## $1-9$ Patterns, Equations, and Graphs

Common Core State Standards
A-REI.D. 10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Also A-CED.A. 2
MP 1, MP 2, MP 3, MP 4, MP 7

Objective To use tables, equations, and graphs to describe relationships

solution of an equation

- inductive reasoning

How can you tell whether an ordered pair is a solution? Replace $x$ with the first value in the ordered pair and $y$ with the second value in the ordered pair. Is the resulting equation true?

In the Solve It, you may have described the pattern using words. You can also use an equation or a graph to describe a pattern.

Essential Understanding Sometimes the value of one quantity can be found if you know the value of another. You can represent the relationship between the quantities in different ways, including tables, equations, and graphs.

You can use an equation with two variables to represent the relationship between two varying quantities. A solution of an equation with two variables $x$ and $y$ is any ordered pair $(x, y)$ that makes the equation true.

## Problem 1 Identifying Solutions of a Two-Variable Equation

Is $(3,10)$ a solution of the equation $y=4 x$ ?
$y=4 x$
$10 \stackrel{?}{=} 4 \cdot 3$ Substitute 3 for $x$ and 10 for $y$.
$10 \neq 12 \quad$ So, $(3,10)$ is not a solution of $y=4 x$.


Got $l i$ ? 1. Is the ordered pair a solution of the equation $y=4 x$ ?
a. $(5,20)$
b. $(-5,-20)$
c. $(-20,-5)$
d. $(1.5,6)$

Why does it make sense to connect the points on the graph? A person's age can be any positive real number, and the ages of the girls are always 2 years apart. So every point on the line makes sense in this situation.

You can represent the same relationship between two variables in several different ways.

## Problem 2 Using a Table, an Equation, and a Graph

Ages Both Carrie and her sister Kim were born on October 25, but Kim was born 2 years before Carrie. How can you represent the relationship between Carrie's age and Kim's age in different ways?

## Know

Kim was born 2 years before Carrie.

## Need

Different ways to represent the relationship

## Plan

Use a table, an equation, and a graph.

Step 1 Make a table.

> Carrie's and Kim's Ages (years)

| Carrie's Age | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| Kim's Age | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

Step 2 Write an equation.
Let $x=$ Carrie's age. Let $y=$ Kim's age. From the table, you can see that $y$ is always 2 greater than $x$.
So $y=x+2$.
Step 3 Draw a graph.


Got lt? 2. a. Will runs 6 laps before Megan joins him at the track. They then run together at the same pace. How can you represent the relationship between the number of laps Will runs and the number of laps Megan runs in different ways? Use a table, an equation, and a graph.
b. Reasoning Describe how the graph in Problem 2 above would change if the difference in ages were 5 years instead of 2 years.
should you connect the points on the graph with a solid line? No. The number of tiles must be a whole number. use a dotted line to see the trend.

Inductive reasoning is the process of reaching a conclusion based on an observed pattern. You can use inductive reasoning to predict values.

## Problem 3 Extending a Pattern

The table shows the relationship between the number of blue tiles and the total number of tiles in each figure. Extend the pattern. What is the total number of tiles in a figure with 8 blue tiles?


Tiles


Method 1 Draw a graph.
Number of Tiles


Step 2 The points fall on a line. Extend the pattern with a dashed line.

Step 3 Find the point on the line with $x$-coordinate 8. The $y$-coordinate of this point is 72 .

The total number of tiles is 72 .

Method 2 Write an equation.

$$
\begin{aligned}
y & =9 x & & \text { The total number of tiles is } 9 \text { times the number of blue tiles. } \\
& =9(8) & & \text { Substitute } 8 \text { for } x . \\
& =72 & & \text { Simplify. }
\end{aligned}
$$

The total number of tiles is 72 .

Got lt? 3. Use the tile figure from Problem 3.
a. Make a table showing the number of orange tiles and the total number of tiles in each figure. How many tiles in all will be in a figure with 24 orange tiles?
b. Make a table showing the number of blue tiles and the number of yellow tiles in each figure. How many yellow tiles will be in a figure with 24 blue tiles?

## Lesson Check

## Do you know HOW?

1. Is $(2,4)$ a solution of the equation $y=x-2$ ?
2. Is $(-3,-9)$ a solution of the equation $y=3 x$ ?
3. Drinks at the fair cost $\$ 2.50$. Use a table, an equation, and a graph to represent the relationship between the number of drinks bought and the cost.
4. Exercise On a treadmill, you burn 11 Cal in 1 min , 22 Cal in $2 \mathrm{~min}, 33 \mathrm{Cal}$ in 3 min , and so on. How many Calories do you burn in 10 min ?

## Do you UNDERSTAND?

5. Vocabulary Describe the difference between inductive reasoning and deductive reasoning.6. Compare and Contrast How is writing an equation to represent a situation involving two variables similar to writing an equation to represent a situation involving only one variable? How are they different?
7. Reasoning Which of $(3,5),(4,6),(5,7)$, and $(6,8)$ are solutions of $y=x+2$ ? What is the pattern in the solutions of $y=x+2$ ?

