

Objectives To write sets and identify subsets
To find the complement of a set



How can you organize the choice of letters to help you solve this problem?



Getting Ready!

Most numbers on a telephone keypad correspond to a set of letters. Suppose you're sending a text message. You press 4, 6, 6, and 3, in that order—one number for each letter. What word might your telephone think you're trying to spell? What words could you be trying to spell? Explain your reasoning.

1	2 ABC	3 DEF
4 GHI	5 JKL	6 MNO
7 PQRS	8 TUV	9 WXYZ
*	0 +	#



MATHEMATICAL PRACTICES

Recall from Lesson 1-3 that a *set* is a collection of distinct elements. A *subset* contains elements from a set. For example, the number 6 on the telephone keypad corresponds to the set {M, N, O}. The set {M, O} is one subset of this set.

Essential Understanding Sets are the basis of mathematical language. You can write sets in different ways and form smaller sets of elements from a larger set. You can also describe the elements that are *not* in a given set.

Roster form is one way to write sets. Roster form lists the elements of a set within braces, { }. For example, you write the set containing 1 and 2 as {1, 2}, and you write the set of multiples of 2 as {2, 4, 6, 8, ...}.

Set-builder notation is another way to write sets. It describes the properties an element must have to be included in a set. For example, you can write the set {2, 4, 6, 8, ...} in set-builder notation as { x | x is a multiple of 2}. You read this as “the set of all real numbers x , such that x is a multiple of 2.”



Lesson Vocabulary

- roster form
- set-builder notation
- empty set
- universal set
- complement of a set

Plan

How are roster form and set-builder notation different?

Roster form *lists* the elements of a set. Set-builder notation *describes* the properties of those elements.



Problem 1 Using Roster Form and Set-Builder Notation

How do you write “ T is the set of natural numbers that are less than 6” in roster form? In set-builder notation?

Roster form

Write “ T is” as “ $T =$.”

List all natural numbers that are less than 6.

$$T = \{1, 2, 3, 4, 5\}$$

Set-builder notation

Use a variable.

Describe the limits on the variable.

$$T = \{x \mid x \text{ is a natural number, } x < 6\}$$

- Got It?** 1. N is the set of even natural numbers that are less than or equal to 12. How do you write N in roster form? In set-builder notation?

You can use set-builder notation to write the solutions of a linear inequality.

Problem 2 Inequalities and Set-Builder Notation

Multiple Choice In set-builder notation, how do you write the solutions of $-5x + 7 \leq 17$?

- (A) $x \geq -2$ (C) $\{-2, -1, 0, \dots\}$
 (B) $\{x \mid x \geq -2\}$ (D) $\{x \mid x \leq -2\}$

$$-5x + 7 \leq 17$$

$$-5x + 7 - 7 \leq 17 - 7 \quad \text{Subtract 7 from each side.}$$

$$-5x \leq 10 \quad \text{Simplify.}$$

$$\frac{-5x}{-5} \geq \frac{10}{-5} \quad \text{Divide each side by } -5. \text{ Reverse the inequality symbol.}$$

$$x \geq -2 \quad \text{Simplify.}$$

In set-builder notation, the solutions are given by $\{x \mid x \geq -2\}$. The answer is B.

- Got It?** 2. In set-builder notation, how do you write the solutions of $9 - 4n > 21$?

You know that a set A is a subset of a set B if each element of A is also an element of B . For example, if $B = \{-2, -1, 0, 1, 2, 3\}$ and $A = \{-1, 0, 2\}$, then A is a subset of B . You can write this relationship as $A \subseteq B$.

The **empty set**, or **null set**, is the set that contains no elements. The empty set is a subset of every set. Use \emptyset or $\{\}$ to represent the empty set.

Problem 3 Finding Subsets

What are all the subsets of the set $\{3, 4, 5\}$?

- \emptyset Start with the empty set.
 $\{3\}, \{4\}, \{5\}$ List the subsets with one element.
 $\{3, 4\}, \{3, 5\}, \{4, 5\}$ List the subsets with two elements.
 $\{3, 4, 5\}$ List the original set. It is always considered a subset.

The eight subsets of $\{3, 4, 5\}$ are $\emptyset, \{3\}, \{4\}, \{5\}, \{3, 4\}, \{3, 5\}, \{4, 5\},$ and $\{3, 4, 5\}$.

