7-2 Multiplying Powers With the Same Base

Common Core State Standards

N-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values . . .

MP 1, MP 2, MP 3, MP 4, MP 7

Objective To multiply powers with the same base

MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

Scientists estimate that there are about 10^{20} stars in the universe. A cubic meter of beach sand contains about 10^9 grains of sand. Suppose all of the sand from the world's beaches is combined into one large beach, as shown below. Are there more stars in the universe or grains of sand on the world's beaches? Explain your reasoning.

All of the numbers in the Solve It are powers of 10. In this lesson, you will learn a method for multiplying powers that have the same base.

Essential Understanding You can use a property of exponents to multiply powers with the same base.

You can write a product of powers with the same base, such as $3^4 \cdot 3^2$, using one exponent.

$$3^4 \cdot 3^2 = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3) = 3^6$$

Notice that the sum of the exponents in the expression $3^4 \cdot 3^2$ equals the exponent of 3^6.

In general, an equation such as $3^4 \cdot 3^2 = 3^6$ can be written using variables: $a^m \cdot a^n = a^{m+n}$.

Here's Why It Works You can use repeated multiplication to rewrite a product of powers.

$$a^m \cdot a^n = \underbrace{(a \cdot a \cdot \ldots \cdot a)}_{m \text{ factors of } a} \cdot \underbrace{(a \cdot a \cdot \ldots \cdot a)}_{n \text{ factors of } a} = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m+n \text{ factors of } a} = a^{m+n}$$

Problem 1 Multiplying Powers

Think

When can you use the property for multiplying powers?
You can use the property for multiplying powers when the bases of the powers are the same.

What is each expression written using each base only once?

A $12^4 \cdot 12^3 = 12^{4+3}$ Add the exponents of the powers with the same base.

$= 12^7$ Simplify the exponent.

B $(-5)^{-2}(-5)^7 = (-5)^{-2+7}$ Add the exponents of the powers with the same base.

$= (-5)^5$ Simplify the exponent.

Got It? 1. What is each expression written using each base only once?

a. $8^3 \cdot 8^6$ b. $(0.5)^{-3}(0.5)^{-8}$ c. $9^{-3} \cdot 9^2 \cdot 9^6$

When variable factors have more than one base, be careful to combine only those powers with the same base.

Problem 2 Multiplying Powers in Algebraic Expressions

Plan

Which parts of the expression can you combine?
You can group the coefficients and multiply. You can also write any powers that have the same base with a single exponent.

What is the simplified form of each expression?

A $4z^5 \cdot 9z^{-12} = (4 \cdot 9)(z^5 \cdot z^{-12})$ Commutative and associative properties of multiplication

$= 36(z^{5+(-12)})$ Multiply the coefficients. Add the exponents of the powers with the same base.

$= 36z^{-7}$ Simplify the exponent.

$= \frac{36}{z^7}$ Rewrite using a positive exponent.

B $2a \cdot 9b^4 \cdot 3a^2 = (2 \cdot 9 \cdot 3)(a \cdot a^2)(b^4)$ Commutative and associative properties of multiplication

$a = a^1$

$= 54(a^1 \cdot a^2)(b^4)$ Multiply the coefficients. Write a as a^1.

$= 54(a^{1+2})(b^4)$ Add exponents of powers with the same base.

$= 54a^3b^4$ Simplify.

Got It? 2. What is the simplified form of each expression in parts (a)–(c)?

a. $5x^4 \cdot x^9 \cdot 3x$ b. $-4c^3 \cdot 7d^2 \cdot 2c^{-2}$ c. $j^2 \cdot k^{-2} \cdot 12j$

