4-7 Arithmetic Sequences

Common Core State Standards

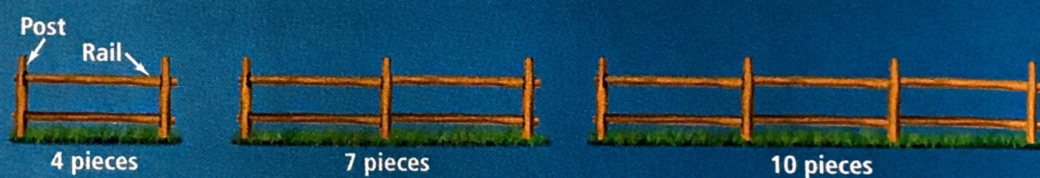
F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, . . . **Also A-SSE.A.1a, F-BF.A.1a, F-BF.A.2, F-LE.A.2**

MP 1, MP 2, MP 3, MP 4, MP 7

Objectives To identify and extend patterns in sequences
To represent arithmetic sequences using function notation

SOLVE IT! **Getting Ready!**

A wooden post-and-rail fence with two rails is made as shown below. Find the number of pieces of wood needed to build a 4-section fence, a 5-section fence, and a 6-section fence. Suppose you want to build a fence with 3 rails. How many pieces of wood are needed for each size fence? Describe the pattern.



Identify the pattern so you can extend it.

MATHEMATICAL PRACTICES

Lesson Vocabulary
- sequence
- term of a sequence
- arithmetic sequence
- common difference
- recursive formula
- explicit formula

In the Solve It, the numbers of pieces of wood used for 1 section of fence, 2 sections of fence, and so on, form a pattern, or a sequence. A **sequence** is an ordered list of numbers that often form a pattern. Each number in the list is called a **term of a sequence.**

Essential Understanding When you can identify a pattern in a sequence, you can use it to extend the sequence. You can also model some sequences with a function rule that you can use to find any term of the sequence.

Problem 1 Extending Sequences

Plan

How can you identify a pattern?
Look at how each term of the sequence is related to the previous term. Your goal is to identify a single rule that you can apply to every term to produce the next term.

Describe a pattern in each sequence. What are the next two terms of each sequence?

A 5, 8, 11, 14, . . .
(+3, +3, +3)

A pattern is "add 3 to the previous term." So the next two terms are $14 + 3 = 17$ and $17 + 3 = 20$.

B 2.5, 5, 10, 20, . . .
(×2, ×2, ×2)

A pattern is "multiply the previous term by 2." So the next two terms are $2(20) = 40$ and $2(40) = 80$.

Got It? **1.** Describe a pattern in each sequence. What are the next two terms of each sequence?

a. 5, 11, 17, 23, . . .

b. 400, 200, 100, 50, . . .

c. 2, −4, 8, −16, . . .

d. −15, −11, −7, −3, . . .

In an **arithmetic sequence,** the difference between consecutive terms is constant. This difference is called the **common difference.**

Problem 2 Identifying an Arithmetic Sequence

Plan

How can you identify an arithmetic sequence?
The difference between every pair of consecutive terms must be the same.

Tell whether the sequence is arithmetic. If it is, what is the common difference?

A 3, 8, 13, 18, . . .

+5 +5 +5

The sequence has a common difference of 5, so it is arithmetic.

B 6, 9, 13, 17, . . .

+3 +4 +4

The sequence does not have a common difference, so it is not arithmetic.

Got It? **2.** Tell whether the sequence is arithmetic. If it is, what is the common difference?

a. 8, 15, 22, 30, . . .

b. 7, 9, 11, 13, . . .

c. 10, 4, −2, −8, . . .

d. 2, −2, 2, −2, . . .

A sequence is a function whose domain is the natural numbers, and whose outputs are the terms of the sequence.

You can write a sequence using a recursive formula. A **recursive formula** is a function rule that relates each term of a sequence after the first to the ones before it. Consider the sequence 7, 11, 15, 19, . . . You can use the common difference of the terms of an arithmetic sequence to write a recursive formula for the sequence. For the sequence 7, 11, 15, 19, . . . , the common difference is 4.

Let n = the term number in the sequence.
Let $A(n)$ = the value of the nth term of the sequence.

value of term 1 = $A(1) = 7$

value of term 2 = $A(2) = A(1) + 4 = 11$

value of term 3 = $A(3) = A(2) + 4 = 15$

value of term 4 = $A(4) = A(3) + 4 = 19$

value of term n = $A(n) = A(n-1) + 4$

The common difference is 4.

