

7-3

More Multiplication Properties of Exponents

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N-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

MP 1, MP 2, MP 3, MP 4, MP 7

Objectives To raise a power to a power
To raise a product to a power

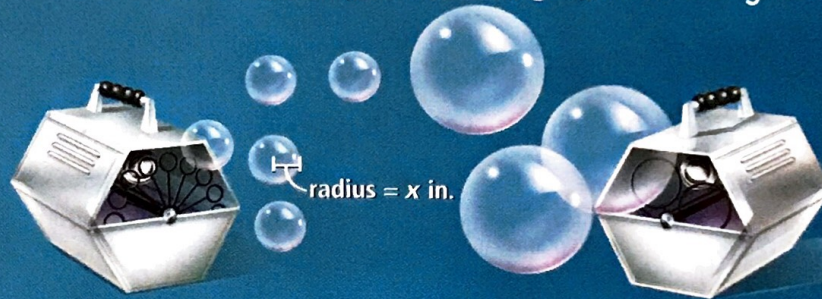


Make a plan. What do you need to know before you can use the volume formula?



Getting Ready!

The radius of a bubble made by the bubble machine on the right is 2.5 times as large as the radius of a bubble made by the bubble machine on the left. What is the volume of a bubble made by the machine on the right? Explain your reasoning. (Hint: $V = \frac{4}{3}\pi r^3$)



In the Solve It, the expression for the volume of the larger bubble involves a product raised to a power. In this lesson, you will use properties of exponents to simplify similar expressions.

Essential Understanding You can use properties of exponents to simplify a power raised to a power or a product raised to a power.

You can use repeated multiplication to simplify a power raised to a power.

$$(x^5)^2 = x^5 \cdot x^5 = x^{5+5} = x^{5 \cdot 2} = x^{10}$$

Notice that $(x^5)^2 = x^{5 \cdot 2}$. Raising a power to a power is the same as raising the base to the product of the exponents.

Take note

Property Raising a Power to a Power

Words To raise a power to a power, multiply the exponents.

Algebra $(a^m)^n = a^{mn}$, where $a \neq 0$ and m and n are rational numbers

Examples $(5^4)^2 = 5^{4 \cdot 2} = 5^8$

$$(m^3)^5 = m^{3 \cdot 5} = m^{15}$$

$$(a^{\frac{3}{2}})^3 = a^{\frac{3}{2} \cdot 3} = a^{\frac{9}{2}}$$

$$(x^{\frac{1}{2}})^{\frac{3}{5}} = x^{\frac{1}{2} \cdot \frac{3}{5}} = x^{\frac{3}{10}}$$

Think

Should you add or multiply the exponents to simplify the expression?

You multiply the exponents when raising a power to a power.

Problem 1 Simplifying a Power Raised to a Power

A What is the simplified form of $(n^4)^7$?

$$\begin{aligned}(n^4)^7 &= n^{4 \cdot 7} && \text{Multiply exponents when raising a power to a power.} \\ &= n^{28} && \text{Simplify.}\end{aligned}$$

B What is the simplified form of $(x^{\frac{2}{3}})^{\frac{1}{2}}$?

$$\begin{aligned}(x^{\frac{2}{3}})^{\frac{1}{2}} &= x^{\frac{2}{3} \cdot \frac{1}{2}} && \text{Multiply exponents when raising a rational power to a rational power.} \\ &= x^{\frac{1}{3}} = x^{\frac{1}{3}} && \text{Simplify.}\end{aligned}$$

- Got It?** 1. What is the simplified form of each expression in parts (a)–(d)?
- a. $(p^5)^4$ b. $(p^4)^5$ c. $(p^{\frac{1}{2}})^{\frac{1}{4}}$ d. $(p^{\frac{1}{4}})^{\frac{1}{2}}$
- e. **Reasoning** Is $(a^m)^n = (a^n)^m$ true for all integers m and n ? Explain.

Use the order of operations when you simplify an exponential expression.

Problem 2 Simplifying an Expression With Powers

What is the simplified form of $y^3(y^{\frac{5}{2}})^{-2}$?

Plan

What is the first step in simplifying the expression?

By the order of operations, you simplify powers before you multiply.

