10-4 Solving Radical Equations

Common Core State Standards

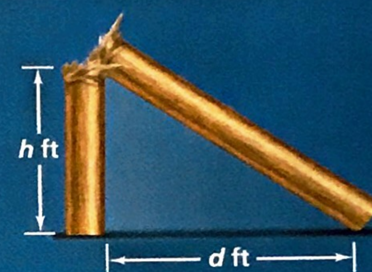
A-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

MP 1, MP 2, MP 3, MP 4

Objectives To solve equations containing radicals
To identify extraneous solutions

SOLVE IT! **Getting Ready!**

A pole 16 ft tall breaks, as shown in the diagram. What is an expression for d in terms of h? Explain your process.



The diagram labels two important features. What is the third?

MATHEMATICAL PRACTICES The expression for d in the Solve It has a variable in a radicand. A **radical equation** is an equation that has a variable in a radicand. Examples include $\sqrt{x} - 5 = 3$ and $\sqrt{x-2} = 1$. To solve a radical equation, get the radical by itself on one side of the equation. Then square both sides. The expression under the radical must be nonnegative.

Lesson Vocabulary
- radical equation
- extraneous solution

Essential Understanding You can solve some radical equations by squaring each side of the equation and testing the solutions.

Problem 1 Solving by Isolating the Radical

Plan

How do you start when solving a radical equation? Use the properties of equality to get the radical by itself on one side of the equation.

What is the solution of $\sqrt{x} + 7 = 16$?

$\sqrt{x} + 7 = 16$

$\sqrt{x} = 9$ Get the radical by itself on one side of the equation.

$(\sqrt{x})^2 = 9^2$ Square each side.

$x = 81$ Simplify.

Check $\sqrt{x} + 7 = 16$

$\sqrt{81} + 7 \stackrel{?}{=} 16$ Substitute 81 for x.

$9 + 7 = 16$ ✔

Got It? **1.** What is the solution of $\sqrt{x} - 5 = -2$?

Problem 2 Using a Radical Equation

Clocks **The time t in seconds it takes for a pendulum of a clock to complete a full swing is approximated by the equation $t = 2\sqrt{\frac{\ell}{3.3}}$, where ℓ is the length of the pendulum, in feet. If the pendulum of a clock completes a full swing in 3 s, what is the length of the pendulum? Round to the nearest tenth of a foot.**

Know	Need	Plan
• A function relating t and ℓ • The value of t	The value for ℓ, the length of the pendulum	Substitute for t in the function and solve for ℓ.

Think

Have you solved problems like this before?
Yes. You have substituted a value for one variable in a function and then solved for the other variable.

$t = 2\sqrt{\frac{\ell}{3.3}}$

$3 = 2\sqrt{\frac{\ell}{3.3}}$ Substitute 3 for t.

$1.5 = \sqrt{\frac{\ell}{3.3}}$ Divide each side by 2 to isolate the radical.

$(1.5)^2 = \left(\sqrt{\frac{\ell}{3.3}}\right)^2$ Square each side.

$2.25 = \frac{\ell}{3.3}$ Simplify.

$7.425 = \ell$ Multiply each side by 3.3.

Check $3 \stackrel{?}{=} 2\sqrt{\frac{7.425}{3.3}}$ Substitute 7.425 for ℓ.

$3 \stackrel{?}{=} 2\sqrt{2.25}$

$3 = 3$ ✔

The pendulum is about 7.4 ft long.

Got It? **2.** How long is a pendulum if each swing takes 1 s?

Problem 3 Solving With Radical Expressions on Both Sides

Think

How can you make the equation simpler to solve?
You can *solve a simpler problem* by squaring each side of the equation. You know how to solve equations like $5t - 11 = t + 5$.

What is the solution of $\sqrt{5t - 11} = \sqrt{t + 5}$?

$\sqrt{5t - 11} = \sqrt{t + 5}$

$(\sqrt{5t - 11})^2 = (\sqrt{t + 5})^2$ Square each side.

$5t - 11 = t + 5$ Simplify.

$4t - 11 = 5$ Subtract t from each side.

$4t = 16$ Add 11 to each side.

$t = 4$ Divide each side by 4.

Check $\sqrt{5(4) - 11} \stackrel{?}{=} \sqrt{4 + 5}$ Substitute 4 for t.

$\sqrt{9} = \sqrt{9}$ ✔

Got It? **3.** What is the solution of $\sqrt{7x - 4} = \sqrt{5x + 10}$?

When you solve an equation by squaring each side, you create a new equation. The new equation may have solutions that do not satisfy the original equation.

Original Equation	Square each side.	New Equation	Apparent Solutions
$x = 3$	$x^2 = 3^2$	$x^2 = 9$	$3, -3$

In the example above, -3 does not satisfy the original equation. It is an *extraneous* solution. An **extraneous solution** is an apparent solution that does not satisfy the original equation. Always substitute each apparent solution into the original equation to check for extraneous solutions.

Problem 4 Identifying Extraneous Solutions

What is the solution of $n = \sqrt{n + 12}$?

