7-1 Zero and Negative Exponents

Common Core State Standards

Prepares for N-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values . . . **Also prepares for N-RN.A.2**

MP 1, MP 2, MP 3, MP 4, MP 7

Objective To simplify expressions involving zero and negative exponents

Look for a pattern in the values in the table.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

Copy and complete the table. Make a conjecture about how the value of an exponential expression (an expression containing an exponent) changes when you decrease the exponent by 1. What do you think the value of 5^{-2} is? Explain your reasoning.

2^x	10^x
$2^4 =$ ■	$10^4 =$ ■
$2^3 =$ ■	$10^3 =$ ■
$2^2 =$ ■	$10^2 =$ ■
$2^1 =$ ■	$10^1 =$ ■
$2^0 =$ ■	$10^0 =$ ■
$2^{-1} =$ ■	$10^{-1} =$ ■
$2^{-2} =$ ■	$10^{-2} =$ ■

The patterns you found in the Solve It illustrate the definitions of zero and negative exponents.

Essential Understanding You can extend the idea of exponents to include zero and negative exponents.

Consider 3^3, 3^2, and 3^1. Decreasing the exponents by 1 is the same as dividing by 3. If you continue the pattern, 3^0 equals 1 and 3^{-1} equals $\frac{1}{3}$.

take note

Properties Zero and Negative Exponents

Zero as an Exponent For every nonzero number a, $a^0 = 1$.

Examples $4^0 = 1$ $(-3)^0 = 1$ $(5.14)^0 = 1$

Negative Exponent For every nonzero number a and integer n, $a^{-n} = \frac{1}{a^n}$.

Examples $7^{-3} = \frac{1}{7^3}$ $(-5)^{-2} = \frac{1}{(-5)^2}$

Why can't you use 0 as a base with zero exponents? The first property on the previous page implies the following pattern.

$3^0 = 1$ $\quad 2^0 = 1$ $\quad 1^0 = 1$ $\quad 0^0 = 1$

However, consider the following pattern.

$0^3 = 0$ $\quad 0^2 = 0$ $\quad 0^1 = 0$ $\quad 0^0 = 0$

It is not possible for 0^0 to equal both 1 and 0. Therefore 0^0 is undefined.

Why can't you use 0 as a base with a negative exponent? Using 0 as a base with a negative exponent will result in division by zero, which is undefined.

Problem 1 Simplifying Powers

What is the simplified form of each expression?

Think

Can you use the definition of zero as an exponent when the base is a negative number?
Yes, the definition of zero as an exponent is true for all nonzero bases.

A 9^{-2}

$9^{-2} = \frac{1}{9^2}$ Use the definition of negative exponent.

$= \frac{1}{81}$ Simplify.

B $(-3.6)^0 = 1$ Use the definition of zero as an exponent.

Got It? **1.** What is the simplified form of each expression?

a. 4^{-3} **b.** $(-5)^0$ **c.** 3^{-2} **d.** 6^{-1} **e.** $(-4)^{-2}$

An algebraic expression is in simplest form when powers with a variable base are written with only positive exponents.

Problem 2 Simplifying Exponential Expressions

What is the simplified form of each expression?

Think

Which part of the expression do you need to rewrite?
The base b has a negative exponent, so you need to rewrite it with a positive exponent.

A $5a^3b^{-2}$

$5a^3b^{-2} = 5a^3\left(\frac{1}{b^2}\right)$ Use the definition of negative exponent.

$= \frac{5a^3}{b^2}$ Simplify.

B $\frac{1}{x^{-5}}$

$\frac{1}{x^{-5}} = 1 \div x^{-5}$ Rewrite using a division symbol.

$= 1 \div \frac{1}{x^5}$ Use the definition of negative exponent.

$= 1 \cdot x^5$ Multiply by the reciprocal of $\frac{1}{x^5}$, which is x^5.

$= x^5$ Identity Property of Multiplication

Got It? **2.** What is the simplified form of each expression?

a. x^{-9} **b.** $\frac{1}{n^{-3}}$ **c.** $4c^{-3}b$ **d.** $\frac{2}{a^{-3}}$ **e.** $\frac{n^{-5}}{m^2}$

When you evaluate an exponential expression, you can simplify the expression before substituting values for the variables.

Problem 3 Evaluating an Exponential Expression

Plan

How do you simplify the expression?
Use the definition of negative exponent to rewrite the expression with only positive exponents.

What is the value of $3s^3t^{-2}$ for $s = 2$ and $t = -3$?