Think

You multiply exponents when raising a power to a power.

You add exponents when multiplying powers with the same base.

Write the expression using only positive exponents.

Write

$$\begin{aligned}y^3(y^{\frac{5}{2}})^{-2} &= y^3y^{\frac{5}{2} \cdot (-2)} \\ &= y^3y^{-\frac{10}{2}} \\ &= y^{3+(-5)} \\ &= y^{-2} \\ &= \frac{1}{y^2}\end{aligned}$$

- Got It?** 2. What is the simplified form of each expression?
- a. $x^2(x^6)^{-4}$ b. $w^{-2}(w^{\frac{5}{3}})^3$ c. $(s^{-5})^{-\frac{1}{2}}(s^{\frac{3}{2}})$

You can use repeated multiplication to simplify an expression like $(4m^{\frac{1}{2}})^3$.

$$\begin{aligned}(4m^{\frac{1}{2}})^3 &= 4m^{\frac{1}{2}} \cdot 4m^{\frac{1}{2}} \cdot 4m^{\frac{1}{2}} \\ &= 4 \cdot 4 \cdot 4 \cdot m^{\frac{1}{2}} \cdot m^{\frac{1}{2}} \cdot m^{\frac{1}{2}} \\ &= 4^3m^{\frac{3}{2}} \\ &= 64m^{\frac{3}{2}}\end{aligned}$$

Notice that $(4m^{\frac{1}{2}})^3 = 4^3m^{\frac{3}{2}}$. This example illustrates another property of exponents.

take note

Property Raising a Product to a Power

Words To raise a product to a power, raise each factor to the power and multiply.

Algebra $(ab)^n = a^n b^n$, where $a \neq 0$, $b \neq 0$, and n is a rational number

Examples $(3x)^4 = 3^4 x^4 = 81x^4$ $(4b)^{\frac{3}{2}} = 4^{\frac{3}{2}} b^{\frac{3}{2}} = 8b^{\frac{3}{2}}$



Problem 3 Simplifying a Product Raised to a Power

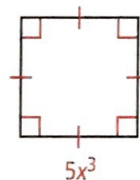
Multiple Choice Which expression represents the area of the square?

(A) $10x^3$

(C) $25x^5$

(B) $5x^6$

(D) $25x^6$



$$(5x^3)^2 = 5^2(x^3)^2 \quad \text{Raise each factor to the second power.}$$

$$= 5^2 x^6 \quad \text{Multiply the exponents of a power raised to a power.}$$

$$= 25x^6 \quad \text{Simplify.}$$

The correct answer is D.



Got It? 3. What is the simplified form of each expression?

a. $(7m^9)^3$

b. $(2z)^{-4}$

c. $(3g^4)^{-2}$

Plan

How do you find the area of the square?

The area of a square with side length s is s^2 . Square the side length of the square to find the area.



Problem 4 Simplifying an Expression With Products

What is the simplified form of $(n^{\frac{1}{2}})^{10}(4mn^{-\frac{2}{3}})^3$?

$$(n^{\frac{1}{2}})^{10}(4mn^{-\frac{2}{3}})^3 = (n^{\frac{1}{2}})^{10}4^3 m^3 (n^{-\frac{2}{3}})^3 \quad \text{Raise each factor of } 4mn^{-\frac{2}{3}} \text{ to the third power.}$$

$$= n^5 4^3 m^3 n^{-2} \quad \text{Multiply the exponents of a power raised to a power.}$$

$$= 4^3 m^3 n^5 n^{-2} \quad \text{Commutative Property of Multiplication}$$

$$= 4^3 m^3 n^{5+(-2)} \quad \text{Add the exponents of powers with the same base.}$$

$$= 64m^3 n^3 \quad \text{Simplify.}$$



Got It? 4. What is the simplified form of each expression?

a. $(x^{-2})^2(3xy^5)^4$

b. $(3c^{\frac{5}{2}})^4(c^2)^3$

c. $(6ab)^3(5a^{-3})^2$

Think

What is the exponent of m ?

It has an implied exponent of 1. Similar to coefficients, exponents of 1 don't need to be written.

