt{12}$

take note

Concept Summary Real Numbers



An **inequality** is a mathematical sentence that compares the values of two expressions using an inequality symbol. The symbols are

$<$, less than	$\leq$, less than or equal to
$>$, greater than	$\geq$, greater than or equal to

Plan

How can you compare numbers?
Write the numbers in the same form, such as decimal form.

Problem 4 Comparing Real Numbers

What is an inequality that compares the numbers $\sqrt{17}$ and $4\frac{1}{3}$?

$\sqrt{17} = 4.12310\ldots$	Write the square root as a decimal.
$4\frac{1}{3} = 4.\overline{3}$	Write the fraction as a decimal.
$\sqrt{17} < 4\frac{1}{3}$	Compare using an inequality symbol.

Got It? 4. a. What is an inequality that compares the numbers $\sqrt{129}$ and 11.52?
b. Reasoning In Problem 4, is there another inequality you can write that compares the two numbers? Explain.

You can graph and order all real numbers using a number line.

Problem 5 Graphing and Ordering Real Numbers

Multiple Choice What is the order of $\sqrt{4}$, 0.4, $-\frac{2}{3}$, $\sqrt{2}$, and -1.5 from least to greatest?

Ⓐ $-\frac{2}{3}, 0.4, -1.5, \sqrt{2}, \sqrt{4}$

Ⓑ $-1.5, \sqrt{2}, 0.4, \sqrt{4}, -\frac{2}{3}$

Ⓒ $-1.5, -\frac{2}{3}, 0.4, \sqrt{2}, \sqrt{4}$

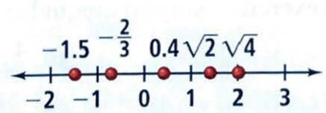
Ⓓ $\sqrt{4}, \sqrt{2}, 0.4, -\frac{2}{3}, -1.5$

Know	Need	Plan
Five real numbers	Order of numbers from least to greatest	Graph the numbers on a number line.

Think

Why is it useful to rewrite numbers in decimal form?
It allows you to compare numbers whose values are close, like $\frac{1}{4}$ and 0.26.

First, write the numbers that are not in decimal form as decimals: $\sqrt{4} = 2$, $-\frac{2}{3} \approx -0.67$, and $\sqrt{2} \approx 1.41$. Then graph all five numbers on the number line to order the numbers, and read the graph from left to right.



From least to greatest, the numbers are -1.5, $-\frac{2}{3}$, 0.4, $\sqrt{2}$, and $\sqrt{4}$. The correct answer is C.

Got It? 5. Graph 3.5, -2.1, $\sqrt{9}$, $-\frac{7}{2}$, and $\sqrt{5}$ on a number line. What is the order of the numbers from least to greatest?

Lesson Check

Do you know HOW?

Name the subset(s) of the real numbers to which each number belongs.

1. $\sqrt{11}$

2. -7

3. Order $\frac{47}{10}$, 4.1, -5, and $\sqrt{16}$ from least to greatest.

4. A square card has an area of 15 in.^{2}. What is the approximate side length of the card?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary What are the two subsets of the real numbers that form the set of real numbers?

6. Vocabulary Give an example of a rational number that is not an integer.

Reasoning **Tell whether each square root is *rational* or *irrational*. Explain.**

7. $\sqrt{100}$

8. $\sqrt{0.29}$

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Simplify each expression. See Problem 1.

9. $\sqrt{36}$ **10.** $\sqrt{169}$ **11.** $\sqrt{16}$ **12.** $\sqrt{900}$ **13.** $\sqrt{\frac{36}{49}}$

14. $\sqrt{\frac{25}{81}}$ **15.** $\sqrt{\frac{1}{9}}$ **16.** $\sqrt{\frac{121}{16}}$ **17.** $\sqrt{1.96}$ **18.** $\sqrt{0.25}$

Estimate the square root. Round to the nearest integer. See Problem 2.

19. $\sqrt{17}$ **20.** $\sqrt{35}$ **21.** $\sqrt{242}$ **22.** $\sqrt{61}$ **23.** $\sqrt{320}$

Find the approximate side length of each square figure to the nearest whole unit.

24. a mural with an area of 18 m^2

25. a game board with an area of 160 in.^{2}

26. a helicopter launching pad with an area of 3000 ft^2

Name the subset(s) of the real numbers to which each number belongs. See Problem 3.

27. $\frac{2}{3}$ **28.** 13 **29.** -1 **30.** $-\frac{19}{100}$ **31.** π

32. -2.38 **33.** $\frac{17}{4573}$ **34.** $\sqrt{144}$ **35.** $\sqrt{113}$ **36.** $\frac{59}{2}$

Compare the numbers in each exercise using an inequality symbol. See Problem 4.

37. $5\frac{2}{3}, \sqrt{29}$ **38.** $-3.1, -\frac{16}{5}$ **39.** $\frac{4}{3}, \sqrt{2}$ **40.** $9.6, \sqrt{96}$

41. $-\frac{7}{11}, -0.63$ **42.** $\sqrt{115}, 10.72104\ldots$ **43.** $-\frac{22}{25}, -0.\overline{8}$ **44.** $\sqrt{184}, 15.56987\ldots$

Order the numbers in each exercise from least to greatest. See Problem 5.