The value of the previous term plus 4

The recursive formula for the arithmetic sequence above is $A(n) = A(n-1) + 4$, where $A(1) = 7$.

Problem 3 Writing a Recursive Formula

Write a recursive formula for the arithmetic sequence below. What is the value of the 8th term?

$$70, \quad 77, \quad 84, \quad 91, \ldots$$

$+7 \quad +7 \quad +7$

Step 1	$A(1) = 70$	First term of the sequence
	$A(2) = A(1) + 7 = 70 + 7 = 77$	$A(2)$ is found by adding 7 to $A(1)$.
	$A(3) = A(2) + 7 = 77 + 7 = 84$	$A(3)$ is found by adding 7 to $A(2)$.
	$A(4) = A(3) + 7 = 84 + 7 = 91$	$A(4)$ is found by adding 7 to $A(3)$.
	$A(n) = A(n - 1) + 7$	$A(n)$ is found by adding 7 to $A(n - 1)$.

The recursive formula for the arithmetic sequence is $A(n) = A(n - 1) + 7$, where $A(1) = 70$.

Step 2 To find the value of the 8th term, you need to extend the pattern.

$A(5) = A(4) + 7 = 91 + 7 = 98$

$A(6) = A(5) + 7 = 98 + 7 = 105$

$A(7) = A(6) + 7 = 105 + 7 = 112$

$A(8) = A(7) + 7 = 112 + 7 = 119$

The value of the 8th term is 119.

Got It? **3.** Write a recursive formula for each arithmetic sequence. What is the 9th term of each sequence?

a. 3, 9, 15, 21, . . .

b. 23, 35, 47, 59, . . .

c. 7.3, 7.8, 8.3, 8.8, . . .

d. 97, 88, 79, 70, . . .

e. Reasoning Is a recursive formula a useful way to find the value of an arithmetic sequence? Explain.

You can find the value of any term of an arithmetic sequence using a recursive formula. You can also write a sequence using an explicit formula. An **explicit formula** is a function rule that relates each term of a sequence to the term number.

take note

Key Concept Explicit Formula For an Arithmetic Sequence

The nth term of an arithmetic sequence with first term $A(1)$ and common difference d is given by

$$A(n) = A(1) + (n - 1)d$$

nth term ↑ $A(n)$; first term ↑ $A(1)$; term number ↑ n; common difference ↖ d

Problem 4 Writing an Explicit Formula

Online Auction **An online auction works as shown below. Write an explicit formula to represent the bids as an arithmetic sequence. What is the twelfth bid?**

Make a table of the bids. Identify the first term and common difference.

Term Number, n	1	2	3	4
Value of Term, $A(n)$	200	210	220	230

The first term $A(1)$ is 200.

+10 +10 +10

The common difference d is 10.

> **Plan**
> **What information do you need to write a rule for an arithmetic sequence?**
> You need the first term of the sequence and the common difference.

Substitute $A(1) = 200$ and $d = 10$ into the formula $A(n) = A(1) + (n - 1)d$. The explicit formula $A(n) = 200 + (n - 1)10$ represents the arithmetic sequence of the auction bids. To find the twelfth bid, evaluate $A(n)$ for $n = 12$.

$$A(12) = 200 + (12 - 1)10 = 310$$

The twelfth bid is $310.

Got It? **4. a.** A subway pass has a starting value of $100. After one ride, the value of the pass is $98.25. After two rides, its value is $96.50. After three rides, its value is $94.75. Write an explicit formula to represent the remaining value on the card as an arithmetic sequence. What is the value of the pass after 15 rides?

b. Reasoning How many rides can be taken with the $100 pass?

You can write an explicit formula from a recursive formula and vice versa.

Problem 5 Writing an Explicit Formula From a Recursive Formula

An arithmetic sequence is represented by the recursive formula $A(n) = A(n - 1) + 12$. If the first term of the sequence is 19, write the explicit formula.

The first term is 19, so $A(1) = 19$.

$A(n) = A(n - 1) + 12$ — Adding 12 to the previous term means that the common difference d is 12

$A(n) = A(1) + (n - 1)d$ — General form of an explicit formula

$A(n) = 19 + (n - 1)12$ — Substitute 19 for $A(1)$ and 12 for d.

The explicit formula $A(n) = 19 + (n - 1)12$ represents the arithmetic sequence.