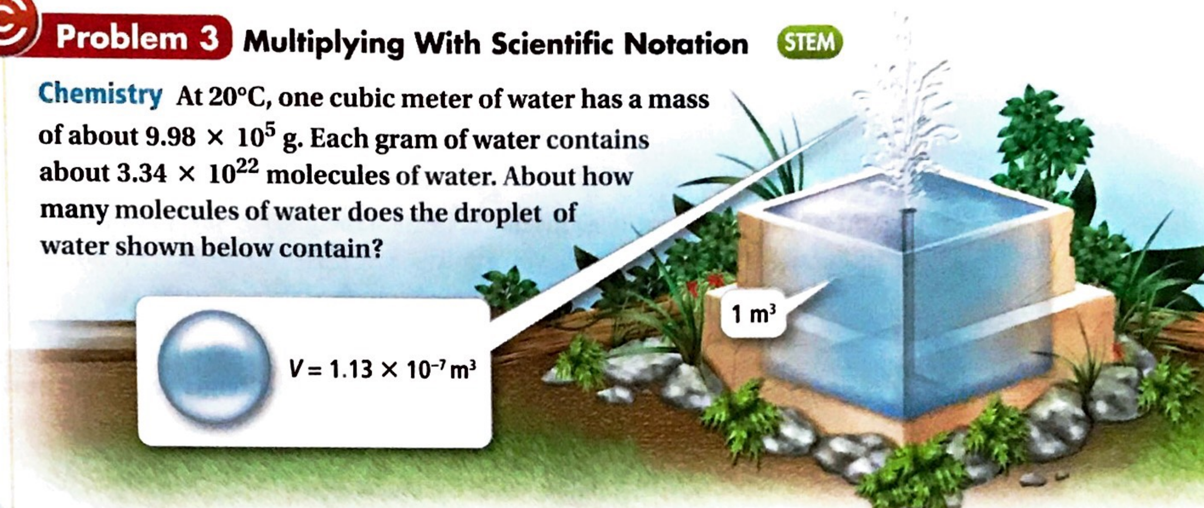
d. **Reasoning** Explain how to simplify the expression $x^a \cdot x^b \cdot x^c$.

You can use the property for multiplying powers with the same base to multiply two numbers written in scientific notation.

Recall that you can use powers of 10 to make writing very large and very small numbers more convenient. In scientific notation, you can write any number as $a \times 10^b$, where $1 \leq |a| < 10$. For example, 256,000 is written in scientific notation as 2.56×10^5.

Problem 3 Multiplying With Scientific Notation STEM

Chemistry **At 20°C, one cubic meter of water has a mass of about 9.98×10^5 g. Each gram of water contains about 3.34×10^{22} molecules of water. About how many molecules of water does the droplet of water shown below contain?**



Plan
How do you find the number of molecules?
Use unit analysis. Divide out the common units.

$$\text{molecules of water} = \text{cubic meters} \cdot \frac{\text{grams}}{\text{cubic meters}} \cdot \frac{\text{molecules}}{\text{grams}}$$ Use unit analysis.

$= (1.13 \times 10^{-7}) \cdot (9.98 \times 10^5) \cdot (3.34 \times 10^{22})$ Substitute.

$= (1.13 \cdot 9.98 \cdot 3.34) \times (10^{-7} \cdot 10^5 \cdot 10^{22})$ Commutative and associative properties of multiplication

$\approx 37.7 \times 10^{-7+5+22}$ Multiply. Add exponents.

$= 37.7 \times 10^{20}$ Simplify.

$= 3.77 \times 10^{21}$ Write in scientific notation.

The droplet contains about 3.77×10^{21} molecules of water.

Got It? **3.** About how many molecules of water are in a swimming pool that holds 200 m^3 of water? Write your answer in scientific notation.

Exponents can also be expressed as fractions. Fractional exponents are called rational exponents.

Recall that 3^2 means $3 \cdot 3$, which equals 9. You can write the same expression using rational exponents: $9^{\frac{1}{2}}$. The equation $9^{\frac{1}{2}} = b$ indicates that b is the positive number that when multiplied by itself, equals 9.

$$9^{\frac{1}{2}} = 3 \text{ since } 3 \cdot 3 = 9.$$

In general, $a^{\frac{1}{m}} = b$ means that b multiplied as a factor m times equals a.

Problem 4 Simplifying Expressions With Rational Exponents

Think

What number multiplied by itself 4 times equals 81?

$9 \cdot 9 = 81$ and $(3 \cdot 3)(3 \cdot 3) = 81$.

Simplify the expression $81^{\frac{1}{4}}$.

$81^{\frac{1}{4}}$ Find the number that when multiplied by itself four times gives 81.