Method 1 Simplify first.

$$3s^3t^{-2} = \frac{3(s)^3}{t^2}$$

$$= \frac{3(2)^3}{(-3)^2}$$

$$= \frac{24}{9} = 2\frac{2}{3}$$

Method 2 Substitute first.

$$3s^3t^{-2} = 3(2)^3(-3)^{-2}$$

$$= \frac{3(2)^3}{(-3)^2}$$

$$= \frac{24}{9} = 2\frac{2}{3}$$

Got It? **3.** What is the value of each expression in parts (a)–(d) for $n = -2$ and $w = 5$?

a. $n^{-4}w^0$ **b.** $\frac{n^{-1}}{w^2}$ **c.** $\frac{n^0}{w^6}$ **d.** $\frac{1}{nw^{-1}}$

e. Reasoning Is it easier to evaluate n^0w^0 for $n = -2$ and $w = 3$ by simplifying first or by substituting first? Explain.

Problem 4 Using an Exponential Expression STEM

Population Growth **A population of marine bacteria doubles every hour under controlled laboratory conditions. The number of bacteria is modeled by the expression $1000 \cdot 2^h$, where h is the number of hours after a scientist measures the population size. Evaluate the expression for $h = 0$ and $h = -3$. What does each value of the expression represent in the situation?**

Know
$1000 \cdot 2^h$ models the population.

Need
Values of the expression for $h = 0$ and $h = -3$

Plan
Substitute each value of h into the expression and simplify.

$$1000 \cdot 2^h = 1000 \cdot 2^0 \quad \text{Substitute 0 for } h.$$

$$= 1000 \cdot 1 = 1000 \quad \text{Simplify.}$$

The value of the expression for $h = 0$ is 1000. There were 1000 bacteria at the time the scientist measured the population.

$$1000 \cdot 2^h = 1000 \cdot 2^{-3} \quad \text{Substitute } -3 \text{ for } h.$$

$$= 1000 \cdot \frac{1}{8} = 125 \quad \text{Simplify.}$$

The value of the expression for $h = -3$ is 125. There were 125 bacteria 3 h before the scientist measured the population.

Got It? **4.** A population of insects triples every week. The number of insects is modeled by the expression $5400 \cdot 3^w$, where w is the number of weeks after the population was measured. Evaluate the expression for $w = -2$, $w = 0$, and $w = 1$. What does each value of the expression represent in the situation?

Lesson Check

Do you know HOW?

Simplify each expression.

1. 2^{-5}
2. m^0
3. $5s^2t^{-1}$
4. $\frac{4}{x^{-3}}$

Evaluate each expression for $a = 2$ and $b = -4$.

5. a^3b^{-1}
6. $2a^{-4}b^0$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. **Vocabulary** A positive exponent shows repeated multiplication. What repeated operation does a negative exponent show?

8. **Error Analysis** A student incorrectly simplified $\frac{x^n}{a^{-n}b^0}$ as shown below. Find and correct the student's error.

$$\frac{x^n}{a^{-n}b^0} = \frac{a^n x^n}{b^0}$$
$$= \frac{a^n x^n}{0} \text{ undefined}$$

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Simplify each expression. See Problem 1.

9. 3^{-2}
10. $(-4.25)^0$
11. $(-5)^{-2}$
12. -5^{-2}
13. $(-4)^{-2}$
14. 2^{-6}
15. -3^0
16. -12^{-1}
17. $\frac{1}{2^0}$
18. 58^{-1}
19. 1.5^{-2}
20. $(-5)^{-3}$

Simplify each expression. See Problem 2.

21. $4ab^0$
22. $\frac{1}{x^{-7}}$
23. $5x^{-4}$
24. $\frac{1}{c^{-1}}$
25. $\frac{3^{-2}}{n}$
26. $k^{-4}j^0$
27. $\frac{3x^{-2}}{y}$
28. $\frac{7ab^{-2}}{3w}$
29. $c^{-5}d^{-7}$
30. $c^{-5}d^7$
31. $\frac{8}{2s^{-3}}$
32. $\frac{7s}{5t^{-3}}$
33. $\frac{6a^{-1}c^{-3}}{d^0}$
34. $2^{-3}x^2z^{-7}$
35. $12^0t^7u^{-11}$
36. $\frac{7s^0t^{-5}}{2^{-1}m^2}$

Evaluate each expression for $r = -3$ and $s = 5$.

See Problem 3.

37. r^{-3} **38.** s^{-3} **39.** $\frac{3r}{s^{-2}}$ **40.** $\frac{s^0}{r^{-2}}$

41. $4s^{-1}$ **42.** r^0s^{-2} **43.** $r^{-4}s^2$ **44.** $2^{-4}r^3s^{-2}$

See Problem 4.

45. Internet Traffic The number of visitors to a certain Web site triples every month. The number of visitors is modeled by the expression $8100 \cdot 3^m$, where m is the number of months after the number of visitors was measured. Evaluate the expression $m = -4$. What does the value of the expression represent in the situation?

