1-4 Properties of Real Numbers

Common Core State Standards

Prepares for N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational . . .

MP 1, MP 2, MP 3, MP 4, MP 6, MP 7

Objective To identify and use properties of real numbers

Remember that ≠ means "is not equal to."

SOLVE IT!

Getting Ready!

Tell whether each pair of expressions is equal by completing each statement with = or ≠. Explain your answers.

$34 + 12 \stackrel{?}{=} 12 + 34$ $\quad 18 \div \frac{1}{18} \stackrel{?}{=} 1$

$100 - 1 \stackrel{?}{=} 1 - 100$ $\quad 45 - 1 \stackrel{?}{=} 45$

$0 + 180 \stackrel{?}{=} 180$ $\quad 6 \times \frac{1}{6} \stackrel{?}{=} 1$

MATHEMATICAL PRACTICES The Solve It illustrates numerical relationships that are always true for real numbers.

Essential Understanding Relationships that are always true for real numbers are called *properties*, which are rules used to rewrite and compare expressions.

Lesson Vocabulary
- equivalent expressions
- deductive reasoning
- counterexample

Two algebraic expressions are **equivalent expressions** if they have the same value for all values of the variable(s). The following properties show expressions that are equivalent for all real numbers.

take note

Properties **Properties of Real Numbers**

Let a, b, and c be any real numbers.

Commutative Properties of Addition and Multiplication

Changing the order of the addends does not change the sum. Changing the order of the factors does not change the product.

	Algebra	Example
Addition	$a + b = b + a$	$18 + 54 = 54 + 18$
Multiplication	$a \cdot b = b \cdot a$	$12 \cdot \frac{1}{2} = \frac{1}{2} \cdot 12$

Associative Properties of Addition and Multiplication

Changing the grouping of the addends does not change the sum. Changing the grouping of the factors does not change the product.

Addition	$(a + b) + c = a + (b + c)$	$(23 + 9) + 4 = 23 + (9 + 4)$
Multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(7 \cdot 9) \cdot 10 = 7 \cdot (9 \cdot 10)$

take note

Properties Properties of Real Numbers

Let a be any real number.

Identity Properties of Addition and Multiplication

The sum of any real number and 0 is the original number. The product of any real number and 1 is the original number.

	Algebra	Example
Addition	$a + 0 = a$	$5\frac{3}{4} + 0 = 5\frac{3}{4}$
Multiplication	$a \cdot 1 = a$	$67 \cdot 1 = 67$

Zero Property of Multiplication

The product of a and 0 is 0.

$a \cdot 0 = 0$ $\quad 18 \cdot 0 = 0$

Multiplication Property of -1

The product of -1 and a is $-a$.

$-1 \cdot a = -a$ $\quad -1 \cdot 9 = -9$

Problem 1 Identifying Properties

Think

What math symbols give you clues about the properties? Parentheses, operation symbols, and the numbers 0 and 1 may indicate certain properties.

What property is illustrated by each statement?

A $42 \cdot 0 = 0$ Zero Property of Multiplication

B $(y + 2.5) + 28 = y + (2.5 + 28)$ Associative Property of Addition

C $10x + 0 = 10x$ Identity Property of Addition

Got It? 1. What property is illustrated by each statement?

a. $4x \cdot 1 = 4x$ b. $x + (\sqrt{y} + z) = x + (z + \sqrt{y})$

You can use properties to help you solve some problems using mental math.

Problem 2 Using Properties for Mental Calculations

Plan

How can you make the addition easier? Look for numbers having decimal parts you can add easily, such as 0.75 and 0.25.

Movies **A movie ticket costs \$7.75. A drink costs \$2.40. Popcorn costs \$1.25. What is the total cost for a ticket, a drink, and popcorn? Use mental math.**

$(7.75 + 2.40) + 1.25 = (2.40 + 7.75) + 1.25$ Commutative Property of Addition

$= 2.40 + (7.75 + 1.25)$ Associative Property of Addition

$= 2.40 + 9$ Simplify inside parentheses.

$= 11.40$ Add.

The total cost is \$11.40.

Got It? 2. A can holds 3 tennis balls. A box holds 4 cans. A case holds 6 boxes. How many tennis balls are in 10 cases? Use mental math.

Problem 3 Writing Equivalent Expressions

Simplify each expression.

A $5(3n)$

Know An expression

Need Groups of numbers that can be simplified

Plan Use properties to group or reorder parts of the expression.

$5(3n) = (5 \cdot 3)n$ Associative Property of Multiplication

$= 15n$ Simplify.

B $(4 + 7b) + 8$

$(4 + 7b) + 8 = (7b + 4) + 8$ Commutative Property of Addition

$= 7b + (4 + 8)$ Associative Property of Addition

$= 7b + 12$ Simplify.

