Multiplying and Dividing Real Numbers

Objective To find products and quotients of real numbers

SOLVE IT!

Getting Ready!

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Use patterns to complete the table and answer the questions below. Explain your reasoning.

- What is the sign of the product of a positive number and a negative number?
- What is the sign of the product of two negative numbers?

$2 \cdot 3 = 6$	$-2 \cdot 3 = -6$
$2 \cdot 2 = 4$	$-2 \cdot 2 = -4$
$2 \cdot 1 = 2$	$-2 \cdot 1 = -2$
$2 \cdot 0 = \blacksquare$	$-2 \cdot 0 = \blacksquare$
$2 \cdot (-1) = \blacksquare$	$-2 \cdot (-1) = \blacksquare$
$2 \cdot (-2) = \blacksquare$	$-2 \cdot (-2) = \blacksquare$

You may not know the answer, but you can make a conjecture.

© MATHEMATICAL PRACTICES

Lesson Vocabulary
- multiplicative inverse
- reciprocal

The patterns in the Solve It suggest rules for multiplying real numbers.

Essential Understanding The rules for multiplying real numbers are related to the properties of real numbers and the definitions of operations.

You know that the product of two positive numbers is positive. For example, $3(5) = 15$. You can think about the product of a positive number and a negative number in terms of groups of numbers. For example, $3(-5)$ means 3 groups of -5. So, $3(-5) = (-5) + (-5) + (-5)$, or $3(-5) = -15$.

You can also derive the product of two negative numbers, such as $-3(-5)$.

$3(-5) = -15$	Start with the product $3(-5) = -15$.
$-[3(-5)] = -(-15)$	The opposites of two equal numbers are equal.
$-1[3(-5)] = -(-15)$	Multiplication Property of -1
$[-1(3)](-5) = -(-15)$	Associative Property of Multiplication
$-3(-5) = -(-15)$	Multiplication Property of -1
$-3(-5) = 15$	The opposite of -15 is 15.

These discussions illustrate the following rules for multiplying real numbers.
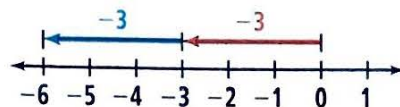
take note

Key Concept   Multiplying Real Numbers

Words The product of two real numbers with *different* signs is *negative*.

Examples $2(-3) = -6$ $-2 \cdot 3 = -6$

Model $2(-3) = -6$

Words The product of two real numbers with the *same* sign is *positive*.

Examples $2 \cdot 3 = 6$ $-2(-3) = 6$

Model $2 \cdot 3 = 6$

What is your first step in finding a product of real numbers?
Identify the signs of the factors. Then determine the sign of the product.

Problem 1   Multiplying Real Numbers

What is each product?

A $12(-8) = -96$ The product of two numbers with different signs is negative.

B $24(0.5) = 12$ The product of two numbers with the same sign is positive.

C $-\frac{3}{4} \cdot \frac{1}{2} = -\frac{3}{8}$ The product of two numbers with different signs is negative.

D $(-3)^2 = (-3)(-3) = 9$ The product of two numbers with the same sign is positive.

Got It? **1.** What is each product?

     a. $6(-15)$ **b.** $12(0.2)$ **c.** $-\frac{7}{10}\left(\frac{3}{5}\right)$ **d.** $(-4)^2$

Notice that $(-3)^2 = 9$ in part (d) of Problem 1. Recall from Lesson 1-3 that a is a square root of b if $a^2 = b$. So, -3 is a square root of 9. A negative square root is represented by $-\sqrt{\ }$. Every positive real number has a positive and a negative square root. The symbol $\pm$ in front of the radical indicates both square roots.

How can you find a negative square root?
Look for a negative number that you can multiply by itself to get a product that is equal to the radicand.

Problem 2   Simplifying Square Root Expressions

What is the simplified form of each expression?

A $-\sqrt{25} = -5$ $(-5)^2 = 25$, so $-\sqrt{25} = -5$.

