2-6 Ratios, Rates, and Conversions

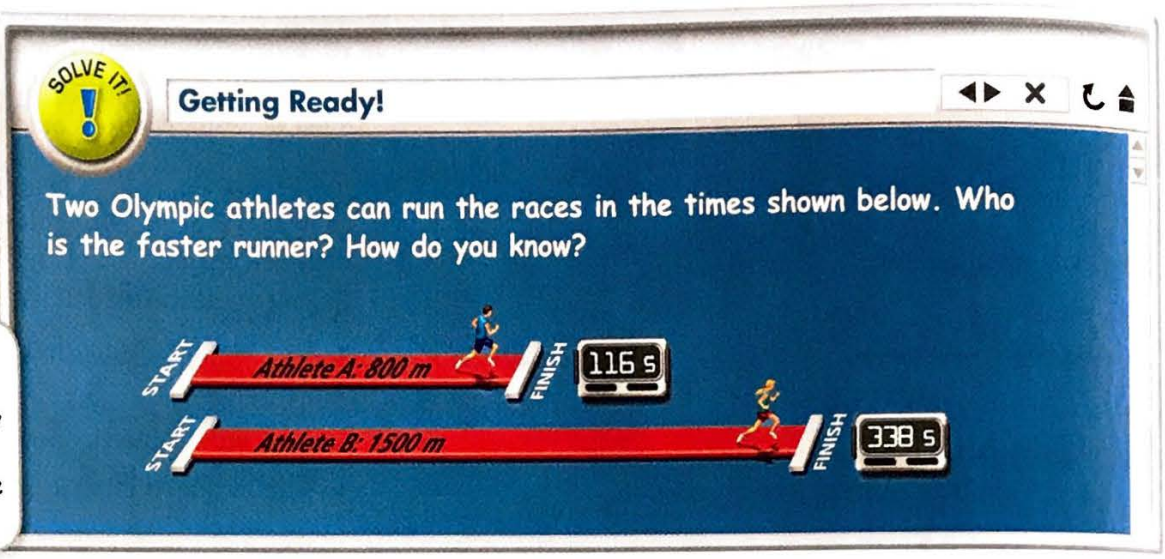
Common Core State Standards

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas . . . **Also N-Q.A.2**

MP 2, MP 3, MP 4, MP 5, MP 6

Objectives To find ratios and rates
To convert units and rates

Whoa—we're comparing times and distances. Why is it important to be precise with the units?



MATHEMATICAL PRACTICES

A **ratio** compares two numbers by division. The ratio of two numbers a and b, where $b \neq 0$, can be written in three ways: $\frac{a}{b}$, $a : b$, and a to b. For every a units of one quantity, you have b units of another quantity.

Lesson Vocabulary
- ratio
- rate
- unit rate
- conversion factor
- unit analysis

You can also think of a ratio as a multiplicative relationship. For example, if the ratio of the number of boys to the number of girls in a class is 2 : 1, then the number of boys is *two times* the number of girls.

A ratio that compares quantities measured in different units is called a **rate.** A rate with a denominator of 1 unit is a **unit rate.** In the Solve It, you can express each athlete's speed as the number of meters traveled per 1 second of time. This is an example of a unit rate.

Essential Understanding You can write ratios and find unit rates to compare quantities. You can also convert units and rates to solve problems.

Problem 1 Comparing Unit Rates

Think

How can estimation help you?
Use estimation to *solve a simpler problem.* You can use the given information to estimate the unit rates. The estimates can help you find the solution.

Shopping **You are shopping for T-shirts. Which store offers the best deal?**

Store A: $25 for 2 shirts **Store B: $45 for 4 shirts** **Store C: $30 for 3 shirts**

Write each price as a ratio. Then write the ratio as a unit rate to compare.

Store A

$$\frac{\$25}{2 \text{ shirts}} = \frac{\$12.50}{1 \text{ shirt}}$$

Store B

$$\frac{\$45}{4 \text{ shirts}} = \frac{\$11.25}{1 \text{ shirt}}$$

Store C

$$\frac{\$30}{3 \text{ shirts}} = \frac{\$10}{1 \text{ shirt}}$$

Store C has the best deal because its unit rate is the lowest.

Got It? **1.** If Store B lowers its price to $42 for 4 shirts, does the solution to Problem 1 change? Explain.

To convert from one unit to another, such as feet to inches, you multiply the original unit by a *conversion factor* that produces the desired unit. A **conversion factor** is a ratio of two equivalent measures in different units. A conversion factor is always equal to 1, such as $\frac{1\text{ ft}}{12\text{ in.}}$. See the table on page 814 for some common equivalent units of measure.

Problem 2 Converting Units

What is the given amount converted to the given units?

Choose and multiply by the appropriate conversion factor. The appropriate factor will allow you to divide out the common units and simplify.

Plan

How do you choose the conversion factor?
Write a conversion factor that has the desired units in the numerator and the original units in the denominator.

