

2-7

Solving Proportions

© Common Core State Standards

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Also N-Q.A.1, A-CED.A.1

MP 1, MP 3, MP 4, MP 8

Objective To solve and apply proportions

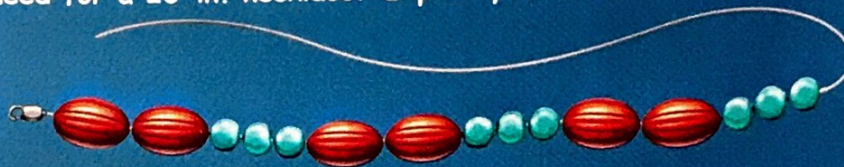


You've used ratios to compare. This problem involves equal ratios.



Getting Ready!

You are making beaded necklaces, using a pattern in which 2 large red beads are followed by 3 small blue beads. Each set of 5 beads adds $1\frac{1}{4}$ in. to the length of the necklace. How many of each color bead will you need for a 20-in. necklace? Explain your answer.



In the Solve It, the number of red beads and the number of blue beads are quantities that have a proportional relationship. This means that the ratio of the quantities is constant even though the quantities themselves can change. For example, as you are making the necklace you will have 2 red beads and 3 blue beads, then 4 red beads and 6 blue beads, then 6 red beads and 9 blue beads, and so on. At each stage, the ratio of red beads to blue beads remains constant, 2 : 3.

A proportional relationship can produce an infinite number of equivalent ratios. Any two of these can be used to write a proportion. A **proportion** is an equation that states that two ratios are equal. For example, $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, is a proportion. You read this as “ a is to b as c is to d .”

Essential Understanding If two ratios are equal and a quantity in one of the ratios is unknown, you can write and solve a proportion to find the unknown quantity.



Lesson Vocabulary

- proportion
- cross products
- Cross Products Property

Think

How is this problem related to problems you've solved before?

Solving this proportion is similar to solving a one-step equation using multiplication. You can simply multiply by 12 to isolate m .



Problem 1 Solving a Proportion Using the Multiplication Property


What is the solution of the proportion $\frac{7}{8} = \frac{m}{12}$?

$$\frac{7}{8} = \frac{m}{12}$$

$$12 \cdot \frac{7}{8} = 12 \cdot \frac{m}{12} \quad \text{Multiply each side by 12.}$$

$$\frac{84}{8} = m \quad \text{Simplify.}$$

$$10.5 = m \quad \text{Divide.}$$

 **Got It?** 1. What is the solution of the proportion $\frac{x}{7} = \frac{4}{5}$?

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products ad and bc are called **cross products**. You can use the following property of cross products to solve proportions.

Take note

Property Cross Products Property of a Proportion

Words The cross products of a proportion are equal.

Algebra If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

Example $\frac{3}{4} = \frac{9}{12}$, so $3(12) = 4(9)$, or $36 = 36$.

Here's Why It Works You can use the Multiplication Property of Equality to prove the Cross Products Property.

$$\begin{aligned}\frac{a}{b} &= \frac{c}{d} && \text{Assume this equation is true.} \\ bd \cdot \frac{a}{b} &= bd \cdot \frac{c}{d} && \text{Multiplication Property of Equality} \\ \cancel{bd} \cdot \frac{a}{\cancel{b}} &= \cancel{bd} \cdot \frac{c}{\cancel{d}} && \text{Divide the common factors.} \\ da &= bc && \text{Simplify.} \\ ad &= bc && \text{Commutative Property of Multiplication}\end{aligned}$$

For this proportion, a and d are called the *extremes* of the proportion and b and c are called the *means*. Notice that in the Cross Products Property the product of the means equals the product of the extremes.



Problem 2 Solving a Proportion Using the Cross Products Property

What is the solution of the proportion $\frac{4}{3} = \frac{8}{x}$?

$$\frac{4}{3} = \frac{8}{x}$$


$$4x = 3(8) \quad \text{Cross Products Property}$$

$$4x = 24 \quad \text{Multiply.}$$

$$x = 6 \quad \text{Divide each side by 4 and simplify.}$$



Got It? 2. a. What is the solution of the proportion $\frac{y}{3} = \frac{3}{5}$?