B $\pm\sqrt{\frac{4}{49}} = \pm\frac{2}{7}$ $\left(\frac{2}{7}\right)^2 = \frac{4}{49}$ and $\left(-\frac{2}{7}\right)^2 = \frac{4}{49}$, so $\pm\sqrt{\frac{4}{49}} = \pm\frac{2}{7}$.

Got It? **2.** What is the simplified form of each expression?

     a. $\sqrt{64}$ **b.** $\pm\sqrt{16}$ **c.** $-\sqrt{121}$ **d.** $\pm\sqrt{\frac{1}{36}}$

Essential Understanding Rules for dividing real numbers are related to the rules for multiplying real numbers.

For any real numbers a, b, and c where $a \neq 0$, if $a \cdot b = c$, then $b = c \div a$. For instance, $-8(-2) = 16$, so $-2 = 16 \div (-8)$. Similarly $-8(2) = -16$, so $2 = -16 \div (-8)$. These examples illustrate the following rules.

take note

Key Concept   Dividing Real Numbers

Words   The quotient of two real numbers with *different* signs is *negative*.
Examples $-20 \div 5 = -4$ $20 \div (-5) = -4$

Words   The quotient of two real numbers with the *same* sign is *positive*.
Examples $20 \div 5 = 4$ $-20 \div (-5) = 4$

Division Involving 0

Words   The quotient of 0 and any nonzero real number is 0. The quotient of any real number and 0 is undefined.

Examples $0 \div 8 = 0$ $8 \div 0$ is undefined.

© **Problem 3** **Dividing Real Numbers**

How is dividing similar to multiplying?
You find the sign of a quotient using the signs of the numbers you're dividing, just as you find the sign of a product using the signs of the factors.

Sky Diving A sky diver's elevation changes by -3600 ft in 4 min after the parachute opens. What is the average change in the sky diver's elevation each minute?

$-3600 \div 4 = -900$ The numbers have different signs, so the quotient is negative.

The sky diver's average change in elevation is -900 ft per minute.

✓ **Got It? 3.** You make five withdrawals of equal amounts from your bank account. The total amount you withdraw is $360. What is the change in your account balance each time you make a withdrawal?

The Inverse Property of Multiplication describes the relationship between a number and its multiplicative inverse.

take note

Property   Inverse Property of Multiplication

Words   For every nonzero real number a, there is a **multiplicative inverse** $\frac{1}{a}$ such that $a\left(\frac{1}{a}\right) = 1$.

Examples   The multiplicative inverse of -4 is $-\frac{1}{4}$ because $-4\left(-\frac{1}{4}\right) = 1$.

The **reciprocal** of a nonzero real number of the form $\frac{a}{b}$ is $\frac{b}{a}$. The product of a number and its reciprocal is 1, so the reciprocal of a number is its multiplicative inverse. This suggests a rule for dividing fractions.

Here's Why It Works Let a, b, c, and d be nonzero integers.

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}}$$

Write the expression as a fraction.

$$= \frac{\frac{a}{b} \cdot \frac{d}{c}}{\frac{c}{d} \cdot \frac{d}{c}}$$

Multiply the numerator and denominator by $\frac{d}{c}$. Since this is equivalent to multiplying by 1, it does not change the quotient.

$$= \frac{\frac{a}{b} \cdot \frac{d}{c}}{1}$$

Inverse Property of Multiplication

$$= \frac{a}{b} \cdot \frac{d}{c}$$

Simplify.

This shows that dividing by a fraction is equivalent to **multiplying by the reciprocal of the fraction.**

© **Problem 4** Dividing Fractions

Multiple Choice What is the value of $\frac{x}{y}$ when $x = -\frac{3}{4}$ and $y = -\frac{2}{3}$?

Ⓐ $-\frac{9}{8}$ Ⓑ $-\frac{1}{2}$ Ⓒ $\frac{1}{2}$ Ⓓ $\frac{9}{8}$

Think	Write
Rewrite the expression.	$\frac{x}{y} = x \div y$
Substitute $-\frac{3}{4}$ for x and $-\frac{2}{3}$ for y.	$= -\frac{3}{4} \div \left(-\frac{2}{3}\right)$
Multiply by the reciprocal of $-\frac{2}{3}$.	$= -\frac{3}{4} \cdot \left(-\frac{3}{2}\right)$
Simplify. Since both factors are negative, the product is positive.	$= \frac{9}{8}$

The correct answer is D.

