4-3 Patterns and Nonlinear Functions

Common Core State Standards

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **Also F-IF.B.4**

MP 1, MP 2, MP 3, MP 4, MP 6

Objective To identify and represent patterns that describe nonlinear functions

Language is important! Make sure you know the definition of a function.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

The table shows the relationship between the number of steps in the staircase below and the number of blocks needed to build the staircase. Copy and complete the table. Is the relationship a function? If so, is it a linear function? Explain.

1 step 2 steps 3 steps

Number of Steps	Number of Blocks	Ordered Pair
1	1	(1, 1)
2	3	(2, 3)
3	6	(3, 6)
4	■	■
5	■	■

Lesson Vocabulary
- nonlinear function

The relationship in the Solve It is an example of a nonlinear function. A **nonlinear function** is a function whose graph is not a line or part of a line.

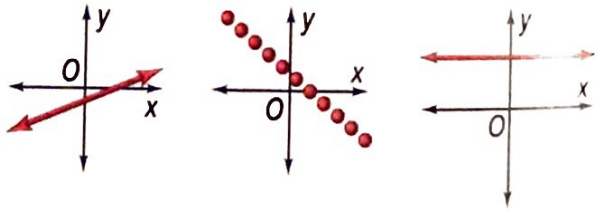
Essential Understanding Just like linear functions, nonlinear functions can be represented using words, tables, equations, sets of ordered pairs, and graphs.

take note

Concept Summary Linear and Nonlinear Functions

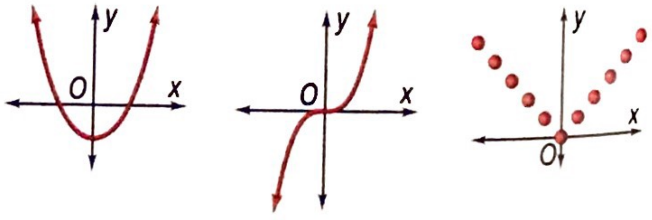
Linear Function

A linear function is a function whose graph is a nonvertical line or part of a nonvertical line.



Nonlinear Function

A nonlinear function is a function whose graph is not a line or part of a line.



Problem 1 Classifying Functions as Linear or Nonlinear

Pizza The area A, in square inches, of a pizza is a function of its radius r, in inches. The cost C, in dollars, of the sauce for a pizza is a function of the weight w, in ounces, of sauce used. Graph these functions shown by the tables below. Is each function *linear* or *nonlinear*?

Pizza Area

Radius (in.), r	Area (in.^{2}), A
2	12.57
4	50.27
6	113.10
8	201.06
10	314.16

Sauce Cost

Weight (oz), w	Cost, C
2	\$.80
4	\$1.60
6	\$2.40
8	\$3.20
10	\$4.00

Know
The relationships shown in the tables are functions.

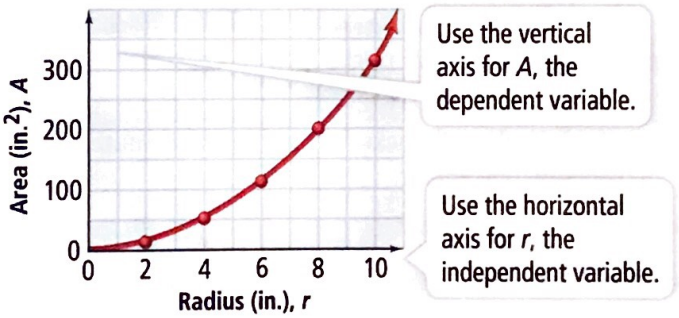
Need
To classify the functions as *linear* or *nonlinear*

Plan
Use the tables to make graphs.

Think
How can a graph tell you if a function is linear or nonlinear?
The graph of a linear function is a nonvertical line or part of a line, but the graph of a nonlinear function is not.

Graph A as a function of r.

