

1-7

The Distributive Property

Common Core State Standards

A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MP 1, MP 2, MP 3, MP 4, MP 6, MP 7

Objective To use the Distributive Property to simplify expressions

There's more than one way to figure this out.



Getting Ready!

In your favorite video game, you rotate shapes as they fall to make them fit together in a rectangle. When you complete an entire row, you score 1 point for each square in that row.

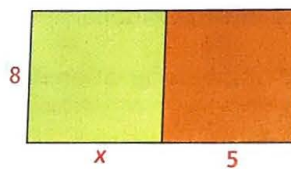
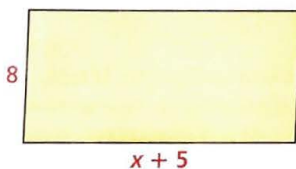
The screen at the right shows your latest game in pause mode. Using only the shapes shown, what is the maximum possible score for this game? Explain your reasoning.



Lesson Vocabulary

- Distributive Property
- term
- constant
- coefficient
- like terms

To solve problems in mathematics, it is often useful to rewrite expressions in simpler forms. The **Distributive Property**, illustrated by the area model below, is another property of real numbers that helps you to simplify expressions.



The model shows that $8(x + 5) = 8(x) + 8(5)$.

Essential Understanding You can use the Distributive Property to simplify the product of a number and a sum or difference.

take note

Property Distributive Property

Let a , b , and c be real numbers.

Algebra

$$\begin{aligned} a(b + c) &= ab + ac \\ (b + c)a &= ba + ca \\ a(b - c) &= ab - ac \\ (b - c)a &= ba - ca \end{aligned}$$

Examples

$$\begin{aligned} 4(20 + 6) &= 4(20) + 4(6) \\ (20 + 6)4 &= 20(4) + 6(4) \\ 7(30 - 2) &= 7(30) - 7(2) \\ (30 - 2)7 &= 30(7) - 2(7) \end{aligned}$$

Think

How do you read expressions like $3(x + 8)$?
Read an expression inside parentheses as "the quantity." Read $3(x + 8)$ as "3 times the quantity x plus 8."

C Problem 1 Simplifying Expressions

What is the simplified form of each expression?

A $3(x + 8)$

$$\begin{aligned} 3(x + 8) &= 3(x) + 3(8) && \text{Distributive Property} \\ &= 3x + 24 && \text{Simplify.} \end{aligned}$$

B $(5b - 4)(-7)$

$$\begin{aligned} (5b - 4)(-7) &= 5b(-7) - 4(-7) \\ &= -35b + 28 \end{aligned}$$

Got It? 1. What is the simplified form of each expression?

a. $5(x + 7)$

b. $12\left(3 - \frac{1}{6}t\right)$

c. $(0.4 + 1.1c)3$

d. $(2y - 1)(-y)$

Recall that a fraction bar may act as a grouping symbol. A fraction bar indicates division. Any fraction $\frac{a}{b}$ can also be written as $a \cdot \frac{1}{b}$. You can use this fact and the Distributive Property to rewrite some fractions as sums or differences.

C Problem 2 Rewriting Fraction Expressions

What sum or difference is equivalent to $\frac{7x + 2}{5}$?

$$\frac{7x + 2}{5} = \frac{1}{5}(7x + 2) \quad \text{Write division as multiplication.}$$

$$= \frac{1}{5}(7x) + \frac{1}{5}(2) \quad \text{Distributive Property}$$

$$= \frac{7}{5}x + \frac{2}{5} \quad \text{Simplify.}$$

Got It? 2. What sum or difference is equivalent to each expression?

a. $\frac{4x - 16}{3}$

b. $\frac{11 + 3x}{6}$

c. $\frac{15 + 6x}{12}$

d. $\frac{4 - 2x}{8}$

Think

How can you get started?
Think of division as multiplying by the reciprocal. So change the division by 5 to multiplication by $\frac{1}{5}$.

The Multiplication Property of -1 states that $-1 \cdot x = -x$. To simplify an expression such as $-(x + 6)$, you can rewrite the expression as $-1(x + 6)$.

C Problem 3 Using the Multiplication Property of -1

Multiple Choice What is the simplified form of $-(2y - 3x)$?

