

4-6

Formalizing Relations and Functions

Common Core State Standards

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range . . . **Also F-IF.A.2**

MP 1, MP 2, MP 3, MP 4, MP 6

Objectives To determine whether a relation is a function
To find domain and range and use function notation



Where you land is related to where you start.



Getting Ready!

Your friend is playing a board game. He is on the space shown in the diagram at the right. He rolled a 3 to get to that space. Where could he have started? Explain your reasoning.



Lesson Vocabulary

- relation
- domain
- range
- vertical line test
- function notation

A **relation** is a pairing of numbers in one set, called the **domain**, with numbers in another set, called the **range**. A relation is often represented as a set of ordered pairs (x, y) . In this case, the domain is the set of x -values and the range is the set of y -values.

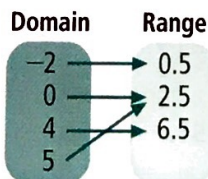
Essential Understanding A function is a special type of relation in which each value in the domain is paired with exactly one value in the range.



Problem 1 Identifying Functions Using Mapping Diagrams

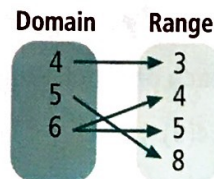
Identify the domain and range of each relation. Represent the relation with a mapping diagram. Is the relation a function?

- A** $\{(-2, 0.5), (0, 2.5), (4, 6.5), (5, 2.5)\}$
The domain is $\{-2, 0, 4, 5\}$.
The range is $\{0.5, 2.5, 6.5\}$.



Each domain value is mapped to only one range value. The relation is a function.

- B** $\{(6, 5), (4, 3), (6, 4), (5, 8)\}$
The domain is $\{4, 5, 6\}$.
The range is $\{3, 4, 5, 8\}$.



The domain value 6 is mapped to two range values. The relation is not a function.

Think

When is a relation not a function?

A function maps each domain value to exactly one range value. So a relation that maps a domain value to more than one range value cannot be a function.

- Got It?** 1. Identify the domain and range of each relation. Represent the relation with a mapping diagram. Is the relation a function?

a. $\{(4.2, 1.5), (5, 2.2), (7, 4.8), (4.2, 0)\}$ b. $\{(-1, 1), (-2, 2), (4, -4), (7, -7)\}$

Another way to decide if a relation is a function is to analyze the graph of the relation using the **vertical line test**. If any vertical line passes through more than one point of the graph, then for some domain value there is more than one range value. So the relation is not a function.

Problem 2 Identifying Functions Using the Vertical Line Test

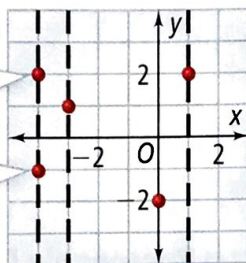
Is the relation a function? Use the vertical line test.

A $\{(-4, 2), (-3, 1), (0, -2), (-4, -1), (1, 2)\}$ **B** $y = -x^2 + 3$

Think

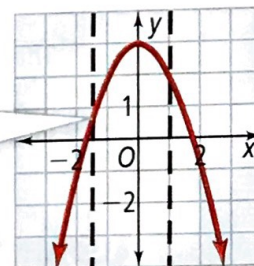
Use a pencil as a vertical line. Place the pencil parallel to the y -axis and slide it across the graph. See if the pencil intersects more than one point at any time.

The domain value -4 corresponds to two range values, 2 and -1 .



The relation is not a function.

There is no vertical line that passes through more than one point of the graph.



The relation is a function.

- Got It?** 2. Is the relation a function? Use the vertical line test.

a. $\{(4, 2), (1, 2), (0, 1), (-2, 2), (3, 3)\}$ b. $\{(0, 2), (1, -1), (-1, 4), (0, -3), (2, 1)\}$

You have seen functions represented as equations involving x and y , such as $y = -3x + 1$. Below is the same equation written using **function notation**.

$$f(x) = -3x + 1$$

Notice that $f(x)$ replaces y . It is read “ f of x .” The letter f is the name of the function, not a variable. Function notation is used to emphasize that the function value $f(x)$ depends on the independent variable x . Other letters besides f can also be used, such as g and h .