- Got It?** 3. a. What are the subsets of the set $P = \{a, b\}$? Of the set $S = \{a, b, c\}$?
 (C) b. **Reasoning** Let $A = \{x \mid x < -3\}$ and $B = \{x \mid x \leq 0\}$. Is A a subset of B ? Explain your reasoning.

Plan

How is this problem similar to others you've solved?

It requires using properties of inequality to solve a multi-step inequality, as you did in Lesson 3-4.

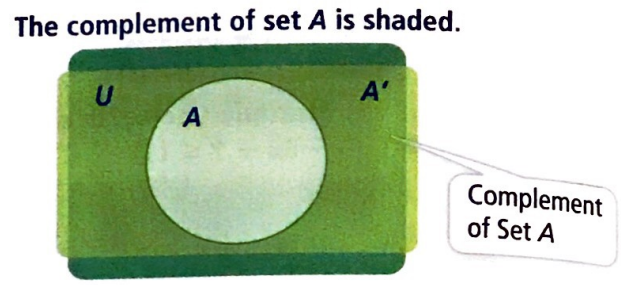
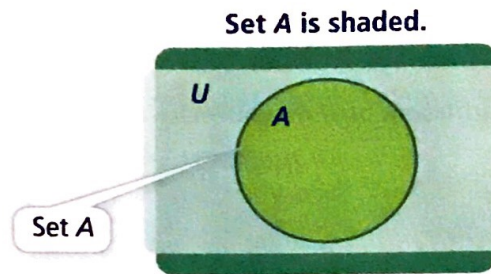
Think

Why is the original set considered a subset?

It's a subset because it contains elements from the original set. In this case, it's the subset that contains all three elements.

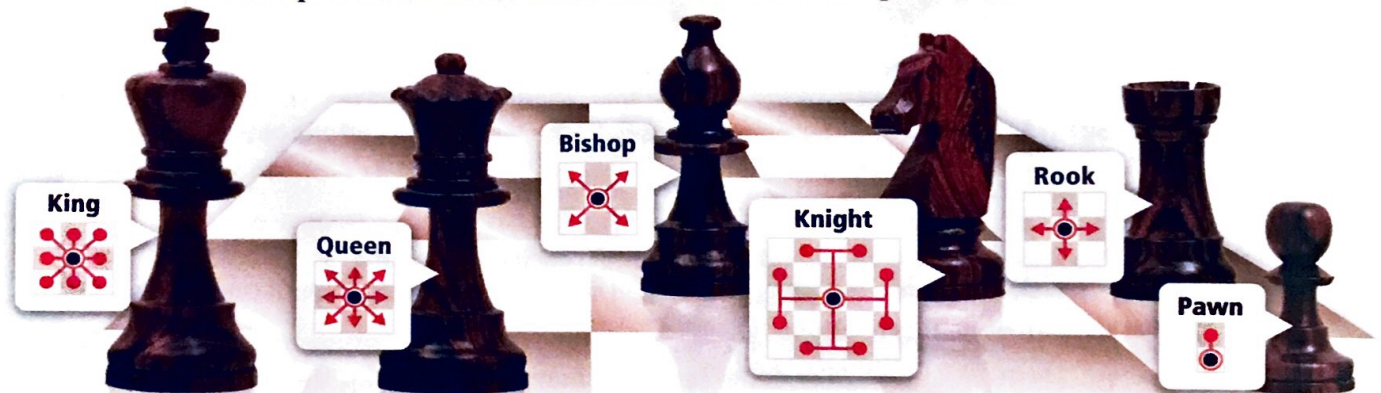
When working with sets, you call the largest set you are using the **universal set**, or universe. The **complement of a set** is the set of all elements in the universal set that are *not* in the set. You denote the complement of A by A' .

In the Venn diagrams below, U represents the universal set. Notice that $A \subseteq U$ and $A' \subseteq U$.



Problem 4 Finding the Complement of a Set

Universal set $U = \{\text{king, queen, bishop, knight, rook, pawn}\}$ and set A is the set of chess pieces that move side to side. What is the complement of set A ?



Know

- The elements of set U
- The elements of set A

Need

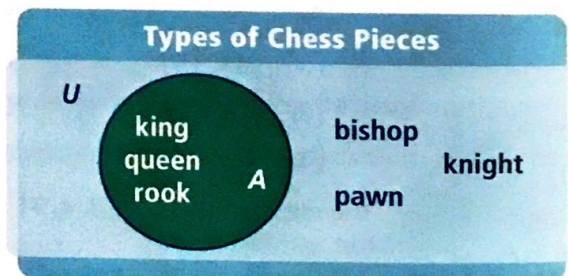
- The elements of A'

Plan

Use a Venn diagram to find all the elements in set U that are *not* in set A .

