

Some equations, such as $|2x - 5| = 13$, have variable expressions within absolute value symbols. The equation $|2x - 5| = 13$ means that the distance on a number line from $2x - 5$ to 0 is 13 units. There are two points that are 13 units from 0: 13 and -13 . So to find the values of x , solve the equations $2x - 5 = 13$ and $2x - 5 = -13$. You can generalize this process as follows.

Take note

Key Concept Solving Absolute Value Equations

To solve an equation in the form $|A| = b$, where A represents a variable expression and $b > 0$, solve $A = b$ and $A = -b$.



Problem 2 Solving an Absolute Value Equation

Multiple Choice Starting from 100 ft away, your friend skates toward you and then passes by you. She skates at a constant speed of 20 ft/s. Her distance d from you in feet after t seconds is given by $d = |100 - 20t|$. At what times is she 40 ft from you?

- (A) -2 s and 8 s (B) -3 s and 7 s (C) 3 s and 7 s (D) 2 s and 8 s

$$100 - 20t = 40 \qquad \leftarrow \text{Write two equations.} \rightarrow \qquad 100 - 20t = -40$$

$$-20t = -60 \qquad \leftarrow \text{Subtract 100 from each side.} \rightarrow \qquad -20t = -140$$

$$t = 3 \qquad \leftarrow \text{Divide each side by } -20. \rightarrow \qquad t = 7$$

The solutions are 3 s and 7 s. The correct answer is C.



Got It? 2. Another friend's distance d from you (in feet) after t seconds is given by $d = |80 - 5t|$. What does the 80 in the equation represent? What does the 5 in the equation represent? At what times is she 60 ft from you?

Recall that absolute value represents distance from 0 on a number line. Distance is always nonnegative. So any equation that states that the absolute value of an expression is negative has no solutions.



Problem 3 Solving an Absolute Value Equation With No Solution

What are the solutions of $3|2z + 9| + 12 = 10$?

$$3|2z + 9| + 12 = 10$$

$$3|2z + 9| = -2 \quad \text{Subtract 12 from each side.}$$

$$|2z + 9| = -\frac{2}{3} \quad \text{Divide each side by 3.}$$

The absolute value of an expression cannot be negative, so there is no solution.



Got It? 3. What are the solutions of $|3x - 6| - 5 = -7$?

Plan

What must be true of the expression $100 - 20t$?

Its absolute value is 40, so $100 - 20t$ must equal either 40 or -40 . Use this fact to write and solve two equations.

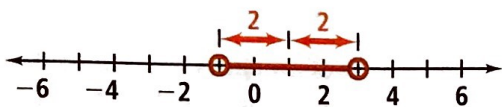
Plan

How can you make the equation look like one you've solved before?

Use properties of equality to isolate the absolute value expression on one side of the equal sign.

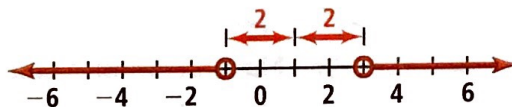
You can write absolute value inequalities as compound inequalities. The graphs below show two absolute value inequalities.

$$|n - 1| < 2$$



$|n - 1| < 2$ represents all numbers with a distance from 1 that is less than 2 units. So $|n - 1| < 2$ means $-2 < n - 1 < 2$.

$$|n - 1| > 2$$



$|n - 1| > 2$ represents all numbers with a distance from 1 that is greater than 2 units. So $|n - 1| > 2$ means $n - 1 < -2$ or $n - 1 > 2$.

Take note

Key Concept Solving Absolute Value Inequalities

To solve an inequality in the form $|A| < b$, where A is a variable expression and $b > 0$, solve the compound inequality $-b < A < b$.



To solve an inequality in the form $|A| > b$, where A is a variable expression and $b > 0$, solve the compound inequality $A < -b$ or $A > b$.



Similar rules are true for $|A| \leq b$ or $|A| \geq b$.



Problem 4 Solving an Absolute Value Inequality Involving \geq

What are the solutions of $|8n| \geq 24$? Graph the solutions.

Think

The inequality says that $8n$ is at least 24 units from 0 on a number line.

To be at least 24 units from 0, $8n$ can be less than or equal to -24 or greater than or equal to 24.

You need to isolate n . Undo multiplication by dividing each side by the same number.

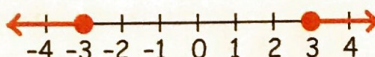
Write

$$|8n| \geq 24$$

$$8n \leq -24 \text{ or } 8n \geq 24$$

$$\frac{8n}{8} \leq \frac{-24}{8} \text{ or } \frac{8n}{8} \geq \frac{24}{8}$$

$$n \leq -3 \text{ or } n \geq 3$$





4. What are the solutions of $|2x + 4| \geq 5$? Graph the solutions.



Problem 5

Solving an Absolute Value Inequality Involving \leq STEM

Manufacturing A company makes boxes of crackers that should weigh 213 g. A quality-control inspector randomly selects boxes to weigh. Any box that varies from the weight by more than 5 g is sent back. What is the range of allowable weights for a box of crackers?