$81^{\frac{1}{4}} = 3$ $3 \cdot 3 \cdot 3 \cdot 3 = 81$

Got It? 4. Simplify each expression.

a. $16^{\frac{1}{4}}$ **b.** $27^{\frac{1}{3}}$ **c.** $64^{\frac{1}{2}}$

You can also have expressions like $9^{\frac{3}{2}}$, which means $9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}}$. Consider each factor individually. Because $9^{\frac{1}{2}} = 3$, you know $9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} = 3 \cdot 3 \cdot 3 = 27$. So, $9^{\frac{3}{2}} = 27$.

Problem 5 Simplifying Expressions With Rational Exponents

Think

How can the fractional exponent be rewritten?

Exponents can be rewritten as multiple factors if the base of each exponential factor is the same.

Simplify the expression $64^{\frac{3}{2}}$.

$64^{\frac{3}{2}} = 64^{\frac{1}{2}} \cdot 64^{\frac{1}{2}} \cdot 64^{\frac{1}{2}}$ Rewrite the expression.

$= 8 \cdot 8 \cdot 8$ Substitute 8 for $64^{\frac{1}{2}}$.

$= 512$ Simplify.

Got It? 5. Simplify each expression.

a. $25^{\frac{3}{2}}$ **b.** $27^{\frac{2}{3}}$ **c.** $16^{\frac{3}{4}}$

You can use the properties of multiplying powers with the same base to simplify expressions with rational exponents.

take note

Property Multiplying Powers With the Same Base

Words To multiply powers with the same base, add the exponents.

Algebra $a^m \cdot a^n = a^{m+n}$, where $a \neq 0$ and m and n are rational numbers

Examples $4^{\frac{1}{3}} \cdot 4^{\frac{1}{3}} = 4^{\frac{1}{3}+\frac{1}{3}} = 4^{\frac{2}{3}}$ $b^7 \cdot b^{-4} = b^{7+(-4)} = b^3$

Problem 6 Simplifying Expressions With Rational Exponents

Think

Why must like variables be grouped together?
To simplify by adding exponents, the bases must be the same.

Simplify the expression $\left(2a^{\frac{2}{3}} \cdot 3b^{\frac{1}{4}}\right)\left(a^{\frac{1}{3}} \cdot 5b^{\frac{1}{2}}\right)$.

$= (2 \cdot 3 \cdot 5)\left(a^{\frac{2}{3}} \cdot a^{\frac{1}{3}}\right)\left(b^{\frac{1}{4}} \cdot b^{\frac{1}{2}}\right)$	Commutative and associative properties of multiplication
$= 30\left(a^{\frac{2}{3}} \cdot a^{\frac{1}{3}}\right)\left(b^{\frac{1}{4}} \cdot b^{\frac{1}{2}}\right)$	Simplify.
$= 30\left(a^{\frac{3}{3}}\right)\left(b^{\frac{3}{4}}\right)$	Add exponents that have the same base.
$= 30ab^{\frac{3}{4}}$	Simplify.

Got It? **6.** Simplify each expression.

a. $2c^{\frac{3}{5}} \cdot 2c^{\frac{1}{5}}$

b. $n^{\frac{1}{3}} \cdot n^{\frac{4}{3}}$

c. $\left(b^{\frac{2}{3}} \cdot c^{\frac{2}{5}}\right)\left(b^{\frac{4}{9}} \cdot c^{\frac{9}{10}}\right)$

d. $\left(3j^{\frac{2}{3}} \cdot 7m^{\frac{1}{4}}\right)\left(3j^{\frac{1}{6}} \cdot 7m^{\frac{3}{2}}\right)$

Lesson Check

Do you know HOW?

1. What is $8^4 \cdot 8^8$ written using each base only once?

2. What is the simplified form of $2n^{\frac{2}{3}} \cdot 3n^{\frac{3}{4}}$?

3. What is $(3 \times 10^5)(8 \times 10^4)$ written in scientific notation?

4. **Measurement** The diameter of a penny is about 1.9×10^{-5} km. It would take about 2.1×10^9 pennies placed end to end to circle the equator once. What is the approximate length of the equator?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Writing** Can $x^8 \cdot y^3$ be written as a single power? Explain your reasoning.