Got It? 5. For each recursive formula, find an explicit formula that represents the same sequence.

a. $A(n) = A(n-1) + 2;\ A(1) = 21$

b. $A(n) = A(n-1) + 7;\ A(1) = 2$

Problem 6 Writing a Recursive Formula From an Explicit Formula

An arithmetic sequence is represented by the explicit formula $A(n) = 32 + (n - 1)(22)$. What is the recursive formula?

$$A(n) = 32 + (n-1)(22)$$

32 is the first term.

22 is the common difference.

A recursive formula relates the value of the term to the previous term using the common difference. Use $A(n)$ to represent the value of the term and $A(n-1)$ to represent the value of the previous term.

The arithmetic sequence is represented by the recursive formula $A(n) = A(n-1) + 22;$ $A(1) = 32$.

Got It? 6. For each explicit formula, find a recursive formula that represents the same sequence.

a. $A(n) = 76 + (n-1)(10)$

b. $A(n) = 1 + (n-1)(3)$

Lesson Check

Do you know HOW?

Describe a pattern in each sequence. Then find the next two terms of the sequence.

1. 3, 11, 19, 27, . . .
2. 3, −6, 12, −24, . . .

Tell whether the sequence is arithmetic. If it is, identify the common difference.

3. 1, −7, −14, −21, . . .
4. 11, 20, 29, 38, . . .
5. Write a recursive and an explicit formula for the arithmetic sequence.

   9, 7, 5, 3, 1, . . .

Do you UNDERSTAND?

MATHEMATICAL PRACTICES

6. **Vocabulary** Consider the following arithmetic sequence: 25, 19, 13, 7, . . . Is the common difference 6 or −6? Explain.
7. **Error Analysis** Describe and correct the error below in finding the tenth term of the arithmetic sequence 4, 12, 20, 28, . . .

   first term = 4
   common difference = 8
   tenth term = 4 + 10(8) = 84

8. **Reasoning** Can you use the explicit formula below to find the nth term of an arithmetic sequence with a first term $A(1)$ and a common difference d? Explain.

$$A(n) = A(1) + nd - d$$

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Describe a pattern in each sequence. Then find the next two terms of the sequence.

See Problem 1.

9. 6, 13, 20, 27, . . .

10. 8, 4, 2, 1, . . .

11. 2, 6, 10, 14, . . .

12. 10, 4, −2, −8, . . .

13. 13, 11, 9, 7, . . .

14. 2, 20, 200, 2000, . . .

15. 1.1, 2.2, 3.3, 4.4, . . .

16. 99, 88, 77, 66, . . .

17. 4.5, 9, 18, 36, . . .

Tell whether the sequence is arithmetic. If it is, identify the common difference.

See Problem 2.

18. −7, −3, 1, 5, . . .

19. −9, −17, −26, −33, . . .

20. 19, 8, −3, −14, . . .

21. 2, 11, 21, 32, . . .

22. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}, 0, \ldots$

23. 0.2, 1.5, 2.8, 4.1, . . .

24. 10, 8, 6, 4, . . .

25. 10, 24, 36, 52, . . .

26. 3, 6, 12, 24, . . .

27. 15, 14.5, 14, 13.5, 13, . . .

28. 4, 4.4, 4.44, 4.444, . . .

29. −3, −7, −10, −14, . . .

Write a recursive formula for each sequence.

See Problem 3.

30. 1.1, 1.9, 2.7, 3.5, . . .

31. 99, 88, 77, 66, . . .

32. 23, 38, 53, 68, . . .

33. 13, 10, 7, 4, . . .

34. 2.3, 2.8, 3.3, 3.8, . . .

35. 4.6, 4.7, 4.8, 4.9, . . .

See Problem 4.

36. Garage After one customer buys 4 new tires, a garage recycling bin has 20 tires in it. After another customer buys 4 new tires, the bin has 24 tires in it. Write an explicit formula to represent the number of tires in the bin as an arithmetic sequence. How many tires are in the bin after 9 customers buy all new tires?

37. Cafeteria You have a cafeteria card worth \$50. After you buy lunch on Monday, its value is \$46.75. After you buy lunch on Tuesday, its value is \$43.50. Write an explicit formula to represent the amount of money left on the card as an arithmetic sequence. What is the value of the card after you buy 12 lunches?

Write an explicit formula for each recursive formula.

See Problem 5.

38. $A(n) = A(n-1) + 12; A(1) = 12$

39. $A(n) = A(n-1) + 3.4; A(1) = 7.3$

40. $A(n) = A(n-1) + 3; A(1) = 6$

41. $A(n) = A(n-1) - 0.3; A(1) = 0.3$

Write a recursive formula for each explicit formula.

See Problem 6.