Relate difference between actual and ideal weights is at most 5 g

Define Let w = the actual weight in grams.

Write $|w - 213| \leq 5$

$$|w - 213| \leq 5$$

$$-5 \leq w - 213 \leq 5 \quad \text{Write a compound inequality.}$$

$$208 \leq w \leq 218 \quad \text{Add 213 to each expression.}$$

The weight of a box of crackers must be between 208 g and 218 g, inclusive.

Think

How else could you write this inequality?

You could break the compound inequality into two parts:
 $w - 213 \geq -5$ and
 $w - 213 \leq 5$.



5. a. A food manufacturer makes 32-oz boxes of pasta. Not every box weighs exactly 32 oz. The allowable difference from the ideal weight is at most 0.05 oz. Write and solve an absolute value inequality to find the range of allowable weights.
- b. **Reasoning** In Problem 5, could you have solved the inequality $|w - 213| \leq 5$ by first adding 213 to each side? Explain your reasoning.



Lesson Check

Do you know HOW?

Solve and graph each equation or inequality.

- $|x| = 5$
- $|n| - 3 = 4$
- $|2t| = 6$
- $|h - 3| < 5$
- $|x + 2| \geq 1$

Do you UNDERSTAND? M A T H E M A T I C E S

- Reasoning** How many solutions do you expect to get when you solve an absolute value equation? Explain.
- Writing** Explain why the absolute value equation $|3x| + 8 = 5$ has no solution.
- Compare and Contrast** Explain the similarities and differences in solving the equation $|x - 1| = 2$ with solving the inequalities $|x - 1| \leq 2$ and $|x - 1| \geq 2$.

A Practice

Solve each equation. Graph and check your solutions.

◀ See Problem 1.

- | | | | |
|------------------------|-----------------|-------------------|-------------------|
| 9. $ b = \frac{1}{2}$ | 10. $4 = y $ | 11. $ n + 3 = 7$ | 12. $7 = s - 3$ |
| 13. $ x - 10 = -2$ | 14. $5 d = 20$ | 15. $-3 m = -9$ | 16. $ y + 3 = 3$ |

Solve each equation. If there is no solution, write *no solution*.

◀ See Problems 2 and 3.

- | | | |
|------------------------|--------------------------|-------------------------|
| 17. $ r - 8 = 5$ | 18. $ c + 4 = 6$ | 19. $2 = g + 3 $ |
| 20. $3 = m + 2 $ | 21. $-2 7d = 14$ | 22. $-3 2w = -12$ |
| 23. $3 v - 3 = 9$ | 24. $2 d + 4 = 8$ | 25. $ 4f + 1 - 2 = 5$ |
| 26. $ 3t - 2 + 6 = 2$ | 27. $4 2y - 3 - 1 = 11$ | 28. $3 x + 2 + 4 = 13$ |
| 29. $-4 k = 12$ | 30. $ -3n - 2 = 4$ | 31. $-4 k + 1 = 16$ |

Solve and graph each inequality.

◀ See Problems 4 and 5.

- | | | |
|------------------------|------------------------|------------------------|
| 32. $ x \geq 3$ | 33. $ x < 5$ | 34. $ x + 3 < 5$ |
| 35. $ y + 8 \geq 3$ | 36. $ y - 2 \leq 1$ | 37. $ p - 7 \leq 3$ |
| 38. $ 2c - 5 < 9$ | 39. $ 3t + 1 > 8$ | 40. $ 4w + 1 > 11$ |
| 41. $ 5t - 4 \geq 16$ | 42. $ 4x + 7 > 19$ | 43. $ 2v - 1 \leq 9$ |
| 44. $ 3d - 7 > 28$ | 45. $ 2f + 9 \leq 13$ | 46. $ 5m - 9 \geq 24$ |

47. **Quality Control** The ideal length of one type of model airplane is 90 cm. The actual length may vary from ideal by at most 0.05 cm. What are the acceptable lengths for the model airplane?

48. **Basketball** The ideal circumference of a women's basketball is 28.75 in. The actual circumference may vary from the ideal by at most 0.25 in. What are the acceptable circumferences for a women's basketball?

B Apply

Solve each equation or inequality. If there is no solution, write *no solution*.

- | | | |
|---------------------------------------|--------------------------|---|
| 49. $ 2d + 3 = 21$ | 50. $1.2 5p = 3.6$ | 51. $ d + \frac{1}{2} + \frac{3}{4} = 0$ |
| 52. $ f - \frac{2}{3} = \frac{5}{6}$ | 53. $3 5y - 7 - 6 = 24$ | 54. $ t + 2.7 = 4.5$ |
| 55. $-2 c - 4 = -8.4$ | 56. $\frac{ y }{-3} = 5$ | 57. $ n - \frac{5}{4} < 5$ |
| 58. $\frac{7}{8} < c + 7 $ | 59. $4 - 3 m + 2 > -14$ | 60. $ -3d \geq 6.3$ |

© 61. **Think About a Plan** The monthly average temperature T for San Francisco, California, is usually within 7.5°F of 56.5°F , inclusive. What is the monthly average temperature in San Francisco?