6. **Reasoning** Suppose $a \times 10^m$ and $b \times 10^n$ are two numbers in scientific notation. Is their product $ab \times 10^{m+n}$ *always, sometimes,* or *never* a number in scientific notation? Justify your answer.

7. **Error Analysis** Your friend says $4a^{\frac{1}{2}} \cdot 3a^{\frac{1}{5}} = 7a^{\frac{1}{7}}$. Explain your friend's error. What is the correct answer?

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Rewrite each expression using each base only once. See Problem 1.

8. $7^3 \cdot 7^4$

9. $(-6)^{12} \cdot (-6)^5 \cdot (-6)^2$

10. $9^6 \cdot 9^{-4} \cdot 9^{-2}$

11. $2^2 \cdot 2^7 \cdot 2^0$

12. $5^{-2} \cdot 5^{-4} \cdot 5^8$

13. $(-8)^5 \cdot (-8)^{-5}$

Simplify each expression. See Problem 2.

14. m^3m^4

15. $5c^4 \cdot c^6$

16. $4t^{-5} \cdot 2t^{-3}$

17. $(x^5y^2)(x^{-6}y)$

18. $(5x^5)(3y^6)(3x^2)$

19. $-m^2 \cdot 4r^3 \cdot 12r^{-4} \cdot 5m$

Write each answer in scientific notation. See Problem 3.

STEM **20.** **Biology** A human body contains about 2.7×10^4 microliters (μL) of blood for each pound of body weight. Each microliter of blood contains about 7×10^4 white blood cells. About how many white blood cells are in the body of a 140-lb person?

STEM **21. Astronomy** The distance light travels in one second (one light-second) is about 1.86×10^5 mi. Saturn is about 475 light-seconds from the sun. About how many miles from the sun is Saturn?

Simplify each expression. See Problem 4.

22. $8^{\frac{1}{3}}$ **23.** $625^{\frac{1}{4}}$ **24.** $1000^{\frac{1}{3}}$

Simplify each expression. See Problem 5.

25. $16^{\frac{3}{4}}$ **26.** $9^{\frac{5}{2}}$ **27.** $64^{\frac{7}{3}}$

Simplify each expression. See Problem 6.

28. $\left(8b^{\frac{2}{3}} \cdot 9t^{\frac{1}{5}}\right)\left(8b^{\frac{5}{3}} \cdot 9t^{\frac{3}{5}}\right)$ **29.** $\left(7d^{\frac{3}{2}} \cdot 2g^{\frac{5}{6}}\right)\left(2g^{\frac{3}{2}} \cdot 7d^{\frac{5}{6}}\right)$ **30.** $\left(4r^{\frac{2}{5}} \cdot 5s^{\frac{2}{7}}\right)\left(5s^{\frac{5}{7}} \cdot 4r^{\frac{3}{5}}\right)$

B Apply

Complete each equation.

31. $5^2 \cdot 5^{\blacksquare} = 5^{11}$ **32.** $m^{\blacksquare} \cdot m^{-4} = m^{-9}$ **33.** $2^{\blacksquare} \cdot 2^{\frac{1}{2}} = 2^1$

34. $a^{\blacksquare} \cdot a^4 = 1$ **35.** $a^{\frac{2}{3}} \cdot a^{\blacksquare} = a^{\frac{5}{6}}$ **36.** $x^3y^{\blacksquare} \cdot x^{\blacksquare} = y^2$

C **37. Think About a Plan** A liter of water contains about 3.35×10^{25} molecules. The Mississippi River discharges about 1.7×10^7 L of water every second. About how many molecules does the Mississippi River discharge every minute? Write your answer in scientific notation.

- How can you use unit analysis to help you find the answer?
- What properties can you use to make the calculation easier?

38. When you simplify an algebraic expression like $c^{\frac{3}{5}} \cdot c^{\frac{1}{2}}$, you know that the bases of the expressions must be the same. You also need to rewrite the exponents so that they have a common denominator.

a. Explain why you need to find the common denominator to simplify.

b. Simplify the expression $c^{\frac{3}{5}} \cdot c^{\frac{1}{2}}$.

Simplify each expression. Write each answer in scientific notation.