$n = \sqrt{n + 12}$

$n^2 = (\sqrt{n + 12})^2$ Square each side.

$n^2 = n + 12$ Simplify.

$n^2 - n - 12 = 0$ Subtract $n + 12$ from each side.

$(n - 4)(n + 3) = 0$ Factor the quadratic equation.

$n - 4 = 0$ or $n + 3 = 0$ Use the Zero-Product Property.

$n = 4$ or $n = -3$ Solve for n.

Check $4 \stackrel{?}{=} \sqrt{4 + 12}$ Substitute 4 and -3 for n. $-3 \stackrel{?}{=} \sqrt{-3 + 12}$

$4 = 4$ ✔ $-3 \neq 3$

The solution of the original equation is 4. The value -3 is an extraneous solution.

Think

Does an extraneous solution solve the problem?

No. An extraneous solution solves only the new equation formed after squaring both sides. It is not a solution to the problem.

Got It? **4.** What is the solution of $-y = \sqrt{y + 6}$?

Sometimes you get only extraneous solutions after squaring each side of an equation. In that case, the original equation has no solution.

Problem 5 Identifying Equations With No Solution

What is the solution of $\sqrt{3y} + 8 = 2$?

$\sqrt{3y} + 8 = 2$

$\sqrt{3y} = -6$ Subtract 8 from each side.

$3y = 36$ Square each side.

$y = 12$ Divide each side by 3.

Check $\sqrt{3(12)} + 8 \stackrel{?}{=} 2$ Substitute 12 for y.

$14 \neq 2$ $y = 12$ does not satisfy the original equation.

The apparent solution 12 is extraneous. The original equation has no solution.

Think

Have you seen other equations with no solutions?

Yes. You learned that equations such as $x + 1 = x$ have no solution.

Got It? 5. a. What is the solution of $6 - \sqrt{2x} = 10$?

b. **Reasoning** How can you determine that the equation $\sqrt{x} = -5$ does not have a solution without going through all the steps of solving the equation?

Lesson Check

Do you know HOW?

Solve each radical equation. Check your solution. If there is no solution, write *no solution*.

1. $\sqrt{3x} + 10 = 16$
2. $\sqrt{r+5} = 2\sqrt{r-1}$
3. $\sqrt{2x-1} = x$
4. $\sqrt{x-3} = \sqrt{x+5}$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** Which is an extraneous solution of $s = \sqrt{s+2}$?

(A) 2
(B) 0
(C) −1
(D) −2

6. **Reasoning** What is the converse of the conditional statement "If $x = y$, then $x^2 = y^2$"? Is the converse of this statement always true? Explain.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Solve each radical equation. Check your solution. See Problem 1.

7. $\sqrt{x} + 3 = 5$
8. $\sqrt{t} + 2 = 9$
9. $\sqrt{z} - 1 = 5$
10. $\sqrt{n} - 3 = 6$
11. $\sqrt{2b} + 4 = 8$
12. $3 - \sqrt{t} = -2$
13. $\sqrt{3a+1} = 7$
14. $\sqrt{10b+6} = 6$
15. $1 = \sqrt{-2v-3}$
16. $\sqrt{x-3} = 4$

See Problem 2.

17. **Recreation** You are making a tire swing for a playground. The time t in seconds for the tire to make one swing is given by $t = 2\sqrt{\frac{\ell}{3.3}}$, where ℓ is the length of the swing in feet. You want one swing to take 2.5 s. How many feet long should the swing be?

18. **Geometry** The length s of one edge of a cube is given by $s = \sqrt{\frac{A}{6}}$, where A represents the cube's surface area. Suppose a cube has an edge length of 9 cm. What is its surface area? Round to the nearest hundredth.

Solve each radical equation. Check your solution. See Problem 3.

19. $\sqrt{3x+1} = \sqrt{5x-8}$
20. $\sqrt{2y} = \sqrt{9-y}$
21. $\sqrt{7v-4} = \sqrt{5v+10}$
22. $\sqrt{s+10} = \sqrt{6-s}$
23. $\sqrt{n+5} = \sqrt{5n-11}$
24. $\sqrt{3m+1} = \sqrt{7m-9}$

Tell which solutions, if any, are extraneous for each equation. See Problems 4 and 5.

25. $-z = \sqrt{-z+6}$; $z = -3$, $z = 2$

26. $\sqrt{12-n} = n$; $n = -4$, $n = 3$

27. $y = \sqrt{2y}$; $y = 0$, $y = 2$

28. $2a = \sqrt{4a+3}$; $a = \frac{3}{2}$, $a = -\frac{1}{2}$

29. $x = \sqrt{28-3x}$; $x = 4$, $x = -7$

30. $-t = \sqrt{-6t-5}$; $t = -5$, $t = -1$

Solve each radical equation. Check your solution. If there is no solution, write *no solution*.