## Practice and Problem-Solving Exercises

Tell whether the given equation has the ordered pair as a solution.
8. $y=x+6 ;(0,6)$
9. $y=1-x ;(2,1)$
10. $y=-x+3 ;(4,1)$
11. $y=6 x ;(3,16)$
12. $-x=y ;(-3.1,3.1)$
13. $y=-4 x ;(-2,8)$
14. $y=x+\frac{2}{3} ;\left(1, \frac{1}{3}\right)$
15. $y=x-\frac{3}{4} ;\left(2,1 \frac{1}{4}\right)$
16. $\frac{x}{5}=y ;(-10,-2)$

Use a table, an equation, and a graph to represent each relationship.
See Problem 2.
17. Ty is 3 years younger than Bea.
18. The number of checkers is 24 times the number of checkerboards.
19. The number of triangles is $\frac{1}{3}$ the number of sides.
20. Gavin makes $\$ 8.50$ for each lawn he mows.

Use the table to draw a graph and answer the question.
21. The table shows the height in inches of stacks of tires. Extend the pattern. What is the height of a stack of 7 tires?

| Stacks of Tires |  |
| :---: | :---: |
| Number <br> of Tires, $x$ | Height of <br> Stack, $y$ |
| 1 | 8 |
| 2 | 16 |
| 3 | 24 |
| 4 | 32 |

22. The table shows the length in centimeters of a scarf you are knitting. Suppose the pattern continues. How long is the scarf after 8 days?

| Knitted Scarf |  |
| :---: | :---: |
| Number <br> of Days, $x$ | Length of <br> Scarf, $y$ |
| 1 | 12.5 |
| 2 | 14.5 |
| 3 | 16.5 |
| 4 | 18.5 |

Use the table to write an equation and answer the question.
23. The table shows the heights in inches of trees after they have been planted. What is the height of a tree that is 64 in . tall in its pot?

| Tree Height |  |
| :---: | :---: |
| Height in Pot, $x$ | Height Without Pot, $y$ |
| 30 | 18 |
| 36 | 24 |
| 42 | 30 |
| 48 | 36 |

24. The table shows amounts earned for pet sitting. How much is earned for a 9-day job?
Pet Sitting

| Days, $x$ | Dollars, $y$ |
| :---: | :---: |
| 1 | 17 |
| 2 | 34 |
| 3 | 51 |
| 4 | 68 |

## Refer to the drawing of houses for Exercises 25 and 26.


25. Patterns Make a table and draw a graph to show the relationship between the number of houses and the number of windows.
a. What is the number of windows in 9 houses?
b. If $n$ houses have $k$ windows, write an expression to represent the number of windows for $n+1$ houses.
26. Bobby says that a subdivision similar to the one above has a total of 202 windows. Is 202 a reasonable number of windows? Explain.

Tell whether the given ordered pair is a solution of the equation.
27. $y=2 x+7 ;(-2,3)$
28. $-\frac{1}{4} x+6=y ;(2,4)$
29. $y=-1.2 x-2.6 ;(3.5,6.8)$
30. Think About a Plan The table shows how long it takes Kayla to learn new songs. How many hours does Kayla need to practice to learn 9 songs?

- From row to row, how much does the number of hours $h$ increase? How much does the number of songs $s$ increase?
- By how many rows would you need to extend the table to solve the problem?

31. Air Travel Use the table at the right. How long will the jet take to travel 5390 mi ?

| Passenger Jet Travel |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Hours, $\boldsymbol{h}$ | 1 | 2 | 3 | 4 |
| Miles, $\boldsymbol{m}$ | 490 | 980 | 1470 | 1960 |

32. Reasoning Savannah looks at the table shown and says the equation $y=x-6$ represents the pattern. Mary says $y=x+(-6)$ represents the pattern. Who is correct? Explain.
33. Open-Ended Think of a real-world pattern. Describe the pattern using words and an equation with two variables. Define the variables.
C Challenge
34. Temperature Suppose the temperature starts at $60^{\circ} \mathrm{F}$ and rises $2^{\circ} \mathrm{F}$ every 45 min . Use a table, an equation, and a graph to describe the relationship between the amount of time that has passed in hours and the temperature.
35. Use a table, a graph, and an equation to represent the ordered pairs ( $2,-5.5$ ), $(-3,-0.5),(1,-4.5),(0,-3.5),(-3.5,0)$, and $(-1,-2.5)$.

## Standardized Test Prep

SAT/ACT
36. Use the graph. What is the total price for 4 bags of seeds?
(A) $\$ .50$
(C)
$\$ 4.00$
(B) $\$ 2.00$
(D) $\$ 8.00$
37. What is the simplified form of the expression $-5(n-2)$ ?
(F) $-7 n$
(H) $-5 n+10$
(G) $-5 n-2$
(I) $n+10$


Number of Bags
38. If $a=3$ and $b=-2$, what does $-2 b-a$ equal?
(A) -9
(B) -7
(C) -1
(D) 1
39. What is the value of $-3^{4}$ ?
(F) -81
(G) -12
(H) 12
81

## Mixed Review

Tell whether the given number is a solution of each equation.
40. $3 x+7=10 ; 0$
41. $80=4 a ; 20$
42. $10=-5 t ;-2$

Give an example that illustrates each property.
See Lesson 1-4.
43. Commutative Property of Addition
44. Associative Property of Multiplication
45. Identity Property of Multiplication
46. Zero Property of Addition

## Get Ready! To prepare for Lesson 2-1, do Exercises 47-54.

Find each sum or difference.
See Lesson 1-5.
47. $12+(-3)$
48. $-7+4$
49. $-8+(-6)$
50. $-42+15$
51. $32-(-8)$
52. $-18-12$
53. $-15-(-14)$
54. $-76-5$