42. $A(n) = 5 + (n-1)(3)$

43. $A(n) = 3 + (n-1)(-5)$

44. $A(n) = -1 + (n-1)(-2)$

45. $A(n) = 4 + (n-1)(1)$

Find the second, fourth, and eleventh terms of the sequence described by each explicit formula.

46. $A(n) = 5 + (n-1)(-3)$

47. $A(n) = -3 + (n-1)(5)$

48. $A(n) = -11 + (n-1)(2)$

49. $A(n) = 9 + (n-1)(8)$

50. $A(n) = 0.5 + (n - 1)(3.5)$

51. $A(n) = -7 + (n - 1)(5)$

52. $A(n) = 1 + (n - 1)(-6)$

53. $A(n) = -2.1 + (n - 1)(-1.1)$

B Apply

Tell whether each sequence is arithmetic. Justify your answer. If the sequence is arithmetic, write a recursive and an explicit formula to represent it.

54. 0.3, 0.9, 1.5, 2.1, . . .

55. −3, −7, −11, −15, . . .

56. 1, 8, 27, 64, . . .

57. −5, 5, −5, 5, . . .

58. 46, 31, 16, 2, . . .

59. 0.2, −0.6, −1.4, −2.2, . . .

Using the recursive formula for each arithmetic sequence, find the second, third, and fourth terms of the sequence. Then write the explicit formula that represents the sequence.

60. $A(n) = A(n - 1) - 4; A(1) = 8$

61. $A(n) = A(n - 1) + 1.2; A(1) = 8.8$

62. $A(n) = A(n - 1) + 3; A(1) = 13$

63. $A(n) = A(n - 1) - 2; A(1) = 0$

64. **Reasoning** An arithmetic sequence can be represented by the explicit function $A(n) = -10 + (n - 1)(4)$. Describe the relationship between the first term and the second term. Describe the relationship between the second term and the third term. Write a recursive formula to represent this sequence.

65. **Open-Ended** Write a function rule for a sequence that has 25 as the sixth term.

Write the first six terms in each sequence. Explain what the sixth term means in the context of the situation.

66. A cane of bamboo is 30 in. tall the first week and grows 6 in. per week thereafter.

67. You borrow $350 from a friend the first week and pay the friend back $25 each week thereafter.

68. **Think About a Plan** Suppose the first Friday of a new year is the fourth day of that year. Will the year have 53 Fridays regardless of whether or not it is a leap year?
- What is a rule that represents the sequence of the days in the year that are Fridays?
- How many full weeks are in a 365-day year?

69. **Look For a Pattern** The first five rows of Pascal's Triangle are shown at the right.
  a. Predict the numbers in the seventh row.
  b. Find the sum of the numbers in each of the first five rows. Predict the sum of the numbers in the seventh row.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

70. **Transportation** Buses run every 9 min starting at 6:00 A.M. You get to the bus stop at 7:16 A.M. How long will you wait for a bus?

71. **Multiple Representations** Use the table at the right that shows an arithmetic sequence.
  a. Copy and complete the table.
  b. Graph the ordered pairs (x, y) on a coordinate plane.
  c. What do you notice about the points on your graph?

x	y
1	5
2	8
3	■
4	■

72. **Number Theory** The Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, . . . After the first two numbers, each number is the sum of the two previous numbers.

a. What is the next term of the sequence? The eleventh term of the sequence?

b. **Open-Ended** Choose two other numbers to start a Fibonacci-like sequence. Write the first seven terms of your sequence.

Challenge **Find the common difference of each arithmetic sequence. Then find the next term.**

73. $4, x + 4, 2x + 4, 3x + 4, \ldots$

74. $a + b + c, 4a + 3b + c, 7a + 5b + c, \ldots$

75. a. **Geometry** Draw the next figure in the pattern.

b. **Reasoning** What is the color of the twentieth figure? Explain.

c. How many sides does the twenty-third figure have? Explain.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 8

Look back to your work from the Apply What You've Learned sections on pages 245 and 259 to find the functions modeling Keiko's and Jayden's blogs. Choose from the following words and numbers to complete the sentences below.

explicit	8	−8	common difference	7
term	9	recursive	10	sequence

a. A(n) __?__ is an ordered list of numbers that often form a pattern.

b. The function modeling Keiko's blog is a(n) __?__ formula.

c. The common difference in the function modeling Jayden's blog is __?__.

d. Jayden's blog will first have at least 100 subscribers in __?__ months.

e. Keiko's blog will first have at least 100 subscribers in __?__ months.

f. A function rule that relates each term of a sequence after the first term to the ones before it is called a(n) __?__ formula.