 **b. Reasoning** Would you rather use the Cross Products Property or the Multiplication Property of Equality to solve $\frac{3}{5} = \frac{13}{b}$? Explain.

Think

Which property should you use?

The Cross Products Property can be easier to use when the variable is in the denominator. If you use the Multiplication Property, you must multiply each side by $3x$.



Problem 3 Solving a Multi-Step Proportion

Think

How is this proportion different from others you've seen?

This proportion looks more complex, but the Cross Products Property is true for any proportion. Treat each numerator as a single variable when you cross-multiply.

What is the solution of the proportion $\frac{b-8}{5} = \frac{b+3}{4}$?

$$\frac{b-8}{5} = \frac{b+3}{4}$$

$$4(b-8) = 5(b+3) \quad \text{Cross Products Property}$$

$$4b - 32 = 5b + 15 \quad \text{Distributive Property}$$

$$4b - 32 - 4b = 5b + 15 - 4b \quad \text{Subtract } 4b \text{ from each side.}$$

$$-32 = b + 15 \quad \text{Simplify.}$$

$$-47 = b \quad \text{Subtract 15 from each side and simplify.}$$



Got It? 3. What is the solution of the proportion $\frac{n}{5} = \frac{2n+4}{6}$?

When you model a real-world situation with a proportion, you must write the proportion carefully. You can write the proportion so that the numerators have the same units and the denominators have the same units.

Correct: $\frac{100 \text{ mi}}{2 \text{ h}} = \frac{x \text{ mi}}{5 \text{ h}}$

Incorrect: $\frac{100 \text{ mi}}{2 \text{ h}} = \frac{5 \text{ h}}{x \text{ mi}}$



Problem 4 Using a Proportion to Solve a Problem

Music A portable media player has 2 gigabytes of storage and can hold about 500 songs. A similar but larger media player has 80 gigabytes of storage. About how many songs can the larger media player hold?

Know

- Smaller media player has 2 gigabytes and can hold 500 songs
- Larger media player has 80 gigabytes

Need

The number of songs the larger player can hold

Plan

Write a proportion to model the situation. You can set up the proportion so that the numerators have the same units and the denominators have the same units. Then solve the proportion.

Think

Is there only one way to write a proportion?

No. You can write other proportions to solve the problem. For example, $\frac{2 \text{ gigabytes}}{80 \text{ gigabytes}} = \frac{500 \text{ songs}}{s \text{ songs}}$ also works.

$$\frac{2 \text{ gigabytes}}{500 \text{ songs}} = \frac{80 \text{ gigabytes}}{s \text{ songs}}$$

Write a proportion.

$$2s = 500(80) \quad \text{Cross Products Property}$$

$$2s = 40,000 \quad \text{Multiply.}$$

$$s = 20,000 \quad \text{Divide each side by 2 and simplify.}$$

The larger media player can hold 20,000 songs.



Got It? 4. An 8-oz can of orange juice contains about 97 mg of vitamin C. About how many milligrams of vitamin C are there in a 12-oz can of orange juice?



Lesson Check

Do you know HOW?

Solve each proportion.

1. $\frac{b}{6} = \frac{4}{5}$

2. $\frac{5}{9} = \frac{15}{x}$

3. $\frac{w+3}{4} = \frac{w}{2}$

4. $\frac{3}{x+1} = \frac{1}{2}$

5. **Music** A band went to a recording studio and recorded 4 songs in 3 h. How long would it take the band to record 9 songs if they record at the same rate?

Do you UNDERSTAND?



6. **Vocabulary** Use the proportion $\frac{m}{n} = \frac{p}{q}$. Identify the following.

- 6. the extremes
- 7. the means
- 8. the cross products

9. **Reasoning** When solving $\frac{x}{5} = \frac{3}{4}$, Lisa's first step was to write $4x = 5(3)$. Jen's first step was to write $20\left(\frac{x}{5}\right) = 20\left(\frac{3}{4}\right)$. Will both methods work? Explain.