A $2y + 3x$

B $-2y + (-3x)$

C $-2y + 3x$

D $2y - 3x$

$$-(2y - 3x) = -1(2y - 3x) \quad \text{Multiplication Property of } -1$$

$$= (-1)(2y) + (-1)(-3x) \quad \text{Distributive Property}$$

$$= -2y + 3x \quad \text{Simplify.}$$

The correct choice is C.

Got It? 3. What is the simplified form of each expression?

a. $-(a + 5)$

b. $-(-x + 31)$

c. $-(4x - 12)$

d. $-(6m - 9n)$

Think

What does the negative sign in front of the parentheses mean?
It indicates the opposite of the entire expression inside the parentheses.

You can use the Distributive Property to make calculations easier to do with mental math. Some numbers can be thought of as simple sums or differences.

Problem 4 Using the Distributive Property for Mental Math

Eating Out Deli sandwiches cost \$4.95 each. What is the total cost of 8 sandwiches? Use mental math.

Know

- Sandwiches cost \$4.95.
- You are buying 8 sandwiches.

Need

Total cost of 8 sandwiches

Plan

Express \$4.95 as a difference and use the Distributive Property.

The total cost is the product of the number of sandwiches you buy, 8, and the cost per sandwich, \$4.95.

$$\begin{aligned} 8(4.95) &= 8(5 - 0.05) && \text{Think of 4.95 as } 5 - 0.05. \\ &= 8(5) - 8(0.05) && \text{Distributive Property} \\ &= 40 - 0.4 && \text{Multiply mentally.} \\ &= 39.6 && \text{Subtract mentally.} \end{aligned}$$

The total cost for 8 sandwiches is \$39.60.

Think

How can you express decimals as simple sums and differences?

Think of a decimal as the sum or difference of its whole number portion and its decimal portion.



Got It? 4. Julia commutes to work on the train 4 times each week. A round-trip ticket costs \$7.25. What is her weekly cost for tickets? Use mental math.

Essential Understanding You can simplify an algebraic expression by combining the parts of the expression that are alike.

In an algebraic expression, a **term** is a number, a variable, or the product of a number and one or more variables. A **constant** is a term that has no variable. A **coefficient** is a numerical factor of a term. Rewrite expressions as sums to identify these parts of an expression.

$$6a^2 - 5ab + 3b - 12 = 6a^2 + (-5ab) + 3b + (-12)$$

$6a^2, -5ab, 3b,$ and -12 are terms.

↑ ↑ ↑ ↑
 coefficients constant

In the algebraic expression $6a^2 - 5ab + 3b - 12$, the terms have coefficients of 6, -5, and 3. The term -12 is a constant.

Like terms have the same variable factors. To identify like terms, compare the variable factors of the terms, as shown below.

Terms	$7a$ and $-3a$	$4x^2$ and $12x^2$	$6ab$ and $-2a$	xy^2 and x^2y
Variable Factors	a and a	x^2 and x^2	ab and a	xy^2 and x^2y
Like Terms?	yes	yes	no	no

An algebraic expression in simplest form has no like terms or parentheses.

Not Simplified	Simplified
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$2(3x - 5 + 4x)$	$14x - 10$
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You can use the Distributive Property to help combine like terms. Think of the Distributive Property as $ba + ca = (b + c)a$.

Plan

What terms can you combine?

You can combine any terms that have exactly the same variables with exactly the same exponents.

Problem 5 Combining Like Terms



What is the simplified form of each expression?

A $8x^2 + 2x^2$

$$\begin{aligned} 8x^2 + 2x^2 &= (8 + 2)x^2 && \text{Distributive Property} \\ &= 10x^2 && \text{Simplify.} \end{aligned}$$

B $5x - 3 - 3x + 6y + 4$

$$\begin{aligned} 5x - 3 - 3x + 6y + 4 &= 5x + (-3) + (-3x) + 6y + 4 && \text{Rewrite as a sum.} \\ &= 5x + (-3x) + 6y + (-3) + 4 && \text{Commutative Property} \\ &= (5 - 3)x + 6y + (-3) + 4 && \text{Distributive Property} \\ &= 2x + 6y + 1 && \text{Simplify.} \end{aligned}$$

-   **Got It?** 5. What is the simplified form of each expression in parts (a)–(c)?
- a. $3y - y$ b. $-7mn^4 - 5mn^4$ c. $7y^3z - 6yz^3 + y^3z$
- d. **Reasoning** Can you simplify $8x^2 - 2x^4 - 2x + 2 + xy$ further? Explain.