The Venn diagram shows the relationship between sets A and U . The elements in set U that are *not* in set A are bishop, knight, and pawn.

So, $A' = \{\text{bishop, knight, pawn}\}$.



- Got It?** 4. Universal set $U = \{\text{months of the year}\}$ and set $A = \{\text{months with exactly 31 days}\}$. What is the complement of set A ? Write your answer in roster form.

Lesson Check

Do you know HOW?

1. How do you write “ G is the set of odd natural numbers that are less than 18” in roster form? In set-builder notation?
2. In set-builder notation, how do you write the solutions of $5 + d \leq 8$?
3. What are all the subsets of $\{4, 8, 12\}$?
4. Given the universal set $U = \{\text{seasons of the year}\}$ and $W = \{\text{winter}\}$, what is W' ?

Do you UNDERSTAND?



5. **Vocabulary** What is the complement of A' ? Explain.
6. Is the first set in each pair a subset of the second set? Explain.
a. $\emptyset; \{1, 3, 5\}$ b. $\{1, 3, 5\}; \{1, 3\}$ c. $\{3\}; \{1, 3, 5\}$
7. **Reasoning** A nonempty set is a set that contains at least one element. Given nonempty sets A and B , suppose that $A \subseteq B$. Is $B \subseteq A$ *always*, *sometimes*, or *never* true?
8. **Error Analysis** A student says sets A and B below are the same. What error did the student make?
 $A = \{x \mid x \text{ is a whole number less than } 5\}$
 $B = \{1, 2, 3, 4\}$



Practice and Problem-Solving Exercises



A Practice

Write each set in roster form and in set-builder notation.

◀ See Problem 1.

9. M is the set of integers that are greater than -1 and less than 4 .
10. N is the set of real numbers that are factors of 12 .
11. P is the set of natural numbers that are less than 11 .
12. R is the set of even natural numbers that are less than 2 .

Write the solutions of each inequality in set-builder notation.

◀ See Problem 2.

- | | | |
|----------------------|---------------------------|------------------------|
| 13. $4y + 7 \geq 23$ | 14. $5r + 8 < 63$ | 15. $13 - 9m < 58$ |
| 16. $7 - 3d \geq 28$ | 17. $2(3p - 11) \geq -16$ | 18. $3(2k + 12) < -42$ |

List all the subsets of each set.

◀ See Problem 3.

- | | | |
|----------------------|-------------------|---------------------------------|
| 19. $\{a, e, i, o\}$ | 20. $\{0, 1, 2\}$ | 21. $\{\text{dog, cat, fish}\}$ |
| 22. $\{-2, 2\}$ | 23. $\{1\}$ | 24. $\{+, -, \times, \div\}$ |

25. Suppose $U = \{1, 2, 3, 4, 5\}$ is the universal set and $A = \{2, 3\}$. What is A' ?

◀ See Problem 4.

26. Suppose $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ is the universal set and $P = \{2, 4, 6, 8\}$. What is P' ?

27. Suppose $U = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ is the universal set and $R = \{\dots, -3, -1, 1, 3, \dots\}$. What is R' ?

28. Suppose $U = \{1, 2\}$ is the universal set and $T = \{1\}$. What is T' ?

- © 29. **Think About a Plan** Universal set U and set A are defined below. What are the elements of A ?

$$U = \{\text{days of the week}\}$$

$$A = \{\text{days of the week that contain the letter N}\}$$

- What are the elements of the universal set?
- What are the elements of set A ?
- How can you find the complement of set A ?

Suppose $U = \{0, 1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\}$, and $B = \{1, 2, 3\}$. Tell whether each statement is *true* or *false*. Explain your reasoning.

30. $A \subseteq U$

31. $U \subseteq B$

32. $B \subseteq A$

33. $\emptyset \subseteq B$

Write each set in set-builder notation.