- Should you model this situation with an equation or an inequality?
- How can you use the given information to write the equation or inequality?

STEM 62. **Biology** A horse's body temperature T is considered to be normal if it is within at least 0.9°F of 99.9°F . Find the range of normal body temperatures for a horse.

63. **Biking** Your friend rides his bike toward you and then passes by you at a constant speed. His distance d (in feet) from you t seconds after he started riding his bike is given by $d = |200 - 18t|$. What does the 200 in the equation represent? What does the 18 in the equation represent? At what time(s) is he 120 ft from you?

Error Analysis Find and correct the mistake in solving each equation or inequality.

64. ~~$$\begin{aligned} |x - 3| &= -2 \\ x - 3 &= -2 \text{ or } x - 3 = 2 \\ x &= 1 \text{ or } x = 5 \end{aligned}$$~~

65. ~~$$\begin{aligned} |y + 7| &\leq 1 \\ y + 7 &\leq -1 \text{ or } y + 7 \geq 1 \\ y &\leq -8 \text{ or } y \geq -6 \end{aligned}$$~~

Open-Ended 66. Write an absolute value equation that has 2 and 6 as solutions.

Reasoning 67. Explain why you can rewrite $|x + 5| > 1$ as a compound inequality involving *or*.

Polling 68. According to a poll for an upcoming school board election, 40% of voters are likely to vote for the incumbent. The poll shows a margin of error of ± 3 percentage points. Write and solve an absolute value equation to find the least and the greatest percents of voters v likely to vote for the incumbent.

Banking 69. The official weight of a nickel is 5 g, but the actual weight can vary from this amount by up to 0.194 g. Suppose a bank weighs a roll of 40 nickels. The wrapper weighs 1.5 g.

a. What is the range of possible weights for the roll of nickels?

b. **Reasoning** If all of the nickels in the roll each weigh the official amount, then the roll's weight is $40(5) + 1.5 = 201.5$ g. Is it possible for a roll to weigh 201.5 g and contain nickels that do not weigh the official amount? Explain.

STEM 70. **Oil Production** An oil refinery aims to process 900,000 barrels of oil per day. The daily production varies by up to 50,000 barrels from this goal, inclusive. What are the minimum and maximum numbers of barrels of oil processed each day?

Write an absolute value inequality that represents each set of numbers.

71. all real numbers less than 4 units from 0

72. all real numbers at most 7 units from 0

73. all real numbers more than 2 units from 6

74. all real numbers at least 2 units from -1

STEM 75. **Manufacturing** The ideal diameter of a piston for one type of car engine is 90.000 mm. The actual diameter can vary from the ideal by at most 0.008 mm. What is the range of acceptable diameters for the piston?

76. **Farm Maintenance** For safety, the recommended height of a horse fence is 5 ft. Because of uneven ground surfaces, the actual height of the fence can vary from this recommendation by up to 3 in. Write and solve an absolute value equation to find the maximum and minimum heights of the fence.

Challenge Solve each equation. Check your solutions.

77. $|x + 4| = 3x$

78. $|4t - 5| = 2t + 1$

79. $\frac{4}{3}|2y + 3| = 4y$

Determine whether each statement is *always*, *sometimes*, or *never* true for real numbers a and b .

80. $|ab| = |a| \cdot |b|$

81. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$

82. $|a + b| = |a| + |b|$

Standardized Test Prep

GRIDDED RESPONSE

83. The expected monthly rainfall in a certain town is shown for June, July, and August. The actual rainfall generally varies from the expected amount by up to 0.015 in. What is the maximum amount of rainfall the town can expect to receive in July?

Expected Monthly Rainfall (inches)		
June	July	August
4.12	4.25	4.41

84. What is the solution of the equation $\frac{x}{4} - 3 = 7$?

85. What is the solution of the equation $3w + 2 = 4w - 3$?

86. Jose is purchasing 4 dress shirts that cost \$28 each and 2 pairs of pants that cost \$38 each. The items are all on sale for 35% off. How much money will Jose save by purchasing them on sale instead of at full price?

87. 75% of what number is 90?

Mixed Review

Write a compound inequality to model each situation.

See Lesson 3-6.

88. The highest elevation in North America is 20,310 ft above sea level at Denali in Alaska. The lowest elevation in North America is 282 ft below sea level at Death Valley, California.

89. Normal human body temperature T is within 0.3°C of 37.2°C .

Simplify each expression.

See Lesson 1-7.

90. $2(x + 5)$

91. $-3(y - 7)$

92. $4(\ell + 3) - 7$

93. $-(m - 4) + 8$

Get Ready! To prepare for Lesson 3-8, do Exercises 94-97.

Write each set in set-builder notation.

See Lesson 3-5.

94. $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

95. $B = \{1, 3, 5, 7\}$

Write each set in roster form.

96. $C = \{n \mid n \text{ is an even number between } -15 \text{ and } -5\}$

97. $D = \{k \mid k \text{ is a composite number between } 7 \text{ and } 17\}$