39. $(9 \times 10^7)(3 \times 10^{-16})$ **40.** $(0.5 \times 10^{-6})(0.3 \times 10^{-2})$ **41.** $(0.2 \times 10^5)(4 \times 10^{-12})$

STEM **42. Chemistry** In chemistry, a *mole* is a unit of measure equal to 6.02×10^{23} atoms of a substance. The mass of a single neon atom is about 3.35×10^{-23} g. What is the mass of 2 moles of neon atoms? Write your answer in scientific notation.

Simplify each expression.

43. $\dfrac{1}{a^4 \cdot a^{-3}}$ **44.** $8m^{\frac{1}{3}}\left(m^{\frac{1}{3}} + 2\right)$ **45.** $-4x^3(3x^3 - 10x)$

C **46. a. Open-Ended** Write y^6 as a product of two powers with the same base in four different ways. Use only positive exponents.

b. Write y^6 as a product of two powers with the same base in four different ways, using negative or zero exponents in each product.

c. Reasoning How many ways can you write y^6 as the product of two powers? Explain your reasoning.

Challenge Simplify each expression.

47. $3^x \cdot 3^{2-x} \cdot 3^2$

48. $2^n \cdot 2^{n+2} \cdot 2$

49. $3^{\frac{1}{4}} \cdot 2^y \cdot 3^2 \cdot 2^x$

50. $(a + b)^2(a + b)^{-3}$

51. $(t + 3)^{\frac{4}{5}}(t + 3)^{\frac{2}{5}}$

52. $5^{x+1} \cdot 5^{1-x}$

53. **Nature** A book shows an enlarged photo of a carpenter bee. A carpenter bee is about 6×10^{-3} m long. The photo is 13.5 cm long. About how many times as long as a carpenter bee is the photo?

Standardized Test Prep

SAT/ACT

54. What is the simplified form of $\left(2x^{\frac{1}{2}}y^{\frac{2}{3}}\right)\left(4x^{\frac{1}{4}}y^{\frac{5}{6}}\right)$?

(A) $6x^{\frac{1}{2}}y^{\frac{2}{3}}$ (B) $6xy$ (C) $8x^{\frac{1}{2}}y^{\frac{7}{9}}$ (D) $8x^{\frac{3}{4}}y^{\frac{3}{2}}$

55. What is the x-intercept of the graph of $5x - 3y = 30$?

(F) -10 (G) -6 (H) 6 (I) 10

56. At the Athens Olympics, the winning time for the women's 100-m hurdles was 2.06×10^{-1} min. Which number is another way to express this time in minutes?

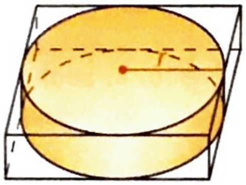
(A) 0.206 (B) 20.6 (C) 206×10^1 (D) 206×10^{-2}

57. What is the solution of $4x - 5 = 2x + 13$?

(F) 3 (G) 4 (H) 9 (I) 32

Extended Response

58. Bill's company packages its circular mirrors in boxes with square bottoms, as shown at the right. Show your work for each answer.



a. What is an expression for the area of the bottom of the box?
b. If the mirror has a radius of 4 in., what is the area of the bottom of the box?
c. The area of the bottom of a second box is 196 in.^{2}. What is the diameter of the largest mirror the box can hold?

Mixed Review

Solve each system. See Lesson 6-3.

59. $2x + 3y = 12$
$-3x + y = -7$

60. $2x - y = -3$
$x - y = 1$

61. $2x + y = 15$
$-\frac{1}{2}x + y = 5$

Find the third, seventh, and tenth terms of the sequence described by each rule. See Lesson 4-7.

62. $A(n) = 10 + (n - 1)(4)$

63. $A(n) = -5 + (n - 1)(2)$

64. $A(n) = 1.2 + (n - 1)(-4)$

Get Ready! **To prepare for Lesson 7-3, do Exercises 65–68.**

Simplify each expression. See Lesson 7-1.

65. $(-2)^{-4}$

66. $5xy^0$

67. $4m^{-1}n^2$

68. $-3x^{\frac{1}{3}}y^{-\frac{1}{2}}z^6$