You can use the property of raising a product to a power to solve problems involving scientific notation. For example, to simplify the expression $(3 \times 10^8)^2$, you raise both 3 and 10^8 to the second power. Then multiply the two powers.

Problem 5 Raising a Number in Scientific Notation to a Power STEM

Aircraft The expression $\frac{1}{2}mv^2$ gives the kinetic energy, in joules, of an object with a mass of m kg traveling at a speed of v meters per second. What is the kinetic energy of an experimental unmanned jet with a mass of 1.3×10^3 kg traveling at a speed of about 3.1×10^3 m/s?

Plan

How do you raise a number in scientific notation to a power?

A number written in scientific notation is a product. Use the property for raising a product to a power.

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{1}{2} \cdot (1.3 \times 10^3)(3.1 \times 10^3)^2 \\ &= \frac{1}{2} \cdot 1.3 \cdot 10^3 \cdot 3.1^2 \cdot (10^3)^2 \\ &= \frac{1}{2} \cdot 1.3 \cdot 10^3 \cdot 3.1^2 \cdot 10^6 \\ &= \frac{1}{2} \cdot 1.3 \cdot 3.1^2 \cdot 10^3 \cdot 10^6 \\ &= \frac{1}{2} \cdot 1.3 \cdot 3.1^2 \cdot 10^{3+6} \\ &= 6.2465 \times 10^9 \end{aligned}$$

Substitute the values for m and v into the expression.

Raise the two factors to the second power.

Multiply the exponents of a power raised to a power.

Use the Commutative Property of Multiplication.

Add exponents of powers with the same base.

Simplify. Write in scientific notation.

The aircraft has a kinetic energy of about 6.2×10^9 joules.



Got It? 5. What is the kinetic energy of an aircraft with a mass of 2.5×10^5 kg traveling at a speed of 3×10^2 m/s?



Lesson Check

Do you know HOW?

Simplify each expression.

1. $(n^3)^6$

2. $(b^{-7})^3$

3. $(3a^{\frac{1}{2}})^4$

4. $(9x^{\frac{1}{2}})^2(x^2)^5$

Simplify each expression. Write each answer in scientific notation.

5. $(4 \times 10^5)^2$

6. $(2 \times 10^{-3})^5$

Do you UNDERSTAND?



7. **Vocabulary** Compare and contrast the property for raising a power to a power and the property for multiplying powers with the same base.
8. **Error Analysis** One student simplified $x^5 + x^5$ to x^{10} . A second student simplified $x^5 + x^5$ to $2x^5$. Which student is correct? Explain.
9. **Open-Ended** Write four different expressions that are equivalent to $(x^{\frac{2}{3}})^3$.



Practice and Problem-Solving Exercises



A Practice

Simplify each expression.

10. $(n^8)^4$

11. $(n^4)^8$

12. $(c^2)^{\frac{1}{4}}$

13. $(x^{\frac{2}{5}})^{10}$

14. $(w^7)^{-1}$

15. $(x^{\frac{3}{5}})^{-\frac{1}{2}}$

16. $d(d^{-2})^{-9}$

17. $(z^8)^0 z^{\frac{1}{2}}$

18. $(a^{\frac{2}{3}})^3 c^4$

19. $(c^3)^{\frac{1}{9}}(d^3)^0$

20. $(t^2)^{-2}(t^2)^{-5}$

21. $(m^3)^{-1}(x^{\frac{1}{3}})^{\frac{1}{4}}$

See Problems 1 and 2.

Simplify each expression.

22. $(3n^{-6})^{-4}$

23. $(7a)^{-2}$

24. $(5y^{\frac{1}{2}})^4$

◀ See Problems 3 and 4.

25. $(36g^4)^{-\frac{1}{2}}$

26. $(2x^{\frac{1}{6}})^3x^2$

27. $(2y^{\frac{7}{9}})^{-3}$

28. $(r^{\frac{2}{5}}s)^5$

29. $(y^2z^{-3})^{\frac{1}{6}}(y^3)^2$

30. $(3b^{-2})^2(a^2b^4)^3$

31. $4j^2k^6(2j^{11})^3k^5$

32. $(mg^4)^{-1}(mg^4)$

33. $(2j^2k^4)^{-5}(k^{-1}j^7)^6$

Simplify. Write each answer in scientific notation.