A **330 min; hours**

$$330\text{ min} \cdot \frac{1\text{ h}}{60\text{ min}} \quad \leftarrow \text{Choose a conversion factor.} \rightarrow \quad 15\text{ kg} \cdot \frac{1000\text{ g}}{1\text{ kg}}$$

$$= 330\text{ min} \cdot \frac{1\text{ h}}{60\text{ min}} \quad \leftarrow \text{Divide out common units.} \rightarrow \quad = 15\text{ kg} \cdot \frac{1000\text{ g}}{1\text{ kg}}$$

$$= 5.5\text{ h} \quad \leftarrow \text{Simplify.} \rightarrow \quad = 15{,}000\text{ g}$$

B **15 kg; grams**

C **5 ft 3 in.; inches**

$$5\text{ ft }3\text{ in.} = 5\text{ ft} + 3\text{ in.}$$

$$= 5\text{ ft} \cdot \frac{12\text{ in.}}{1\text{ ft}} + 3\text{ in.}$$

$$= 60\text{ in.} + 3\text{ in.} = 63\text{ in.}$$

Got It? **2.** What is 1250 cm converted to meters?

In Problem 2, notice that the units for each quantity are included in the calculations to help determine the units for the answers. This process is called **unit analysis**, or *dimensional analysis*.

Problem 3 Converting Units Between Systems STEM

Architecture **The CN Tower in Toronto, Canada, is about 1815 ft tall. About how many meters tall is the tower? Use the fact that 1 m ≈ 3.28 ft.**

Plan

How can you convert units?
Write the conversion factor so that the original units divide out and leave only the desired units.

Multiply by the appropriate conversion factor and divide out common units.

$$1815\text{ ft} \cdot \frac{1\text{ m}}{3.28\text{ ft}} = 1815\text{ ft} \cdot \frac{1\text{ m}}{3.28\text{ ft}} \approx 553\text{ m}$$

The CN Tower is about 553 m tall.

Check Round 1815 to 1800 and 3.28 to 3. Then divide 1800 by 3. $1800 \div 3 = 600$, and 600 is about 553. So, 553 m is a reasonable answer.

Got It? 3. a. A building is 1450 ft tall. How many meters tall is the building? Use the fact that 1 m $\approx$ 3.28 ft.

b. Monetary exchange rates change from day to day. On a particular day, the exchange rate for dollars to euros was about 1 dollar = 0.63 euro. About how many euros could you get for \$325 on that day?

You can also convert rates. For example, you can convert a speed in miles per hour to feet per second. Because rates compare measures in two different units, you must multiply by two conversion factors to change both of the units.

Problem 4 Converting Rates

A student ran the 50-yd dash in 5.8 s. At what speed did the student run in miles per hour? Round your answer to the nearest tenth.

Know
The running speed in yards per second

Need
The running speed in miles per hour

Plan
Write the speed as a ratio. Choose conversion factors so that the original units (yards and seconds) divide out, leaving you with the units you need (miles and hours).

$$\frac{50\text{ yd}}{5.8\text{ s}} \cdot \frac{1\text{ mi}}{1760\text{ yd}} \cdot \frac{3600\text{ s}}{1\text{ h}}$$ Use appropriate conversion factors.

This conversion factor cancels yards and leaves miles.

This conversion factor cancels seconds and leaves hours.

$$= \frac{50\cancel{\text{ yd}}}{5.8\cancel{\text{ s}}} \cdot \frac{1\text{ mi}}{1760\cancel{\text{ yd}}} \cdot \frac{3600\cancel{\text{ s}}}{1\text{ h}}$$ Divide common units.

$$= \frac{180{,}000\text{ mi}}{10{,}208\text{ h}} \approx 17.6\text{ mi/h}$$ Simplify.

The student ran at a speed of about 17.6 mi/h.

Got It? 4. a. An athlete ran a sprint of 100 ft in 3.1 s. At what speed was the athlete running in miles per hour? Round to the nearest mile per hour.

b. **Reasoning** In Problem 4, one student multiplied by the conversion factors $\frac{1\text{ mi}}{1760\text{ yd}}$, $\frac{60\text{ s}}{1\text{ min}}$, and $\frac{60\text{ min}}{1\text{ h}}$ to find the speed. Can this method work? Why or why not?

Lesson Check

Do you know HOW?

1. Which is the better buy, 6 bagels for $3.29 or 8 bagels for $4.15?
2. What is 7 lb 4 oz converted to ounces?
3. Which is longer, 12 m or 13 yd?
4. A car is traveling at 55 mi/h. What is the car's speed in feet per second?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

Vocabulary Tell whether each rate is a unit rate.

5. 20 mi every 3 h
6. 2 dollars per day

7. **Reasoning** Does multiplying by a conversion factor change the amount of what is being measured? How do you know?
8. **Reasoning** If you convert pounds to ounces, will the number of ounces be greater or less than the number of pounds? Explain.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

9. **Running** Trisha ran 10 km in 2.5 h. Jason ran 7.5 km in 2 h. Olga ran 9.5 km in 2.25 h. Who had the fastest average speed? See Problem 1.
10. **Population** Bellingham, Washington, had an area of 25.4 mi^2 and a population of 74,547 during one year. Bakersfield, California, had an area of 113.1 mi^2 and a population of 295,536 during the same year. Which city had a greater number of people per square mile?