45. $\frac{1}{2}, -2, \sqrt{5}, -\frac{7}{4}, 2.4$ **46.** $-3, \sqrt{31}, \sqrt{11}, 5.5, -\frac{60}{11}$ **47.** $-6, \sqrt{20}, 4.3, -\frac{59}{9}$

48. $\frac{10}{3}, 3, \sqrt{8}, 2.9, \sqrt{7}$ **49.** $-\frac{13}{6}, -2.1, -\frac{26}{13}, -\frac{9}{4}$ **50.** $-\frac{1}{6}, -0.3, \sqrt{1}, -\frac{2}{13}, \frac{7}{8}$

B Apply

51. **Think About a Plan** A stage designer paid \$4 per square foot for flooring to be used in a square room. If the designer spent \$600 on the flooring, about how long is a side of the room? Round to the nearest foot.
   - How is the area of a square related to its side length?
   - How can you estimate the length of a side of a square?

Tell whether each statement is *true* or *false*. Explain.

52. All negative numbers are integers.

53. All integers are rational numbers.

54. All square roots are irrational numbers.

55. No positive number is an integer.

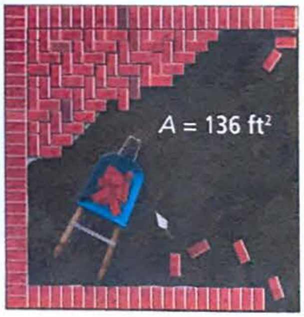
56. **Reasoning** A restaurant owner is going to panel a square portion of the restaurant's ceiling. The portion to be paneled has an area of 185 ft^2. The owner plans to use square tin ceiling panels with a side length of 2 ft. What is the first step in finding out whether the owner will be able to use a whole number of panels?

Show that each number is rational by writing it in the form $\frac{a}{b}$, where a and b are integers.

57. 417 58. 0.37 59. 2.01 60. 2.1 61. 3.06

62. **Error Analysis** A student says that $\sqrt{7}$ is a rational number because you can write $\sqrt{7}$ as the quotient $\frac{\sqrt{7}}{1}$. Is the student correct? Explain.

STEM 63. **Construction** A contractor is tiling a square patio that has the area shown at the right. What is the approximate side length of the patio? Round to the nearest foot.



64. **Open-Ended** You are tutoring a younger student. How would you explain rational numbers, irrational numbers, and how they are different?

65. **Geometry** The irrational number π, equal to 3.14159 . . . , is the ratio of a circle's circumference to its diameter. In the sixth century, the mathematician Brahmagupta estimated the value of π to be $\sqrt{10}$. In the thirteenth century, the mathematician Fibonacci estimated the value of π to be $\frac{864}{275}$. Which is the better estimate? Explain.

66. **Home Improvement** If you lean a ladder against a wall, the length of the ladder should be $\sqrt{(x)^2 + (4x)^2}$ ft to be considered safe. The distance x is how far the ladder's base is from the wall. Estimate the desired length of the ladder when the base is positioned 5 ft from the wall. Round your answer to the nearest tenth.

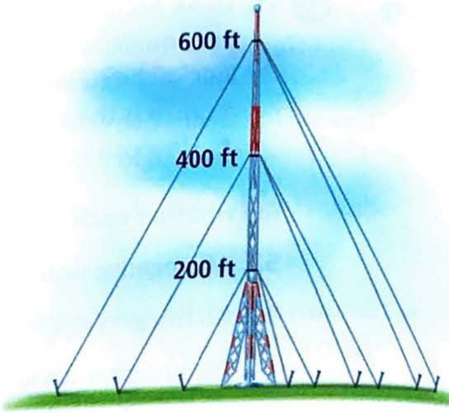
67. **Writing** Is there a greatest integer on the real number line? A least fraction? Explain.

68. **Reasoning** Choose three intervals on the real number line that contain both rational and irrational numbers. Do you think that any given interval on the real number line contains both rational and irrational numbers? Explain.

Challenge

69. **Reasoning** Sometimes the product of two positive numbers is less than either number. Describe the numbers for which this is true.

STEM 70. **Antennas** Guy wires are attached to an antenna tower at the heights h shown at the right. Use the expression $\sqrt{h^2 + (0.55h)^2}$ to estimate the wire length for each height. If three wires are attached at each height, what is the minimum total amount of wire needed?



71. **Cube Roots** The number a is the *cube root* of a number b if $a^3 = b$. For example, the cube root of 8 is 2 because $2^3 = 8$. Find the cube root of each number.

a. 64 **b.** 1000 **c.** 343 **d.** 2197

Standardized Test Prep

SAT/ACT

72. A square picture has an area of 225 in.^{2}. What is the side length of the picture?

(A) 5 in. (B) 15 in. (C) 25 in. (D) 225 in.

73. To simplify the expression $9 \cdot (33 - 5^2) \div 2$, what do you do first?

(F) Divide by 2. (G) Subtract 5. (H) Multiply by 9. (I) Square 5.

74. The table at the right shows the number of pages you can read per minute. Which algebraic expression gives a rule for finding the number of pages read in any number of minutes m?

(A) m (C) $2m$

(B) $m + 2$ (D) $\frac{m}{2}$

Reading

Minutes	Pages Read
1	2
2	4
3	6
m	■

Mixed Review

Evaluate each expression for the given values of the variables.

See Lesson 1-2.

75. $(r - t)^2$; $r = 11$, $t = 7$ **76.** $3m^2 + n$; $m = 5$, $n = 3$ **77.** $(2x)^2y$; $x = 4$, $y = 8$

Write an algebraic expression for each word phrase.

See Lesson 1-1.

78. the sum of 14 and x

79. 4 multiplied by the sum of y and 1

80. 3880 divided by z

81. the product of t and the quotient of 19 and 3

Get Ready! **To prepare for Lesson 1-4, do Exercises 82–85.**

Simplify each expression.

See Lesson 1-2.

82. $4 + 7 \cdot 2$ **83.** $(7 + 1)9$ **84.** $2 + 22 \cdot 20$ **85.** $6 + 18 \div 6$