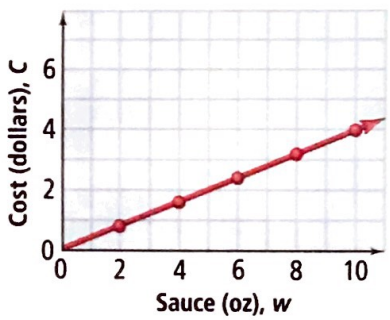
Pizza Area



The graph is a curve, not a line, so the function is nonlinear.

Graph C as a function of w.

Sauce Cost



The graph is a line, so the function is linear.

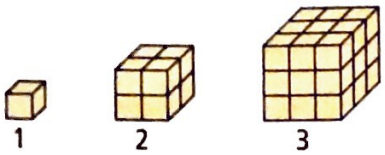
Got It? **1. a.** The table below shows the fraction A of the original area of a piece of paper that remains after the paper has been cut in half n times. Graph the function represented by the table. Is the function *linear or nonlinear*?

Cutting Paper				
Number of Cuts, n	1	2	3	4
Fraction of Original Area Remaining, A	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

b. Reasoning Will the area A in part (a) ever reach zero? Explain.

Problem 2 Representing Patterns and Nonlinear Functions

The table shows the total number of blocks in each figure below as a function of the number of blocks on one edge.



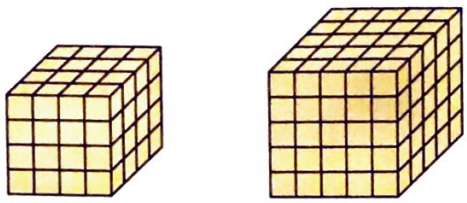
Number of Blocks on Edge, x	Total Number of Blocks, y	Ordered Pair (x, y)
1	1	(1, 1)
2	8	(2, 8)
3	27	(3, 27)
4	■	■
5	■	■

What is a pattern you can use to complete the table? Represent the relationship using words, an equation, and a graph.

Draw the next two figures to complete the table.

Think

How can you use a pattern to complete the table?
You can draw figures with 4 and 5 blocks on an edge. Then analyze the figures to determine the total number of blocks they contain.



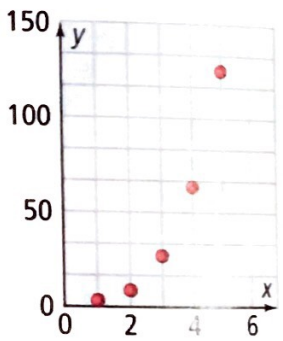
A cube with 4 blocks on an edge contains $4 \cdot 4 \cdot 4 = 64$ blocks. A cube with 5 blocks on an edge contains $5 \cdot 5 \cdot 5 = 125$ blocks.

Number of Blocks on Edge, x	Total Number of Blocks, y	Ordered Pair (x, y)
1	1	(1, 1)
2	8	(2, 8)
3	27	(3, 27)
4	64	(4, 64)
5	125	(5, 125)

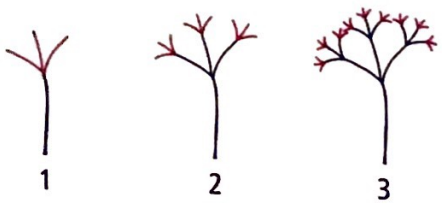
Words The total number of blocks y is the cube of the number of blocks on one edge x.

Equation $y = x^3$

You can use the table to make a graph. The points do not lie on a line. So the relationship between the number of blocks on one edge and the total number of blocks is a nonlinear function.



Got It? **2.** The table shows the number of new branches in each figure of the pattern below. What is a pattern you can use to complete the table? Represent the relationship using words, an equation, and a graph.



Number of Figure, x	1	2	3	4	5
Number of New Branches, y	3	9	27	■	■

A function can be thought of as a rule that you apply to the input in order to get the output. You can describe a nonlinear function with words or with an equation, just as you did with linear functions.

Problem 3 Writing a Rule to Describe a Nonlinear Function

The ordered pairs (1, 2), (2, 4), (3, 8), (4, 16), and (5, 32) represent a function. What is a rule that represents this function?