C $\frac{6xy}{y}$

$\frac{6xy}{y} = \frac{6x \cdot y}{1 \cdot y}$ Rewrite denominator using Identity Property of Multiplication.

$= \frac{6x}{1} \cdot \frac{y}{y}$ Use rule for multiplying fractions: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

$= 6x \cdot 1$ $x \div 1 = x$ and $y \div y = 1$.

$= 6x$ Identity Property of Multiplication

Got It? **3.** Simplify each expression.

a. $2.1(4.5x)$ **b.** $6 + (4h + 3)$ **c.** $\frac{8m}{12mn}$

In Problem 3, reasoning and properties were used to show that two expressions are equivalent. This is an example of *deductive reasoning*. **Deductive reasoning** is the process of reasoning logically from given facts to a conclusion.

To show that a statement is *not* true, find an example for which it is not true. An example showing that a statement is false is a **counterexample.** You need only one counterexample to prove that a statement is false.

Problem 4 Using Deductive Reasoning and Counterexamples

Is the statement *true* or *false*? If it is false, give a counterexample.

A **For all real numbers a and b, $a \cdot b = b + a$.**

False. $5 \cdot 3 \neq 3 + 5$ is a counterexample.

B **For all real numbers a, b, and c, $(a + b) + c = b + (a + c)$.**

True. Use properties of real numbers to show that the expressions are equivalent.

$(a + b) + c = (b + a) + c$ Commutative Property of Addition

$= b + (a + c)$ Associative Property of Addition

Plan

Look for a counterexample to show the statement is false. If you don't find one, try to use properties to show that it is true.

Got It?

4. **Reasoning** Is each statement in parts (a) and (b) *true* or *false*? If it is false, give a counterexample. If true, use properties of real numbers to show the expressions are equivalent.
   - **a.** For all real numbers j and k, $j \cdot k = (k + 0) \cdot j$.
   - **b.** For all real numbers m and n, $m(n + 1) = mn + 1$.
   - **c.** Is the statement in part (A) of Problem 4 false for *every* pair of real numbers a and b? Explain.

Lesson Check

Do you know HOW?

Name the property that each statement illustrates.

1. $x + 12 = 12 + x$
2. $5 \cdot (12 \cdot x) = (5 \cdot 12) \cdot x$
3. You buy a sandwich for \$2.95, an apple for \$.45, and a bottle of juice for \$1.05. What is the total cost?
4. Simplify $\frac{24cd}{c}$.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** Tell whether the expressions in each pair are equivalent.
   - **a.** $5x \cdot 1$ and $1 + 5x$
   - **b.** $1 + (2t + 1)$ and $2 + 2t$
6. Justify each step.

$$\begin{aligned} 3 \cdot (10 \cdot 12) &= 3 \cdot (12 \cdot 10) \\ &= (3 \cdot 12) \cdot 10 \\ &= 36 \cdot 10 \\ &= 360 \end{aligned}$$

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

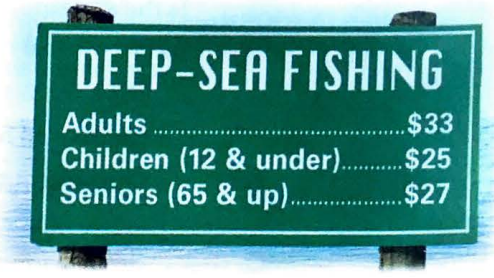
A Practice

Name the property that each statement illustrates. See Problem 1.

7. $75 + 6 = 6 + 75$
8. $\frac{7}{9} \cdot 1 = \frac{7}{9}$
9. $h + 0 = h$
10. $389 \cdot 0 = 0$
11. $27 \cdot \pi = \pi \cdot 27$
12. $9 \cdot (-1 \cdot x) = 9 \cdot (-x)$

Mental Math **Simplify each expression.** See Problem 2.

13. $21 + 6 + 9$
14. $10 \cdot 2 \cdot 19 \cdot 5$
15. $0.1 + 3.7 + 5.9$
16. $4 \cdot 5 \cdot 13 \cdot 5$
17. $55.3 + 0.2 + 23.8 + 0.7$
18. $0.25 \cdot 12 \cdot 4$
19. **Fishing Trip** The sign at the right shows the costs for a deep-sea fishing trip. How much will the total cost be for 1 adult, 2 children, and 1 senior citizen to go on a fishing trip? Use mental math.



Simplify each expression. Justify each step. See Problem 3.

20. $8 + (9t + 4)$ **21.** $9(2x)$ **22.** $(4 + 105x) + 5$ **23.** $(10p)11$

24. $(12 \cdot r) \cdot 13$ **25.** $(2 + 3x) + 9$ **26.** $4 \cdot (x \cdot 6.3)$ **27.** $1.1 + (7d + 0.1)$

28. $\frac{56ab}{b}$ **29.** $\frac{1.5mn}{m}$ **30.** $\frac{13p}{pq}$ **31.** $\frac{33xy}{3x}$

Use deductive reasoning to tell whether each statement is *true* or *false*. If it is false, give a counterexample. If true, use properties of real numbers to show the expressions are equivalent. See Problem 4.