34. $B = \{11, 12, 13, 14, \dots\}$

35. $M = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

36. $S = \{1, 2, 3, 4, 6, 12\}$

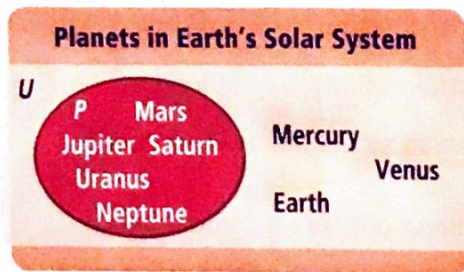
37. $G = \{\dots, -2, -1, 0, 1, 2, \dots\}$

38. Universal set U and set B are defined below. What are the elements of B '?

$$U = \{\text{states of the United States}\}$$

$$B = \{\text{states that do not start with the letter A}\}$$

39. Universal set $U = \{\text{planets in Earth's solar system}\}$ and set $P = \{\text{planets farther from the sun than Earth is from the sun}\}$. What is the complement of set P ? Write your answer in roster form.



Solve each inequality. Write your solutions in set-builder notation.

40. $-2(3x + 7) > -14$

41. $-2(3x + 7) > -14 - 6x$

42. $-2(3x + 7) \geq -14 - 6x$

43. $-3(4x + 8) + 1 \geq -23$

44. $-3(4x + 8) + 1 \geq -23 - 12x$

45. $-3(4x + 8) + 1 < -23 - 12x$

46. Suppose $U = \{x \mid x \text{ is a multiple of } 2, x < 18\}$ is the universal set and $C = \{4, 8, 12, 16\}$. What is C '?

47. Suppose $U = \{x \mid x \text{ is an integer, } x \leq 12\}$ is the universal set and $T = \{x \mid x \text{ is a natural number, } x \leq 12\}$. What is T '?

- © 48. **Open-Ended** Write a two-step inequality with solutions that are given by $\{n \mid n > 0\}$.

49. How many elements are in the set $\{x \mid x \text{ is an even prime number, } x < 100\}$?

- Challenge** 50. **Reasoning** Without listing each subset of a set, how can you determine the number of subsets that the set has?

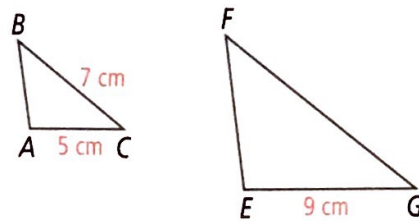
Use your answer from Exercise 50. Determine how many subsets each set has.

51. $R = \{\text{positive even numbers less than 20}\}$ 52. $Q = \{0\}$

Standardized Test Prep

SAT/ACT

53. Let the universal set be $U = \{x \mid x \text{ is a natural number}\}$, and let set $E = \{2, 4, 6, 8, \dots\}$. What is E' ?
- (A) $\{1, 3, 5, 7, \dots\}$ (C) $\{\text{all positive integers}\}$
 (B) $\{0, 2, 4, 6, 8, \dots\}$ (D) $\{2, 4, 6, 8, \dots\}$
54. Which set represents the solutions of $-9x + 17 \geq -64$?
- (F) $\{x \mid x \leq 9\}$ (G) $\{x \mid x \geq 9\}$ (H) $\{x \mid x \leq -\frac{47}{9}\}$ (I) $\{x \mid x \geq -\frac{47}{9}\}$
55. In the diagram below, $\triangle ABC \sim \triangle EFG$. What is FG ?



- (A) $3\frac{8}{9}$ (B) $6\frac{3}{7}$ (C) 11 (D) $12\frac{3}{5}$
56. What is the least whole-number solution of $-10n \leq 5$?
- (F) -1 (G) 0 (H) 1 (I) 2
57. Mum's Florist sells two dozen roses for \$24.60. First Flowers Florist sells 6 roses for \$7.50. Which florist has the lower cost per rose? Explain.

Short Response

Mixed Review

Solve each inequality.

58. $3b + 2 > 26$

59. $2(t + 2) - 3t \geq -1$

60. $6z - 15 < 4z + 11$

See Lesson 3-4.

Evaluate each expression for the given value of the variable.

61. $3n - 6$; $n = 4$

62. $7 - 2b$; $b = 5$

63. $\frac{2d - 3}{5}$; $d = 9$

See Lesson 1-2.

Get Ready! To prepare for Lesson 3-6, do Exercises 64-66.

Graph each pair of inequalities on one number line.

See Lesson 3-1.

64. $c < 8$; $c \geq 10$

65. $t \geq -2$; $t \leq -5$

66. $m \leq 7$; $m > 12$