Practice and Problem-Solving Exercises



A Practice

Solve each proportion using the Multiplication Property of Equality.

See Problem 1.

10. $\frac{q}{8} = \frac{4}{5}$

11. $\frac{-3}{4} = \frac{x}{26}$

12. $\frac{3}{4} = \frac{x}{5}$

13. $\frac{m}{7} = \frac{3}{5}$

14. $\frac{3}{16} = \frac{x}{12}$

15. $\frac{9}{2} = \frac{k}{25}$

16. $\frac{x}{120} = \frac{1}{24}$

17. $\frac{2}{15} = \frac{h}{125}$

Solve each proportion using the Cross Products Property.

See Problem 2.

18. $\frac{3}{v} = \frac{8}{13}$

19. $\frac{15}{a} = \frac{3}{2}$

20. $\frac{3}{8} = \frac{30}{m}$

21. $\frac{2}{7} = \frac{4}{d}$

22. $\frac{-9}{b} = \frac{5}{6}$

23. $\frac{8}{p} = \frac{3}{10}$

24. $\frac{-3}{4} = \frac{m}{22}$

25. $\frac{2}{-5} = \frac{6}{t}$

Solve each proportion using any method.

See Problem 3.

26. $\frac{a-2}{9} = \frac{2}{3}$

27. $\frac{b+4}{5} = \frac{7}{4}$

28. $\frac{3}{7} = \frac{c+4}{35}$

29. $\frac{2c}{11} = \frac{c-3}{4}$

30. $\frac{7}{k-2} = \frac{5}{8}$

31. $\frac{3}{3b+4} = \frac{2}{b-4}$

32. $\frac{q+2}{5} = \frac{2q-11}{7}$

33. $\frac{c+1}{c-2} = \frac{4}{7}$

34. **Gardening** A gardener is transplanting flowers into a flowerbed. She has been working for an hour and has transplanted 14 flowers. She has 35 more flowers to transplant. If she works at the same rate, how many more hours will it take her?

See Problem 4.

35. **Florists** A florist is making centerpieces. He uses 2 dozen roses for every 5 centerpieces. How many dozens of roses will he need to make 20 centerpieces?

36. **Picnics** If 5 lb of pasta salad serves 14 people, how much pasta salad should you bring to a picnic with 49 people?

B Apply

37. **Statistics** Approximately 3 people out of every 30 are left-handed. About how many left-handed people would you expect in a group of 140 people?
38. **Think About a Plan** Maya runs 100 m in 13.4 s. Amy can run 100 m in 14.1 s. If Amy were to finish a 100-m race at the same time as Maya, how much of a head start, in meters, would Amy need?
- What information do you know? What information is unknown?
 - What proportion can you write that will help you solve the problem?
39. **Electricity** The electric bill for Ferguson's Furniture is shown at the right. The cost of electricity per kilowatt-hour and the total charges for one month are given. How many kilowatt-hours of electricity did Ferguson's Furniture use in that month?
40. **Video Downloads** A particular computer takes 15 min to download a 45-min TV show. How long will it take the computer to download a 2-h movie?
41. **Schedules** You want to meet your friend at a park 4 mi away from your house. You are going to bike to the park at an average rate of 10 mi/h. Your friend lives 1.2 mi away from the park and walks at an average rate of 3 mi/h. How many minutes ahead of you should your friend start out so that you meet at the park at the same time?

Centerville Electric	
Account Name:	Ferguson's Furniture
Account Number:	34-14567-89
Cost per kilowatt-hour	\$0.07
Total charges	\$143.32
Previous balance	\$0.00
Total Amount Due	\$143.32

Solve each proportion. Tell whether you used the Multiplication Property of Equality or the Cross Products Property for your first step. Explain your choice.

42. $\frac{p}{4} = \frac{7}{8}$

43. $\frac{m}{4.5} = \frac{2}{5}$

44. $\frac{3}{10} = \frac{b}{7}$

45. $\frac{r}{2.1} = \frac{3.6}{2.8}$

46. $\frac{9}{14} = \frac{3}{n}$

47. $\frac{1.5}{y} = \frac{2.5}{7}$

48. $\frac{b+13}{2} = \frac{-5b}{3}$

49. $\frac{3b}{b-4} = \frac{3}{7}$

50. $\frac{x+2}{2x-6} = \frac{3}{8}$

51. **Error Analysis** Describe and correct the error in solving the proportion at the right.