Make a table to organize the x- and y-values. For each row, identify rules that produce the given y-value when you substitute the x-value. Look for a pattern in the y-values.

x	y
1	2
2	4
3	8
4	16
5	32

What rule produces 2, given an x-value of 1? The rules $y = 2x$, $y = x + 1$, and $y = 2^x$ work for (1, 2).

$y = x + 1$ does not work for (2, 4). $y = 2x$ works for (2, 4), but not for (3, 8). $y = 2^x$ works for all three pairs.

$8 = 2 \cdot 2 \cdot 2$ and $16 = 2 \cdot 2 \cdot 2 \cdot 2$. The pattern of the y-values matches 2^1, 2^2, 2^3, 2^4, 2^5, or $y = 2^x$.

Think

How can you use reasoning to write a rule?
You can *solve a simpler problem* by writing a rule based on the first one or two rows of the table. Then see if the rule works for the other rows.

The function can be represented by the rule $y = 2^x$.

Got It? **3.** What is a rule for the function represented by the ordered pairs (1, 1), (2, 4), (3, 9), (4, 16), and (5, 25)?

Lesson Check

Do you know HOW?

1. Graph the function represented by the table below. Is the function *linear* or *nonlinear*?

x	0	1	2	3	4
y	12	13	14	15	16

2. The ordered pairs (0, −2), (1, 1), (2, 4), (3, 7), and (4, 10) represent a function. What is a rule that represents this function?

3. Which rule could represent the function shown by the table below?

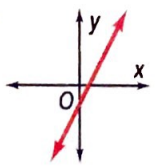
x	0	1	2	3	4
y	0	−1	−4	−9	−16

A. $y = x^2$ **B.** $y = -x^3$ **C.** $y = -x^2$

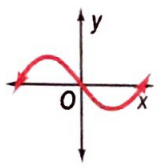
Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary Does the graph represent a *linear function* or a *nonlinear function*? Explain.

a.



b.



5. Error Analysis A classmate says that the function shown by the table at the right can be represented by the rule $y = x + 1$. Describe and correct your classmate's error.

x	y
0	1
1	2
2	5
3	10
4	17

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

See Problem 1.

The cost C, in dollars, for pencils is a function of the number n of pencils purchased. The length L of a pencil, in inches, is a function of the time t, in seconds, it has been sharpened. Graph the function shown by each table below. Tell whether the function is *linear* or *nonlinear*.

6.

Pencil Cost					
Number of Pencils, n	6	12	18	24	30
Cost, C	\$1	\$2	\$3	\$4	\$5

7.

Pencil Sharpening						
Time (s), t	0	3	6	9	12	15
Length (in.), L	7.5	7.5	7.5	7.5	7.4	7.3

Graph the function shown by each table. Tell whether the function is *linear* or *nonlinear*.

8.

x	y
0	5
1	5
2	5
3	5

9.

x	y
0	−4
1	−3
2	0
3	5

10.

x	y
0	0
1	1
2	−5
3	8

11.

x	y
0	0
1	3
2	6
3	9

12. For the diagram below, the table gives the total number of small triangles y in figure number x. What pattern can you use to complete the table? Represent the relationship using words, an equation, and a graph.

See Problem 2.

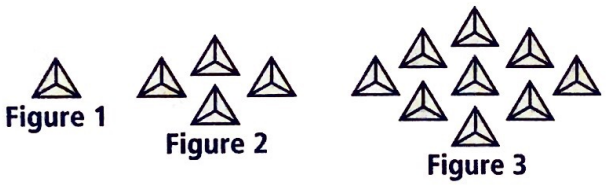


Figure 1 Figure 2 Figure 3

Figure Number, x	Total Small Triangles, y	Ordered Pair (x, y)
1	3	(1, 3)
2	12	(2, 12)
3	27	(3, 27)
4	■	■
5	■	■

Each set of ordered pairs represents a function. Write a rule that represents the function.

See Problem 3.