31. $x = \sqrt{2x+3}$

32. $n = \sqrt{4n+5}$

33. $\sqrt{3b} = -3$

34. $2y = \sqrt{5y+6}$

35. $-2\sqrt{2r+5} = 6$

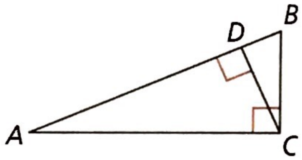
36. $\sqrt{d+12} = d$

B Apply

37. **Error Analysis** A student solved the equation $r = \sqrt{-6r-5}$ and found the solutions -1 and -5. Describe and correct the student's error.

38. **Think About a Plan** The total surface area A of Earth, in square kilometers, is related to Earth's radius r, in kilometers, by $r = \sqrt{\frac{A}{4\pi}}$. Earth's radius is about 6378 km. What is its surface area? Round to the nearest square kilometer.
    - What equation in one variable can you solve to find Earth's surface area?
    - How can you check the reasonableness of your solution?

39. **Geometry** In the right triangle $\triangle ABC$, the altitude $\overline{CD}$ is at a right angle to the hypotenuse. You can use $CD = \sqrt{(AD)(DB)}$ to find missing lengths.
    a. Find AD if $CD = 10$ and $DB = 4$.
    b. Find DB if $AD = 20$ and $CD = 15$.



40. **Packaging** The radius r of a cylindrical can with volume V and height h is given by $r = \sqrt{\frac{V}{\pi h}}$. What is the height of a can with a radius of 2 in. and a volume of 75 in.^{3}?

41. **Writing** Explain how you would solve the equation $\sqrt{2y} - \sqrt{y+2} = 0$.

42. **Open-Ended** Write two radical equations that have 3 for a solution.

Solve each radical equation. Check your solution. If there is no solution, write *no solution*.

43. $\sqrt{5x+10} = 5$

44. $-6 - \sqrt{3y} = -3$

45. $\sqrt{7p+5} = \sqrt{p-3}$

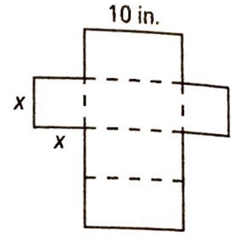
46. $a = \sqrt{7a-6}$

47. $\sqrt{y+12} = 3\sqrt{y}$

48. $3 - \sqrt{4a+1} = 12$

STEM 49. **Physics** The formula $t = \sqrt{\frac{n}{16}}$ gives the time t in seconds for an object that is initially at rest to fall n feet. What is the distance an object falls in the first 10 s?

50. **Packaging** The diagram at the right shows a piece of cardboard that makes a box when sections of it are folded and taped. The ends of the box are x inches by x inches, and the body of the box is 10 in. long.



a. Write an equation for the volume V of the box.
b. Solve the equation in part (a) for x.
c. Find the integer values of x that would give the box a volume between 40 in.^{3} and 490 in.^{3}, inclusive.

Challenge

51. **Reasoning** Explain the difference between squaring $\sqrt{x-1}$ and $\sqrt{x}-1$.

52. a. **Reasoning** What is the solution of $\sqrt{7y+18}=y$? What is the extraneous solution?
b. Multiply one side of $\sqrt{7y+18}=y$ by -1. What is the solution of the new equation? What is an extraneous solution of the new equation?
c. What do you think will happen to the solutions and extraneous solutions of $\sqrt{y+2}=y$ if you multiply one side by -1? Explain.

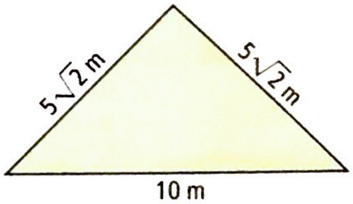
Standardized Test Prep

SAT/ACT

53. What are the solutions of $\sqrt{c^2-17}=8$?

(A) 6, 9 (B) 8, −8 (C) 8, 0 (D) 9, −9

54. Sam is building a fence around a triangular flower garden. What is the perimeter of the garden? Round your answer to the nearest tenth of a meter.



(F) 14.1 m (H) 24.1 m
(G) 20.0 m (I) 50.0 m

55. What is the slope-intercept form of the equation $2x+5y=40$?

(A) $y=-2x+8$ (B) $y=-\frac{2}{5}x+8$ (C) $y=\frac{2}{5}x+8$ (D) $y=2x+8$

Short Response

56. Write the equation of the line passing through $(1,-1)$ with a slope of $\frac{1}{2}$ in three different forms. When would each of the forms be useful?

Mixed Review

Simplify each expression. See Lesson 10-3.

57. $\sqrt{8}+3\sqrt{2}$ 58. $(2\sqrt{5}-6)(9+3\sqrt{5})$ 59. $\frac{2}{\sqrt{3}+\sqrt{8}}$

Use the quadratic formula to solve each equation. See Lesson 9-6.

60. $3a^2+4a+3=0$ 61. $2f^2-8=0$ 62. $6m^2+13m+6=0$

Get Ready! **To prepare for Lesson 10-5, do Exercises 63–65.**

Graph each function by translating $y=|x|$. See Lesson 5-8.

63. $y=|x+2|$ 64. $y=|x|-3$ 65. $y=|x-4|$