© ✓ **Got It?** **4. a.** What is the value of $\frac{3}{4} \div \left(-\frac{5}{2}\right)$?

        b. Reasoning Is $\frac{3}{4} \div \left(-\frac{5}{2}\right)$ equivalent to $-\left(\frac{3}{4} \div \frac{5}{2}\right)$? Explain.

Lesson Check

Do you know HOW?

Find each product. Simplify, if necessary.

1. $-3(-12)$

2. $\frac{5}{8}\left(-\frac{2}{8}\right)$

Find each quotient. Simplify, if necessary.

3. $-48 \div 3$

4. $-\frac{9}{10} \div \left(-\frac{4}{5}\right)$

Do you UNDERSTAND?

MATHEMATICAL PRACTICES

5. Vocabulary What is the reciprocal of $-\frac{1}{5}$?

6. Reasoning Use a number line to explain why $-15 \div 3 = -5$.

7. Reasoning Determine how many real square roots each number has. Explain your answers.

a. 49

b. 0

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

◀ See Problem 1.

Ⓐ Practice Find each product. Simplify, if necessary.

8. $-8(12)$

9. $8(12)$

10. $7(-9)$

11. $5 \cdot 4.1$

12. $-7 \cdot 1.1$

13. $10(-2.5)$

14. $6\left(-\frac{1}{4}\right)$

15. $-\frac{1}{9}\left(-\frac{3}{4}\right)$

16. $-\frac{3}{7} \cdot \frac{9}{10}$

17. $-\frac{2}{11}\left(-\frac{11}{2}\right)$

18. $\left(-\frac{2}{9}\right)^2$

19. $(-1.2)^2$

◀ See Problem 2.

Simplify each expression.

20. $\sqrt{400}$

21. $\sqrt{169}$

22. $-\sqrt{16}$

23. $-\sqrt{900}$

24. $\sqrt{\frac{36}{49}}$

25. $-\sqrt{\frac{25}{81}}$

26. $-\sqrt{\frac{1}{9}}$

27. $-\sqrt{\frac{121}{16}}$

28. $\pm\sqrt{1.96}$

29. $\pm\sqrt{0.25}$

◀ See Problem 3.

Find each quotient. Simplify, if necessary.

30. $48 \div 3$

31. $-84 \div 14$

32. $-39 \div (-13)$

33. $\frac{63}{-21}$

34. $-46 \div (-2)$

35. $-8.1 \div 9$

36. $\frac{-121}{11}$

37. $75 \div (-0.3)$

STEM **38. Scuba Diving** A scuba diver's vertical position in relation to the surface of the water changes by -90 ft in 3 min. What is the average change in the diver's vertical position each minute?

39. Part-Time Job You earn the same amount each week at your part-time job. The total amount you earn in 4 weeks is $460. How much do you earn per week?

◀ See Problem

Find each quotient. Simplify, if necessary.

40. $20 \div \frac{1}{4}$

41. $-5 \div \left(-\frac{5}{3}\right)$

42. $\frac{9}{10} \div \left(-\frac{4}{5}\right)$

43. $-\frac{12}{13} \div \frac{12}{13}$

Find the value of the expression $\frac{x}{y}$ for the given values of x and y. Write your answer in the simplest form.

44. $x = -\frac{2}{3}; y = -\frac{1}{4}$

45. $x = -\frac{5}{6}; y = \frac{3}{5}$

46. $x = \frac{2}{7}; y = -\frac{20}{21}$

47. $x = \frac{3}{8}; y = \frac{3}{4}$

© **48. Think About a Plan** A lumberjack cuts 7 pieces of equal length from a log, as shown at the right. What is the change in the log's length after 7 cuts?

$2\frac{1}{4}$ ft

- What operation can you use to find the answer?
- Will your answer be a positive value or a negative value? How do you know?

49. Farmer's Market A farmer has 120 bushels of beans for sale at a farmer's market. He sells an average of $15\frac{3}{4}$ bushels each day. After 6 days, what is the change in the total number of bushels the farmer has for sale at the farmer's market?