Lesson Check

Do you know HOW?

- What is the simplified form of each expression? Use the Distributive Property.
 - $(j + 2)7$
 - $-8(x - 3)$
 - $-(4 - c)$
 - $-(11 + 2b)$

Rewrite each expression as a sum.

- $-8x^2 + 3xy - 9x - 3$
- $2ab - 5ab^2 - 9a^2b$

Tell whether the terms are like terms.

- $3a$ and $-5a$
- $2xy^2$ and $-x^2y$

Do you UNDERSTAND?



- Vocabulary** Does each equation demonstrate the Distributive Property? Explain.
 - $-2(x + 1) = -2x - 2$
 - $(s - 4)8 = 8(s - 4)$
 - $5n - 45 = 5(n - 9)$
 - $8 + (t + 6) = (8 + t) + 6$
- Mental Math** How can you express 499 to find the product 499×5 using mental math? Explain.
- Reasoning** Is each expression in simplified form? Justify your answer.
 - $4xy^3 + 5x^3y$
 - $-(y - 1)$
 - $5x^2 + 12xy - 3yx$

A Practice

Use the Distributive Property to simplify each expression.

◀ See Problem

- | | | | |
|------------------------|----------------------|--|--|
| 9. $6(a + 10)$ | 10. $8(4 + x)$ | 11. $(5 + w)5$ | 12. $(2t + 3)11$ |
| 13. $10(9 - t)$ | 14. $12(2j - 6)$ | 15. $16(7b + 6)$ | 16. $(1 + 3d)9$ |
| 17. $(3 - 8c)1.5$ | 18. $(5w - 15)2.1$ | 19. $\frac{1}{4}(4f - 8)$ | 20. $6\left(\frac{1}{3}h + 1\right)$ |
| 21. $(-8z - 10)(-1.5)$ | 22. $0(3.7x - 4.21)$ | 23. $1\left(\frac{3}{11} - \frac{7d}{17}\right)$ | 24. $\frac{1}{2}\left(\frac{1}{2}y - \frac{1}{2}\right)$ |

Write each fraction as a sum or difference.

◀ See Problem 2

- | | | | |
|-------------------------|---------------------------|-------------------------|--------------------------|
| 25. $\frac{2x + 7}{5}$ | 26. $\frac{17 + 5n}{4}$ | 27. $\frac{8 - 9x}{3}$ | 28. $\frac{4y - 12}{2}$ |
| 29. $\frac{25 - 8t}{5}$ | 30. $\frac{18x + 51}{17}$ | 31. $\frac{22 - 2n}{2}$ | 32. $\frac{42w + 14}{7}$ |

Simplify each expression.

◀ See Problem 3

- | | | | |
|--------------------|--------------------|---------------------|---------------------|
| 33. $-(20 + d)$ | 34. $-(-5 - 4y)$ | 35. $-(9 - 7c)$ | 36. $-(-x + 15)$ |
| 37. $-(18a - 17b)$ | 38. $-(2.1c - 4d)$ | 39. $-(-m + n + 1)$ | 40. $-(x + 3y - 3)$ |

Use mental math to find each product.

◀ See Problem 4

- | | | | |
|--------------------|---------------------|---------------------|--------------------|
| 41. 5.1×8 | 42. 3×7.25 | 43. 299×3 | 44. 4×197 |
| 45. 3.9×6 | 46. 5×2.7 | 47. 6.15×4 | 48. 6×9.1 |

49. You buy 50 of your favorite songs from a Web site that charges \$.99 for each song. What is the cost of 50 songs? Use mental math.

50. The perimeter of a baseball diamond is about 360 ft. If you take 12 laps around the diamond, what is the total distance you run? Use mental math.

51. One hundred and five students see a play. Each ticket costs \$45. What is the total amount the students spend for tickets? Use mental math.

52. Suppose the distance you travel to school is 5 mi. What is the total distance for 197 trips from home to school? Use mental math.

Simplify each expression by combining like terms.

◀ See Problem 5

- | | | |
|-----------------------|---------------------------|--------------------------|
| 53. $11x + 9x$ | 54. $8y - 7y$ | 55. $5t - 7t$ |
| 56. $-n + 4n$ | 57. $5w^2 + 12w^2$ | 58. $2x^2 - 9x^2$ |
| 59. $-4y^2 + 9y^2$ | 60. $6c - 4 + 2c - 7$ | 61. $5 - 3x + y + 6$ |
| 62. $2n + 1 - 4m - n$ | 63. $-7h + 3h^2 - 4h - 3$ | 64. $10ab + 2ab^2 - 9ab$ |

B Apply

Write a word phrase for each expression. Then simplify each expression.