STEM **46. Population Growth** A Galápagos cactus finch population increases by half every decade. The number of finches is modeled by the expression $45 \cdot 1.5^d$, where d is the number of decades after the population was measured. Evaluate the expression for $d = -2$, $d = 0$, and $d = 1$. What does each value of the expression represent in the situation?

Galápagos cactus finch

B Apply

Mental Math **Is the value of each expression *positive* or *negative*?**

47. -2^2 **48.** $(-2)^2$ **49.** $(-2)^3$ **50.** $(-2)^{-3}$

Write each number as a power of 10 using negative exponents.

51. $\frac{1}{10}$ **52.** $\frac{1}{100}$ **53.** $\frac{1}{1000}$ **54.** $\frac{1}{10{,}000}$

55. a. Patterns Complete the pattern using powers of 5.

$\frac{1}{5^2} = \blacksquare$ $\frac{1}{5^1} = \blacksquare$ $\frac{1}{5^0} = \blacksquare$ $\frac{1}{5^{-1}} = \blacksquare$ $\frac{1}{5^{-2}} = \blacksquare$

b. Write $\frac{1}{5^{-4}}$ using a positive exponent.

c. Rewrite $\frac{1}{a^{-n}}$ as a power of a.

Rewrite each fraction with all the variables in the numerator.

56. $\frac{a}{b^{-2}}$ **57.** $\frac{4g}{h^3}$ **58.** $\frac{5m^6}{3n}$ **59.** $\frac{8c^5}{11d^4e^{-2}}$

60. Think About a Plan Suppose your drama club's budget doubles every year. This year the budget is \$500. How much was the club's budget 2 yr ago?

- What expression models what the budget of the club will be in 1 yr? In 2 yr? In y years?
- What value of y can you substitute into your expression to find the budget of the club 2 yr ago?

61. Copy and complete the table at the right.

n	3	■	■	$\frac{5}{8}$	■
n^{-1}	■	6	$\frac{1}{7}$	■	0.5

62. a. Simplify $a^n \cdot a^{-n}$.

b. Reasoning What is the mathematical relationship between a^n and a^{-n}? Explain.

63. Open-Ended Choose a fraction to use as a value for the variable a. Find the values of a^{-1}, a^2, and a^{-2}.

64. Manufacturing A company is making metal rods with a target diameter of 1.5 mm. A rod is acceptable when its diameter is within 10^{-3} mm of the target diameter. Write an inequality for the acceptable range of diameters.

65. Reasoning Are $3x^{-2}$ and $3x^2$ reciprocals? Explain.

Challenge

Simplify each expression.

66. $\left(\frac{r^{-7}b^{-8}}{t^{-4}w^1}\right)^0$

67. $(-5)^2 - (0.5)^{-2}$

68. $\frac{6}{m^2} + \frac{5m^{-2}}{3^{-3}}$

69. $2^3(5^0 - 6m^2)$

70. $\frac{2x^{-5}y^3}{n^2} \div \frac{r^2y^5}{2n}$

71. $2^{-1} - \frac{1}{3^{-2}} + 5\left(\frac{1}{2^2}\right)$

72. For what value or values of n is $n^{-3} = \left(\frac{1}{n}\right)^5$?

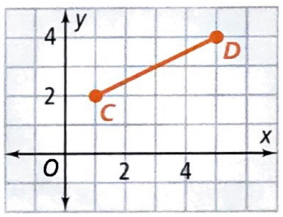
Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

73. What is the simplified form of $-6(-6)^{-1}$?

74. Segment CD represents the flight of a bird that passes through the points $(1, 2)$ and $(5, 4)$. What is the slope of a line that represents the flight of a second bird that flew perpendicular to the first bird?



75. What is the solution of the equation $1.5(x - 2.5) = 3$?

76. What is the simplified form of $|3.5 - 4.7| + 5.6$?

77. What is the y-intercept of the graph of $3x - 2y = -8$?

Mixed Review

Solve each system by graphing. See Lesson 6-6.

78. $y > 3x + 4$
$y \leq -3x + 1$

79. $y \leq -2x + 1$
$y < 2x - 1$

80. $y \geq 0.5x$
$y \leq x + 2$

Write an equation in slope-intercept form for the line with the given slope m and y-intercept b. See Lesson 5-3.

81. $m = -1, b = 4$

82. $m = 5, b = -2$

83. $m = \frac{2}{5}, b = -3$

84. $m = -\frac{3}{11}, b = -17$

85. $m = \frac{5}{9}, b = \frac{1}{3}$

86. $m = 1.25, b = -3.79$

Get Ready! To prepare for Lesson 7-2, do Exercises 87–91.

Simplify each expression. See Lesson 7-1.

87. $6 \cdot 10^4$

88. $7 \cdot 10^{-2}$

89. $8.2 \cdot 10^5$

90. $3 \cdot 10^{-3}$

91. $3.4 \cdot 10^5$