Problem 3 Evaluating a Function

Reading The function $w(x) = 250x$ represents the number of words $w(x)$ you can read in x minutes. How many words can you read in 8 min?

$$w(x) = 250x$$

$$w(8) = 250(8) \quad \text{Substitute 8 for } x.$$

$$w(8) = 2000 \quad \text{Simplify.}$$

You can read 2000 words in 8 min.

Think

How is this function like ones you've seen before?

The function $w(x) = 250x$ can be written as $y = 250x$. Remember that $w(x)$ does not mean w times x .

Got It? 3. Use the function in Problem 3. How many words can you read in 6 min?

Problem 4 Finding the Range of a Function

Multiple Choice The domain of $f(x) = -1.5x + 4$ is $\{1, 2, 3, 4\}$. What is the range?

- (A) $\{-2, -0.5, 1, 2.5\}$ (C) $\{-2.5, -1, -0.5, 2\}$
 (B) $\{-2.5, -1, 0.5, 2\}$ (D) $\{-2.5, -0.5, 1, 2\}$

Think

What is another way to think of the domain and range?

The domain is the set of input values for the function. The range is the set of output values.

Step 1 Make a table. List the domain values as the x -values.

x	$-1.5x + 4$	$f(x)$
1	$-1.5(1) + 4$	2.5
2	$-1.5(2) + 4$	1
3	$-1.5(3) + 4$	-0.5
4	$-1.5(4) + 4$	-2

Step 2 Evaluate $f(x)$ for each domain value. The values of $f(x)$ form the range.

The range is $\{-2, -0.5, 1, 2.5\}$. The correct answer is A.

Got It? 4. The domain of $g(x) = 4x - 12$ is $\{1, 3, 5, 7\}$. What is the range?

Problem 5 Identifying a Reasonable Domain and Range

Painting You have 3 qt of paint to paint the trim in your house. A quart of paint covers 100 ft^2 . The function $A(q) = 100q$ represents the area $A(q)$, in square feet, that q quarts of paint cover. What domain and range are reasonable for the function? What is the graph of the function?

Know

- One quart of paint covers 100 ft^2 .
- You have 3 qt of paint.

Need

Reasonable domain and range values in order to graph the function

Plan

Find the least and greatest amounts of paint you can use and areas of trim you can cover. Use these values to make a graph.

The least amount of paint you can use is none. So the least domain value is 0. You have only 3 qt of paint, so the most paint you can use is 3 qt. The greatest domain value is 3. The domain is $0 \leq q \leq 3$.

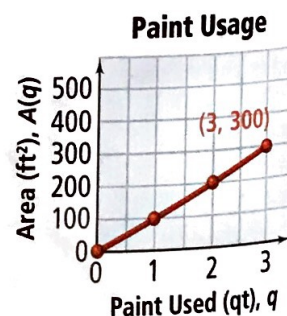
To find the range, evaluate the function using the least and greatest domain values.

$$A(0) = 100(0) = 0 \quad A(3) = 100(3) = 300$$

The range is $0 \leq A(q) \leq 300$.

To graph the function, make a table of values. Choose values of q that are in the domain. The graph is a line segment that extends from $(0, 0)$ to $(3, 300)$.

q	$A(q)$
0	0
1	100
2	200
3	300



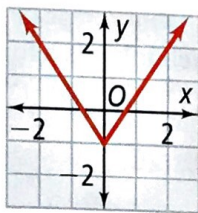


- Got It?** 5. a. If you have 7 qt of paint, what domain and range are reasonable for Problem 5?
 b. **Reasoning** Why does it *not* make sense to have domain values less than 0 or greater than 3 in Problem 5?