Convert the given amount to the given unit. See Problems 2 and 3.

11. 63 yd; feet
12. 168 h; days
13. 2.5 lb; ounces
14. 200 cm; meters
15. 4 min; seconds
16. 1500 mL; liters
17. 9 yd; meters
18. 5 kg; pounds
19. 79 dollars; cents
20. 3 qt; liters
21. 89 cm; inches
22. 2 ft; centimeters

23. **Maintenance** The janitor at a school discovered a slow leak in a pipe. The janitor found that it was leaking at a rate of 4 fl oz per minute. How fast was the pipe leaking in gallons per hour? See Problem 4.
24. **Shopping** Mr. Swanson bought a package of 10 disposable razors for $6.30. He found that each razor lasted for 1 week. What was the cost per day?

B Apply

Copy and complete each statement.

25. 7 ft 3 in. = ■ in.
26. 2.2 kg = ■ lb
27. 2.5 h = ■ min
28. 2 qt/min = ■ gal/s
29. 75 cents/h = ■ dollars/day
30. 60 ft/s = ■ km/h

Choose a Method Choose paper and pencil, mental math, or a calculator to tell which measurement is greater.

31. 640 ft; 0.5 mi **32.** 63 in.; 125 cm **33.** 75 g; 5 oz

34. Think About a Plan A college student is considering a subscription to a social-networking Internet site that advertises its cost as "only 87 cents per day." What is the cost of membership in dollars per year?

- How many conversion factors will you need to use to solve the problem?
- How do you choose the appropriate conversion factors?

35. Recipes Recipe A makes 5 dinner rolls using 1 c of flour. Recipe B makes 24 rolls using $7\frac{1}{2}$ c of flour. Recipe C makes 45 rolls using 10 c of flour. Which recipe requires the most flour per roll?

36. Error Analysis Find the mistake in the conversion below. Explain the mistake and convert the units correctly.

$$9 \text{ yd} = ? \text{ ft}$$

$$9 \text{ yd} \cdot \frac{3 \text{ yd}}{1 \text{ ft}} = 27 \text{ ft}$$

37. Writing Suppose you want to convert kilometers to miles. Which unit should be in the numerator of the conversion factor? Which unit should be in the denominator? Explain how you know.

38. Reasoning Without performing the conversion, determine whether the number of new units will be greater or less than the number of original units.

a. 3 min 20 s converted to seconds

b. 23 cm converted to inches

c. kilometers per hour converted to miles per hour

39. Exchange Rates The table below shows some exchange rates on a particular day. If a sweater sells for \$39.95 in U.S. dollars, what should its price be in rupees and pounds?

U.S. DOLLARS	1.00
INDIAN RUPEES	39.57
ALGERIAN DINARS	64.15
BRITISH POUNDS	.50

40. Estimation Five mi is approximately equal to 8 km. Use mental math to estimate the distance in kilometers to a town that is 30 mi away.

41. Reasoning A carpenter is building an entertainment center. She is calculating the size of the space to leave for the television. She wants to leave about a foot of space on either side of the television. Would measuring the size of the television exactly or estimating the size to the nearest inch be more appropriate? Explain.

42. Reasoning A traveler changed $300 to euros for a trip to Germany, but the trip was canceled. Three months later, the traveler changed the euros back to dollars. Would you expect that the traveler got exactly $300 back? Explain.

Challenge

43. Measurement Dietrich draws a line on the blackboard whose length is given by the expression 1 mm + 1 cm + 1 in. + 1 ft + 1 yd + 1 m. What is the length of the line in millimeters?

44. Square Measurements There are 2.54 cm in 1 in.

a. How many square centimeters are there in 1 $in.^2$? Give your answer to the nearest hundredth of a square centimeter.

b. How many square inches are there in 129 cm^2?

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 6

Look back at the diagram of the monorail route on page 79. In the Apply What You've Learned in Lesson 2-3, you found the average speed of the monorail between the parking garage and Terminal A and the average speed of the monorail between Terminal A and Terminal B. Select all of the following that are true. Explain your reasoning.

A. To convert the monorail's average speed from feet per second to miles per hour, multiply the speed in feet per second by $\frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ h}}{3600\text{ s}}$.

B. Between the parking garage and Terminal A, the monorail's average speed is greater than 25 mi/h.

C. Between Terminal A and Terminal B, the monorail's average speed is greater than 25 mi/h.

D. Between Terminal A and Terminal B, the monorail's average speed is 8.1 mi/h greater than its average speed between the parking garage and Terminal A.

E. If the monorail moved at its average speed between Terminal A and Terminal B for 15 minutes, it would travel about 7.2 miles.

F. If the monorail moved at its average speed between Terminal A and Terminal B, it would take more than an hour to travel 30 miles.