

1-8

An Introduction to Equations

Common Core State Standards

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems.

MP 1, MP 3, MP 4, MP 6, MP 7, MP 8

Objective To solve equations using tables and mental math

The diagrams show one way to look at the problem. Try to think of other ways you could represent it.



Getting Ready!

An animal shelter has a fundraising goal of \$7500. The diagram shows the results for the first three weeks. The first amount is negative due to upfront costs. When will the shelter reach its goal? Make an estimate. State any assumptions and justify your reasoning.



The problem in the Solve It can be modeled by an equation. An **equation** is a mathematical sentence that uses an equal sign (=).

**Lesson Vocabulary**

- equation
- open sentence
- solution of an equation

Essential Understanding You can use an equation to represent the relationship between two quantities that have the same value.

An equation is true if the expressions on either side of the equal sign are equal ($1 + 1 = 2$, $x + x = 2x$). An equation is false if the expressions on either side of the equal sign are not equal ($1 + 1 = 3$, $x + x = 3x$). An equation is an **open sentence** if it contains one or more variables and may be true or false depending on the values of its variables.

Plan

How do you classify an equation?

If an equation contains only numbers, simplify the expressions on either side to determine if they are equal. If there is a variable in the equation, it is open.

**Problem 1 Classifying Equations**

Is the equation *true*, *false*, or *open*? Explain.

A $24 + 18 = 20 + 22$ True, because both expressions equal 42

B $7 \cdot 8 = 54$ False, because $7 \cdot 8 = 56$ and $56 \neq 54$

C $2x - 14 = 54$ Open, because there is a variable



Got It? 1. Is the equation *true*, *false*, or *open*? Explain.

a. $3y + 6 = 5y - 8$

b. $16 - 7 = 4 + 5$

c. $32 \div 8 = 2 \cdot 3$

A **solution of an equation** containing a variable is a value of the variable that makes the equation true.

Plan

How can you tell if a number is a solution of an equation?

Substitute the number for the variable in the equation. Simplify each side to see if you get a true statement.

Problem 2 Identifying Solutions of an Equation

Is $x = 6$ a solution of the equation $32 = 2x + 12$?

$$32 = 2x + 12$$

$$32 \stackrel{?}{=} 2(6) + 12 \quad \text{Substitute 6 for } x.$$

$$32 \neq 24 \quad \text{Simplify.}$$

No, $x = 6$ is not a solution of the equation $32 = 2x + 12$.

Got It? 2. Is $m = \frac{1}{2}$ a solution of the equation $6m - 8 = -5$?

In real-world problems, the word *is* can indicate equality. You can represent some real-world situations using an equation.

Problem 3 Writing an Equation

Multiple Choice An art student wants to make a model of the Mayan Great Ball Court in Chichén Itzá, Mexico. The length of the court is 2.4 times its width. The length of the student's model is 54 in. What should the width of the model be?

- A 2.4 in. C 22.5 in.
 B 11.25 in. D 129.6 in.

Relate The length is 2.4 times the width

Define Let w = the width of the model.

Write $54 = 2.4 \cdot w$

Test each answer choice in the equation to see if it is a solution.

Check A:

$$54 = 2.4w$$

$$54 \stackrel{?}{=} 2.4(2.4)$$

$$54 \neq 5.76$$

Check B:

$$54 = 2.4w$$

$$54 \stackrel{?}{=} 2.4(11.25)$$

$$54 \neq 27$$

Check C:

$$54 = 2.4w$$

$$54 \stackrel{?}{=} 2.4(22.5)$$

$$54 = 54 \checkmark$$

Check D:

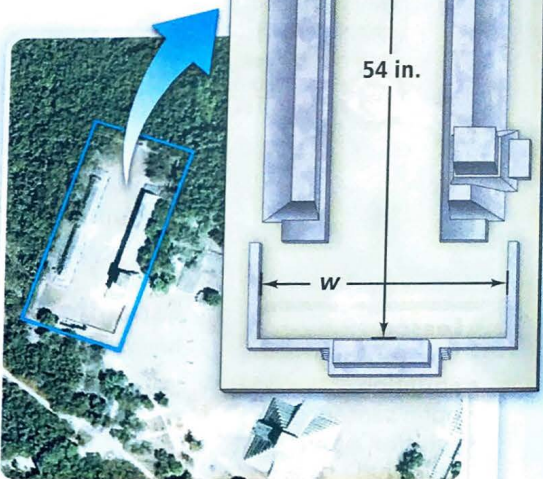
$$54 = 2.4w$$

$$54 \stackrel{?}{=} 2.4(129.6)$$

$$54 \neq 311.04$$

The correct answer is C.

Got It? 3. The length of the ball court at La Venta is 14 times the height of its walls. Write an equation that can be used to find the height of a model that has a length of 49 cm.



Satellite view of Chichén Itzá

Plan

Why do you need to test each answer choice?

You should test each answer choice in case you made a calculation error. If you get two correct answers, then you know you need to double-check your work.