Lesson Check

Do you know HOW?

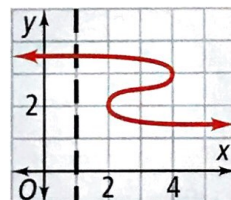
- Identify the domain and range of the relation $\{(-2, 3), (-1, 4), (0, 5), (1, 6)\}$. Represent the relation with a mapping diagram. Is the relation a function?
- Is the relation in the graph shown at the right a function? Use the vertical line test.
- What is $f(2)$ for the function $f(x) = 4x + 1$?
- The domain of $f(x) = \frac{1}{2}x$ is $\{-4, -2, 0, 2, 4\}$. What is the range?



Do you UNDERSTAND?



- Vocabulary** Write $y = 2x + 7$ using function notation.
- Compare and Contrast** You can use a mapping diagram or the vertical line test to tell if a relation is a function. Which method do you prefer? Explain.
- Error Analysis** A student drew the dashed line on the graph shown and concluded that the graph represented a function. Is the student correct? Explain.



Practice and Problem-Solving Exercises



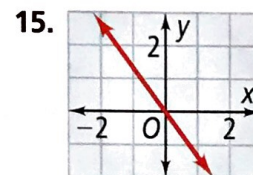
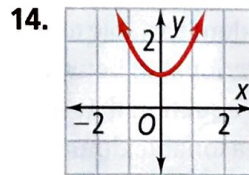
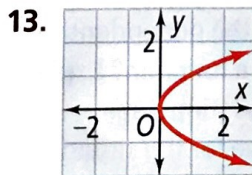
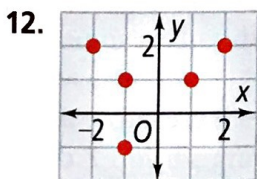
Identify the domain and range of each relation. Use a mapping diagram to determine whether the relation is a function.

- $\{(3, 7), (3, 8), (3, -2), (3, 4), (3, 1)\}$
- $\{(6, -7), (5, -8), (1, 4), (7, 5)\}$
- $\{(0.04, 0.2), (0.2, 1), (1, 5), (5, 25)\}$
- $\{(4, 2), (1, 1), (0, 0), (1, -1), (4, -2)\}$

See Problem 1.

Use the vertical line test to determine whether the relation is a function.

See Problem 2.



- STEM** 16. **Physics** Light travels about 186,000 mi/s. The function $d(t) = 186,000t$ gives the distance $d(t)$, in miles, that light travels in t seconds. How far does light travel in 30 s?

See Problem 3.

17. **Shopping** You are buying orange juice for \$4.50 per container and have a gift card worth \$7. The function $f(x) = 4.50x - 7$ represents your total cost $f(x)$ if you buy x containers of orange juice and use the gift card. How much do you pay to buy 4 containers of orange juice?

Find the range of each function for the given domain.

18. $f(x) = 2x - 7$; $\{-2, -1, 0, 1, 2\}$

19. $g(x) = -4x + 1$; $\{-5, -1, 0, 2, 10\}$

20. $h(x) = x^2$; $\{-1.2, 0, 0.2, 1.2, 4\}$

21. $f(x) = 8x - 3$; $\{-\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}\}$

Find a reasonable domain and range for each function. Then graph the function.

22. **Fuel** A car can travel 32 mi for each gallon of gasoline. The function $d(x) = 32x$ represents the distance $d(x)$, in miles, that the car can travel with x gallons of gasoline. The car's fuel tank holds 17 gal.

23. **Nutrition** There are 98 International Units (IUs) of vitamin D in 1 cup of milk. The function $V(c) = 98c$ represents the amount $V(c)$ of vitamin D, in IUs, you get from c cups of milk. You have a 16-cup jug of milk.

B Apply

Determine whether the relation represented by each table is a function. If the relation is a function, state the domain and range.