32. For all real numbers r, s, and t, $(r \cdot s) \cdot t = t \cdot (s \cdot r)$.

33. For all real numbers p and q, $p \div q = q \div p$.

34. For all real numbers x, $x + 0 = 0$.

35. For all real numbers a and b, $-a \cdot b = a \cdot (-b)$.

B Apply

36. Error Analysis Your friend shows you the problem at the right. He says that the Associative Property allows you to change the order in which you complete two operations. Is your friend correct? Explain.

$$(5 \cdot 11) + 9 = 5 \cdot (11 + 9)$$
$$= 5 \cdot 20$$
$$= 100$$

37. Travel It is 258 mi from Tulsa, Oklahoma, to Dallas, Texas. It is 239 mi from Dallas, Texas, to Houston, Texas.

a. What is the total distance of a trip from Tulsa to Dallas to Houston?

b. What is the total distance of a trip from Houston to Dallas to Tulsa?

c. Explain how you can tell whether the distances described in parts (a) and (b) are equal by using reasoning.

Tell whether the expressions in each pair are equivalent.

38. $2 + h + 4$ and $2 \cdot h \cdot 4$ **39.** $9y \cdot 0$ and 1 **40.** $3x$ and $3x \cdot 1$

41. $m(1 - 1)$ and 0 **42.** $(9 - 7) + \pi$ and 2π **43.** $(3 + 7) + m$ and $m + 10$

44. $\frac{63ab}{7a}$ and $9ab$ **45.** $\frac{11x}{(2 + 5 - 7)}$ and $11x$ **46.** $\frac{7t}{4 - 8 + \sqrt{9}}$ and $7t$

47. Think About a Plan Hannah makes a list of possible gifts for Mary, Jared, and Michael. She has two plans and can spend a total of $75 for all gifts. Which plan(s) can Hannah afford?

- What property can you use to make it easier to find the total cost of different gifts?
- What number do you compare to the total cost of each plan to decide whether it is affordable?

48. Writing Suppose you are mixing red and blue paint in a bucket. Do you think the final color of the mixed paint will be the same whether you add the blue paint or the red paint to the bucket first? Relate your answer to a property of real numbers.

Simplify each expression. Justify each step.

49. $25 \cdot 3.9 \cdot 4$

50. $(4.4 \div 4.4)(x + 7)$

51. $(7^6 - 6^5)(8 - 8)$

Reasoning **Answer each question. Use examples to justify your answers.**

52. Is subtraction commutative?

53. Is subtraction associative?

54. Is division commutative?

55. Is division associative?

Challenge

56. Patterns The Commutative Property of Addition lets you rewrite addition expressions. How many different ways can you write $a + b + c$? Show each way.

57. Reasoning Suppose you know that $a(b + c) = ab + ac$ is true for all real numbers a, b, and c. Use the properties of real numbers to prove that $(b + c)a = ba + ca$ is true for all real numbers a, b, and c.

Standardized Test Prep

SAT/ACT

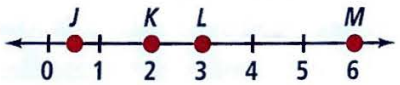
58. What is the simplified form of $(1.2 + 0) + 4.6 + 3.8$?

(A) 1.2 (B) 8.0 (C) 8.4 (D) 9.6

59. Which expression is equal to $3 \cdot 3 \cdot 8 \cdot 8 \cdot 3$?

(F) $3 \cdot 8$ (G) 3^8 (H) $3^3 \cdot 8^2$ (I) $3 \cdot 3 + 2 \cdot 8$

60. There are four points plotted on the number line below.



Which expression represents the greatest amount?

(A) $M \div L$ (B) $M - L$ (C) $J + K$ (D) $L - K$

61. Lane 1 at your local track is 0.25 mi long. You live 0.5 mi away from the track. Which of the following results in the shortest jog?

(F) jogging 6 times around the track in Lane 1
(G) jogging to the track and then 5 times around the track in Lane 1
(H) jogging to the track, 3 times around the track in Lane 1, and then home
(I) jogging 8 times around the track in Lane 1

Mixed Review

Order the numbers in each exercise from least to greatest.

See Lesson 1-3.

62. $-6, 6^3, 1.6, \sqrt{6}$

63. $\frac{8}{5}, 1.4, -17, 10^2$

64. $1.75, -4.5, \sqrt{4}, 14^1$

Get Ready! **To prepare for Lesson 1-5, do Exercises 65–68.**

Find each sum or difference.

See p. 803.

65. $3 + 11$

66. $\frac{3}{8} + \frac{5}{8}$

67. $9.7 - 8.6$

68. $\frac{5}{9} - \frac{5}{10}$