13. (0, 0), (1, 4), (2, 16), (3, 36), (4, 64)

14. $\left(1, \frac{2}{3}\right), \left(2, \frac{4}{9}\right), \left(3, \frac{8}{27}\right), \left(4, \frac{16}{81}\right), \left(5, \frac{32}{243}\right)$

15. (1, 2), (2, 16), (3, 54), (4, 128), (5, 250)

16. (0, 0), (1, 0.5), (2, 2), (3, 4.5), (4, 8)

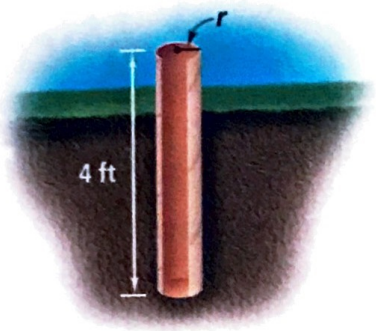
B Apply

17. Writing The rule $V = \frac{4}{3}\pi r^3$ gives the volume V of a sphere as a function of its radius r. Identify the independent and dependent variables in this relationship. Explain your reasoning.

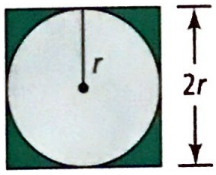
18. Open-Ended Write a rule for a nonlinear function such that y is negative when $x = 1$, positive when $x = 2$, negative when $x = 3$, positive when $x = 4$, and so on.

19. Think About a Plan Concrete forming tubes are used as molds for cylindrical concrete supports. The volume V of a tube is the product of its length ℓ and the area A of its circular base. You can make $\frac{2}{3}$ ft^3 of cement per bag. Write a rule to find the number of bags of cement needed to fill a tube 4 ft long as a function of its radius r. How many bags are needed to fill a tube with a 4-in. radius? A 5-in. radius? A 6-in. radius?

- What is a rule for the volume V of any tube?
- What operation do you use to find the number of bags needed for a given volume?



20. Fountain A designer wants to make a circular fountain inside a square of grass as shown at the right. What is a rule for the area A of the grass as a function of r?



Challenge

21. Reasoning What is a rule for the function represented by $\left(0, \frac{2}{19}\right)$, $\left(1, 1\frac{2}{19}\right)$, $\left(2, 4\frac{2}{19}\right)$, $\left(3, 9\frac{2}{19}\right)$, $\left(4, 16\frac{2}{19}\right)$, and $\left(5, 25\frac{2}{19}\right)$? Explain your reasoning.

22. Reasoning A certain function fits the following description: As the value of x increases by 1 each time, the value of y continually decreases by a smaller amount each time, and never reaches a value as low as 1. Is this function *linear* or *nonlinear*? Explain your reasoning.

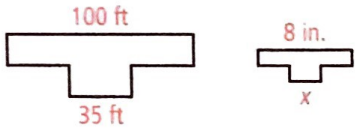
Standardized Test Prep

SAT/ACT

23. The ordered pairs $(-2, 1)$, $(-1, -2)$, $(0, -3)$, $(1, -2)$, and $(2, 1)$ represent a function. Which rule could represent the function?

(A) $y = -3x - 5$ (B) $y = x^2 - 3$ (C) $y = x + 3$ (D) $y = x^2 + 5$

24. You are making a model of the library. The floor plans for the library and the plans for your model are shown. What is the value of x?



(F) 1.4 in. (H) 23.2 in.

(G) 2.8 in. (I) 437.5 in.

Short Response

25. A 15-oz can of tomatoes costs \$.89, and a 29-oz can costs \$1.69. Which can has the lower cost per ounce? Justify your answer.

Mixed Review

26. Determine whether the relationship in the table is a function. Then describe the relationship using words, an equation, and a graph.

x	0	1	2	3
y	3	5	7	9

See Lesson 4-2.

Get Ready! To prepare for Lesson 4-4, do Exercises 27–29.

Evaluate each expression for $x = -3$, $x = 0$, and $x = 2.5$.

See Lesson 1-2.

27. $7x - 3$ **28.** $1 + 4x$ **29.** $-2x^2$