24.

x	0	3	3	5
y	2	1	-1	3

25.

x	-4	-1	0	3
y	-4	-4	-4	-4

26. **Open-Ended** Make a table that represents a relation that is not a function. Explain why the relation is not a function.
27. **Reasoning** If $f(x) = 6x - 4$ and $f(a) = 26$, what is the value of a ? Explain.
28. **Think About a Plan** In a factory, a certain machine needs 10 min to warm up. It takes 15 min for the machine to run a cycle. The machine can operate for as long as 6 h per day including warm-up time. Draw a graph showing the total time the machine operates during 1 day as a function of the number of cycles it runs.
- What domain and range are reasonable?
 - Is the function a linear function?
29. **Carwash** A theater group is having a carwash fundraiser. The group can only spend \$34 on soap, which is enough to wash 40 cars. Each car is charged \$5.
- If c is the total number of cars washed and p is the profit, which is the independent variable and which is the dependent variable?
 - Is the relationship between c and p a function? Explain.
 - Write an equation that shows this relationship.
 - Find a reasonable domain and range for the situation.
30. **Open-Ended** What value of x makes the relation $\{(1, 5), (x, 8), (-7, 9)\}$ a function?

Determine whether each relation is a function. Assume that each different variable has a different value.

31. $\{(a, b), (b, a), (c, c), (e, d)\}$

32. $\{(b, b), (c, d), (d, c), (c, a)\}$

33. $\{(c, e), (c, d), (c, b)\}$

34. $\{(a, b), (b, c), (c, d), (d, e)\}$

35. Reasoning Can the graph of a function be a horizontal line? A vertical line? Explain.



36. To form the inverse of a relation written as a set of ordered pairs, you switch the coordinates of each ordered pair. For example, the inverse of the relation $\{(1, 8), (3, 5), (7, 9)\}$ is $\{(8, 1), (5, 3), (9, 7)\}$. Give an example of a relation that is a function, but whose inverse is *not* a function.

Use the functions $f(x) = 2x$ and $g(x) = x^2 + 1$ to find the value of each expression.

37. $f(3) + g(4)$

38. $g(3) + f(4)$

39. $f(5) - 2 \cdot g(1)$

40. $f(g(3))$

Standardized Test Prep

GRIDDED RESPONSE



41. What is the value of the function $f(x) = 7x$ when $x = 0.75$?

42. Andrew needs x dollars for a snack. Scott needs 2 more dollars than Andrew, but Nick only needs half as many dollars as Andrew. Altogether they need \$17 to pay for their snacks. How many dollars does Nick need?

43. What is the greatest number of \$.43 stamps you can buy for \$5?

44. What is the greatest possible width of the rectangle, to the nearest inch?

$\ell = 35$ in.

$A < 184$ in.²

Mixed Review

Write a function rule to represent each situation.

See Lesson 4-5.

45. You baby-sit for \$5 per hour and get a \$7 tip. Your earnings E are a function of the number of hours h you work.

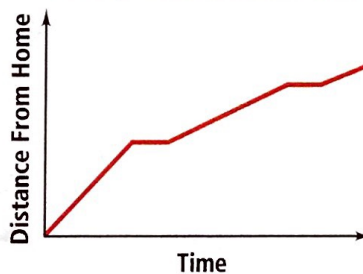
46. You buy several pairs of socks for \$4.50 per pair, plus a shirt for \$10. The total amount a you spend is a function of the number of pairs of socks s you buy.

47. The graph shows a family's distance from home as they drive to the mountains for a vacation.

A Trip to the Mountains

See Lesson 4-1.

- What are the variables in the graph?
- Copy the graph. Describe how the variables are related at various points on the graph.



Get Ready! To prepare for Lesson 4-7, do Exercises 48–50.

Evaluate each expression for $x = 1, 2, 3,$ and 4 .

See Lesson 1-2.

48. $9 + 3(x - 1)$

49. $8 + 7(x - 1)$

50. $0.4 - 3(x - 1)$