Plan

How can you find the solution of an equation?

You can use mental math to find a value that makes the equation true.

Problem 4 Using Mental Math to Find Solutions

What is the solution of each equation? Use mental math.

	Think	Solution	Check
A $x + 8 = 12$	What number plus 8 equals 12?	4	$4 + 8 = 12$ ✓
B $\frac{a}{8} = 9$	What number divided by 8 equals 9?	72	$\frac{72}{8} = 9$ ✓

Got It? 4. What is the solution of $12 - y = 3$? Use mental math.

Think

How can you start?

You can use mental math to quickly check values like 0, 1, and 10. Use these results to choose a reasonable starting value for your table.

Problem 5 Using a Table to Find a Solution

What is the solution of $5n + 8 = 48$? Use a table.

Make a table of values. Choose a starting value using mental math. $5(1) + 8 = 13$ and $5(10) + 8 = 58$, so 1 is too low and 10 is too high.

n	$5n + 8$	Value of $5n + 8$
5	$5(5) + 8$	33
6	$5(6) + 8$	38
7	$5(7) + 8$	43
8	$5(8) + 8$	48

Try $n = 5$ and $n = 6$.

The value of $5n + 8$ increases as n increases, so try greater values of n .

When $n = 8$, $5n + 8 = 48$. So the solution is 8.

Got It? 5. a. What is the solution of $25 - 3p = 55$? Use a table.
b. What is a good starting value to solve part (a)? Explain your reasoning.

Problem 6 Estimating a Solution

What is an estimate of the solution of $-9x - 5 = 28$? Use a table.

To estimate the solution, find the integer values of x between which the solution must lie. $-9(0) - 5 = -5$ and $-9(1) - 5 = -14$. If you try greater values of x , the value of $-9x - 5$ gets farther from 28.

Think

Can identifying a pattern help you make an estimate?

Yes. Identify how the value of the expression changes as you substitute for the variable. Use the pattern you find to work toward the desired value.

x	$-9x - 5$	Value of $-9x - 5$
-1	$-9(-1) - 5$	4
-2	$-9(-2) - 5$	13
-3	$-9(-3) - 5$	22
-4	$-9(-4) - 5$	31

Try lesser values, such as $x = -1$ and $x = -2$.

Now the values of $-9x - 5$ are getting closer to 28.

28 is between 22 and 31, so the solution is between -3 and -4.

Got It? 6. What is the solution of $3x + 3 = -22$? Use a table.

Lesson Check

Do you know HOW?

1. Is $y = -9$ a solution of $y + 1 = 8$?
2. What is the solution of $x - 3 = 12$? Use mental math.
3. **Reading** You can read 1.5 pages for every page your friend can read. Write an equation that relates the number of pages p that you can read and the number of pages n that your friend can read.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** Give an example of an equation that is true, an equation that is false, and an open equation.
5. **Open-Ended** Write an open equation using one variable and division.
6. **Compare and Contrast** Use two different methods to find the solution of the equation $x + 4 = 13$. Which method do you prefer? Explain.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

Practice

Tell whether each equation is *true*, *false*, or *open*. Explain.

 See Problem 1.

- | | | |
|---------------------------|------------------------------------|--|
| 7. $85 + (-10) = 95$ | 8. $225 \div t - 4 = 6.4$ | 9. $29 - 34 = -5$ |
| 10. $-8(-2) - 7 = 14 - 5$ | 11. $4(-4) \div (-8)6 = -3 + 5(3)$ | 12. $91 \div (-7) - 5 = 35 \div 7 + 3$ |
| 13. $4a - 3b = 21$ | 14. $14 + 7 + (-1) = 21$ | 15. $5x + 7 = 17$ |

Tell whether the given number is a solution of each equation.

 See Problem 2.

- | | | |
|------------------------------------|----------------------------------|--|
| 16. $8x + 5 = 29$; 3 | 17. $5b + 1 = 16$; -3 | 18. $6 = 2n - 8$; 7 |
| 19. $2 = 10 - 4y$; 2 | 20. $9a - (-72) = 0$; -8 | 21. $-6b + 5 = 1$; $\frac{1}{2}$ |
| 22. $7 + 16y = 11$; $\frac{1}{4}$ | 23. $14 = \frac{1}{3}x + 5$; 27 | 24. $\frac{3}{2}t + 2 = 4$; $\frac{2}{3}$ |

Write an equation for each sentence.

 See Problem 3.

25. The sum of $4x$ and -3 is 8.
26. The product of 9 and the sum of 6 and x is 1.
27. **Training** An athlete trains for 115 min each day for as many days as possible. Write an equation that relates the number of days d that the athlete spends training when the athlete trains for 690 min.
28. **Salary** The manager of a restaurant earns \$2.25 more each hour than the host of the restaurant. Write an equation that relates the amount h that the host earns each hour when the manager earns \$11.50 each hour.

Use mental math to find the solution of each equation.