52. **Bakery** A bakery sells packages of 10 bagels for \$3.69. If the bakery starts selling the bagels in packages of 12, how much would you expect a package of 12 to cost?

(A) \$3.08

(C) \$4.43

(B) \$4.32

(D) \$4.69

53. **Open-Ended** Write a proportion that contains a variable. Name the extremes, the means, and the cross products. Solve the proportion. Tell whether you used the Multiplication Property of Equality or the Cross Products Property to solve the proportion. Explain your choice.

- STEM** 54. **Biology** Many trees have concentric rings that can be counted to determine the tree's age. Each ring represents one year's growth. A maple tree with a diameter of 12 in. has 32 rings. If the tree continues to grow at about the same rate, how many rings will the tree have when its diameter is 20 in.?

$$\begin{aligned} \frac{8}{3} &= \frac{x+3}{2} \\ 16 &= 3x+3 \\ 13 &= 3x \\ \frac{13}{3} &= x \end{aligned}$$

Challenge

Solve each proportion.

$$55. \frac{4y - 3}{y^2 + 1} = \frac{4}{y}$$

$$56. \frac{w^2 + 3}{2w + 2} = \frac{w}{2}$$

$$57. \frac{5x}{x^3 + 5} = \frac{5}{x^2 - 7}$$

58. **Parade Floats** A group of high school students is making a parade float by stuffing pieces of tissue paper into a wire frame. They use 150 tissues to fill an area 3 ft long and 2 ft wide. The total area they want to fill is 8 ft long and 7 ft wide. What is the total number of tissues they will need?
59. **Insects** It takes an insect 15 s to crawl 1 ft. How many hours would it take the insect to crawl 1 mi if the insect crawls at the same rate?

Standardized Test Prep

SAT/ACT

60. A high school soccer team is making trail mix to sell at a fundraiser. The recipe calls for 3 lb of raisins and 2 lb of peanuts. If the team purchases 54 lb of peanuts, how many pounds of raisins will they need?

(A) 27

(B) 36

(C) 81

(D) 162

61. One day during flu season, $\frac{1}{3}$ of the students in a class were out sick, and only 24 students were left. How many students are in the class?

(F) 16

(G) 30

(H) 36

(I) 72

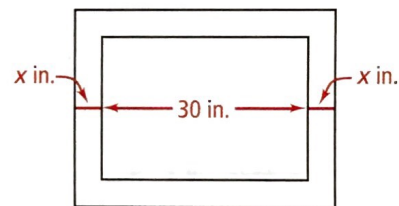
62. An art gallery owner is framing a rectangular painting, as shown. The owner wants the width of the framed painting to be $38\frac{1}{2}$ in. How wide should each of the vertical sections of the frame be?

(A) $4\frac{1}{8}$ in.

(C) $4\frac{1}{2}$ in.

(B) $4\frac{1}{4}$ in.

(D) $8\frac{1}{2}$ in.



Mixed Review

Copy and complete each statement.

◀ See Lesson 2-6.

63. 6 qt = ■ gal

64. 84 in. = ■ ft

65. $2\frac{1}{2}$ yd = ■ in.

66. 3 min 10 s = ■ s

Solve each equation. If the equation is an identity, write *identity*. If it has no solution, write *no solution*.

◀ See Lesson 2-4.

67. $3x - (x - 4) = 2x$

68. $4 + 6c = 6 - 4c$

69. $5a - 2 = 0.5(10a - 4)$

Get Ready! To prepare for Lesson 2-8, do Exercises 70-73.

Solve each proportion.

◀ See Lesson 2-7.

70. $\frac{x}{12} = \frac{7}{30}$

71. $\frac{y}{12} = \frac{8}{45}$

72. $\frac{w}{15} = \frac{12}{27}$

73. $\frac{n}{9} = \frac{n+1}{24}$