- | | | |
|----------------|----------------|---------------------------|
| 65. $3(t - 1)$ | 66. $4(d + 7)$ | 67. $\frac{1}{3}(6x - 1)$ |
|----------------|----------------|---------------------------|

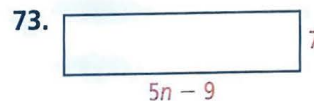
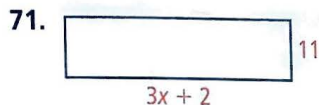
STEM 68. Physiology The recommended heart rate for exercise, in beats per minute, is given by the expression $0.8(200 - y)$ where y is a person's age in years. Rewrite this expression using the Distributive Property. What is the recommended heart rate for a 20-year-old person? For a 50-year-old person? Use mental math.

69. Error Analysis Identify and correct the error shown at the right.

$$\begin{aligned} \cancel{4(x+5)} &= \cancel{(4 \cdot x)(4 \cdot 5)} \\ &= \cancel{80x} \end{aligned}$$

70. Error Analysis A friend uses the Distributive Property to simplify $4(2b - 5)$ and gets $8b - 5$ as the result. Describe and correct the error.

Geometry Write an expression in simplified form for the area of each rectangle.



74. Think About a Plan You are replacing your regular shower head with a water-saving shower head. These shower heads use the amount of water per minute shown. If you take an 8-min shower, how many gallons of water will you save?

- Which would you use to represent water saved each minute, an expression involving addition or an expression involving subtraction?
- How can you use the Distributive Property to find the total amount of water saved?



Simplify each expression.

75. $6yz + 2yz - 8yz$

76. $-2ab + ab + 9ab - 3ab$

77. $-9m^3n + 4m^3n + 5mn$

78. $3(-4cd - 5)$

79. $12x^2y - 8x^2y^2 + 11x^2y - 4x^3y^2 - 9xy^2$

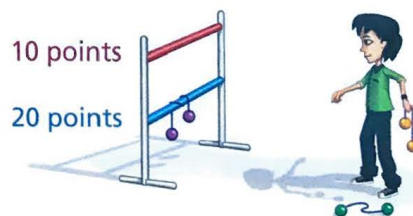
80. $a - \frac{a}{4} + \frac{3}{4}a$

81. Reasoning The Distributive Property also applies to division, as shown.

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

Use the Distributive Property of Division to rewrite $\frac{9 + 12n}{3}$. Then simplify.

82. Lawn Game You play a game where you throw a pair of connected balls at a structure, as shown at the right. When a pair wraps around a bar, you earn the points shown. You toss 3 pairs, and all of them wrap around a bar. Which expression could represent your total score if a pairs of balls wrap around the blue bar?




A $30 + 10a$

C $10a + 20(3 - a)$

B $20a + 3 - 10a$

D $30a + 10$

-  **83. Open-Ended** Suppose you used the Distributive Property to get the expression $3m - 6n - 15$. With what expression could you have started?

-  **Challenge** **84. Writing** Your friend uses the order of operations to find the value of $11(39 - 3)$. Would you prefer to use the Distributive Property instead? Explain.

Simplify each expression.

85. $5(2d + 1) + 7(5d + 3)$

86. $6(4t - 3) + 6(4 - 3t)$

87. $9(5 + t) - 7(t + 3)$

88. $4(r + 8) - 5(2r - 1)$

89. $-(m + 9n - 12)$

90. $-6(3 - 3x - 7y) + 2y - x$



Apply What You've Learned



Look back at the information on page 3 about the walk of fame Naomi is designing. In the Apply What You've Learned in Lesson 1-1, you determined the relationship between n , the number of names on each side of the walk, and each type of tile in the design. You wrote algebraic expressions to represent these relationships.

- Using the expression you wrote for the number of tiles with names inscribed on them, write and simplify an expression for the total cost of the inscribed tiles. List the properties you used to simplify the expression.
- Using the expression you wrote for the number of plain tiles, write and simplify an expression for the total cost of the plain tiles. List the properties you used to simplify the expression.
- Write and simplify an expression for the total cost of all of the tiles. Show your work.