50. Stocks The price per share of a stock changed by −$4.50 on each of 5 consecutive days. If the starting price per share was $67.50, what was the ending price?

© **Open-Ended** Write an algebraic expression that uses x, y, and z and simplifies to the given value when $x = -3$, $y = -2$, and $z = -1$. The expression should involve only multiplication or division.

51. −16 **52.** 1 **53.** 12

Evaluate each expression for $m = -5$, $n = \frac{3}{2}$, and $p = -8$.

54. $-7m - 10n$ **55.** $-3mnp$

56. $8n \div (-6p)$ **57.** $2p^2(-n) \div m$

58. Look for a Pattern Extend the pattern in the diagram to six factors of −2. What rule describes the sign of the product based on the number of negative factors?

$-2(-2) = 4$
$-2(-2)(-2) = -8$
$-2(-2)(-2)(-2) = 16$

STEM **59. Temperature** The formula $F = \frac{9}{5}C + 32$ changes a temperature reading from the Celsius scale C to the Fahrenheit scale F. What is the temperature measured in degrees Fahrenheit when the Celsius temperature is −25°C?

© **60. Reasoning** Suppose a and b are integers. Describe what values of a and b make the statement true.
 a. Quotient $\frac{a}{b}$ is positive.
 b. Quotient $\frac{a}{b}$ is negative.
 c. Quotient $\frac{a}{b}$ is equal to 0.
 d. Quotient $\frac{a}{b}$ is undefined.

© **61. Writing** Explain how to find the quotient of $-1\frac{2}{3}$ and $-2\frac{1}{2}$.

© **62. Reasoning** Do you think a negative number raised to an even power will be positive or negative? Explain.

63. History The Rhind Papyrus is one of the best-known examples of Egyptian mathematics. One problem solved on the Rhind Papyrus is $100 \div 7\frac{7}{8}$. What is the solution of this problem?

© 64. Error Analysis Describe and correct the error in dividing the fractions at the right.

$$-\frac{3}{4} \div \frac{2}{5} = -\frac{4}{3} \cdot \frac{2}{5}$$

$$= -\frac{8}{15} \;\text{✗}$$

© 65. Reasoning You can derive the rule for division involving 0 shown on page 40.
 a. Suppose $0 \div x = y$, where $x \neq 0$. Show that $y = 0$. (*Hint*: If $0 \div x = y$, then $x \cdot y = 0$ by the definition of division.)
 b. If $x \neq 0$, show that there is no value of y such that $x \div 0 = y$. (*Hint*: Suppose there is a value of y such that $x \div 0 = y$. What would this imply about x?)

© Challenge Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

66. The product of a number and its reciprocal is -1.

67. The quotient of a nonzero number and its opposite is -1.

68. If the product of two fractions is negative, then their quotient is positive.

© 69. Reasoning What is the greatest integer n for which $(-n)^3$ is positive and the value of the expression has a 2 in the ones place?

Standardized Test Prep

70. Which expression does NOT have the same value as $-11 + (-11) + (-11)$?

  Ⓐ -33 Ⓑ $3(-11)$ Ⓒ $(-11)^3$ Ⓓ $33 - 66$

71. Miguel measured the area of a piece of carpet and figured out that the approximate error was $3|-0.2|$. What is the decimal form of $3|-0.2|$?

  Ⓕ -0.6 Ⓖ -0.06 Ⓗ 0.06 Ⓘ 0.6

72. What is the perimeter of the triangle shown?

  Ⓐ $6y + 24$

  Ⓑ $21y + 9$

  Ⓒ $15y + 15$

  Ⓓ $30y$
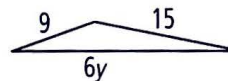
Mixed Review

Find each difference.

73. $46 - 16$ **74.** $34 - 44$ **75.** $-37 - (-27)$

Get Ready! To prepare for Lesson 1-7, do Exercises 76–78.

Name the property that each statement illustrates.

76. $-x + 0 = -x$ **77.** $13(-11) = -11(13)$ **78.** $-5 \cdot (m \cdot 8) = (-5 \cdot m) \cdot 8$