 See Problem 4.

- | | | |
|------------------|-----------------------|-----------------------|
| 29. $x - 3 = 10$ | 30. $4 = 7 - y$ | 31. $18 + d = 24$ |
| 32. $2 - x = -5$ | 33. $\frac{m}{3} = 4$ | 34. $\frac{x}{7} = 5$ |
| 35. $6t = 36$ | 36. $20a = 100$ | 37. $13c = 26$ |

Use a table to find the solution of each equation.

◀ See Problem 5.

38. $2t - 1 = 11$

39. $5x + 3 = 23$

40. $0 = 4 + 2y$

41. $8a - 10 = 38$

42. $12 = 6 - 3b$

43. $8 - 5w = -12$

44. $-48 = -9 - 13n$

45. $\frac{1}{2}x - 5 = -1$

Use a table to find two consecutive integers between which the solution lies.

◀ See Problem 6.

46. $6x + 5 = 81$

47. $3.3 = 1.5 - 0.4y$

48. $-115b + 80 = -489$

B Apply

49. **Bicycle Sales** In the United States, the number y (in millions) of bicycles sold with wheel sizes of 20 in. or greater can be modeled by the equation $y = 0.3x + 15$, where x is the number of years since 1981. In what year were about 22 million bicycles sold?

© 50. **Error Analysis** A student checked whether $d = -2$ is a solution of $-3d + (-4) = 2$, as shown. Describe and correct the student's error.

~~$$\begin{aligned} -3d + (-4) &= 2 \\ -3(-2) + (-4) &\stackrel{?}{=} 2 \\ -6 + (-4) &\stackrel{?}{=} 2 \\ -10 &\neq 2 \quad \times \end{aligned}$$~~

© 51. **Writing** What are the differences between an expression and an equation? Does a mathematical expression have a solution? Explain.

52. **Basketball** A total of 1254 people attend a basketball team's championship game. There are six identical benches in the gymnasium. About how many people would you expect each bench to seat?

Find the solution of each equation using mental math or a table. If the solution lies between two consecutive integers, identify those integers.

53. $x + 4 = -2$

54. $4m + 1 = 9$

55. $10.5 = 3n - 1$

56. $-3 + t = 19$

57. $5a - 4 = -16$

58. $9 = 4 + (-y)$

59. $1 = -\frac{1}{4}n + 1$

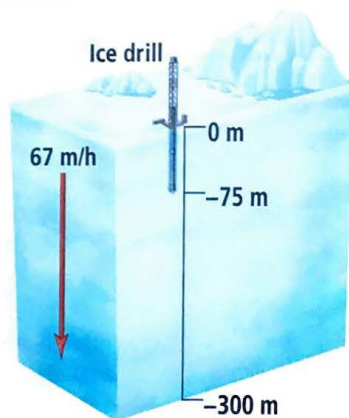
60. $17 = 6 + 2x$

© 61. **Open-Ended** Give three examples of equations that involve multiplication and subtraction and have a solution of -4 .

© 62. **Think About a Plan** Polar researchers drill into an ice sheet. The drill is below the surface at the location shown. The drill advances at a rate of 67 m/h. About how many hours will it take the drill to reach a depth of 300 m?

- What equation models this situation?
- What integers do you need?

63. **Deliveries** The equation $25 + 0.25p = c$ gives the cost c in dollars that a store charges to deliver an appliance that weighs p pounds. Use the equation and a table to find the weight of an appliance that costs \$55 to deliver.



© 64. **Look for a Pattern** Use a table. Evaluate $2x + 2$ for $x = -2, -1, 0, 1, 2$, and 3 . What pattern do you notice in your results? Use this pattern to find the solution of $2x + 2 = 28$. Check your solution.

**Challenge**

65. Reasoning Your friend says that the solution of $15 = 4 + 2t$ is between two consecutive integers, because 15 is an odd number and 4 and 2 are both even numbers. Explain your friend's reasoning.



66. Construction A construction crew needs to install 550 ft of curbing along a street. The crew can install curbing at a rate of 32 ft/h. Yesterday the crew installed 272 ft of curbing. Today it wants to finish the job in at most 10 h, which includes a 15-min drive to the job, an hour lunch break, and 45 min to break down the equipment. Can the crew achieve its goal? Explain.

**Apply What You've Learned****MATHEMATICAL PRACTICES****MP 4**

Look back at the information on page 3 about the walk of fame Naomi is designing, and at your work in the Apply What You've Learned sections in Lessons 1-1 and 1-7. Choose from the following numbers and expressions to complete the sentences below.

$n + 2$	14	10	$30n$	56
$5n + 10$	28	8	16	44

- An equation that can be used to find the value of n that results in a walk costing \$500 is $\underline{\quad} + \underline{\quad} = 500$.
- Solving the equation shows that when n is equal to $\underline{\quad}$, the walkway will cost \$500.
- The number of plain tiles Naomi should buy is $\underline{\quad}$.