◀ See Problem 5.

34. $(3 \times 10^5)^2$

35. $(4 \times 10^2)^5$

36. $(2 \times 10^{-10})^3$

37. $(2 \times 10^{-3})^3$

38. $(7.4 \times 10^4)^2$

39. $(6.25 \times 10^{-12})^{-2}$

40. $(3.5 \times 10^{-4})^3$

41. $(2.37 \times 10^8)^3$

42. **Geometry** The radius of a cylinder is 7.8×10^{-4} m. The height of the cylinder is 3.4×10^{-2} m. What is the volume of the cylinder? Write your answer in scientific notation. (*Hint: $V = \pi r^2 h$*)

B Apply

Complete each equation.

43. $(b^2)^{\blacksquare} = b^8$

44. $(m^{\blacksquare})^{\frac{1}{3}} = m^{-12}$

45. $(x^{\blacksquare})^7 = x^6$

46. $(n^9)^{\blacksquare} = n$

47. $(y^{-4})^{\blacksquare} = y^{\frac{1}{2}}$

48. $7(c^1)^{\blacksquare} = 7c^{\frac{2}{3}}$

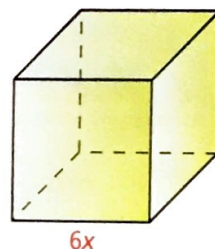
49. $(5x^{\blacksquare})^2 = 25x^{-4}$

50. $(3x^3y^{\blacksquare})^3 = 27x^9$

51. $(m^2n^3)^{\blacksquare} = \frac{1}{m^6n^9}$

© 52. **Think About a Plan** How many times the volume of the small cube is the volume of the large cube?

- What expression can you write for the volume of the small cube? For the volume of the large cube?
- What property of exponents can you use to simplify the volume expressions?



Simplify each expression.

53. $3^2(3x)^3$

54. $(4.1)^5(4.1)^{-5}$

55. $(b^{\frac{1}{6}})^3b^{\frac{1}{6}}$

56. $(-5x)^2 + 5x^2$

57. $(-2a^{\frac{2}{3}}b)^3(ab^{\frac{1}{3}})^3$

58. $(2x^{-3})^2(0.2x)^2$

59. $4xy^20^4(-y)^{-3}$

60. $(10^3)^4(4.3 \times 10^{-8})$

61. $(3^7)^2(3^{-4})^3$

© 62. **Reasoning** Simplify $(x^2)^3$ and x^2^3 . Are the expressions equivalent? Explain.

© 63. a. **Error Analysis** What mistake did the student make in simplifying the expression at the right?

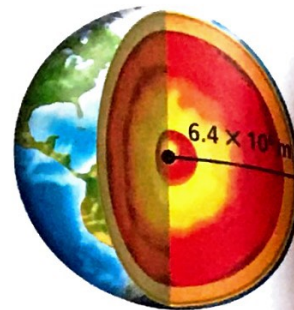
b. What is the correct simplified form of the expression?

$$\begin{aligned} (2 + 3)^2 &= 2^2 + 3^2 \\ &= 4 + 9 \\ &= 13 \end{aligned}$$

STEM 64. **Wind Energy** The power generated by a wind turbine depends on the wind speed. The expression $800v^3$ gives the power in watts for a certain wind turbine at wind speed v in meters per second. If the wind speed triples, by what factor does the power generated by the wind turbine increase?

65. Can you write the expression $49x^2y^2z^2$ using only one exponent? Show how or explain why not.

- STEM** 66. a. **Geography** Earth has a radius of about 6.4×10^6 m. What is the approximate surface area of Earth? Use the formula for the surface area of a sphere, $S.A. = 4\pi r^2$. Write your answer in scientific notation.
- b. Oceans cover about 70% of the surface of the Earth. About how many square meters of Earth's surface are covered by ocean water?
- c. The oceans have an average depth of 3790 m. Estimate the volume of water in Earth's oceans.



Challenge Solve each equation. Use the fact that if $a^x = a^y$, then $x = y$.

67. $5^6 = 25^x$

68. $3^x = 27^4$

69. $8^{\frac{1}{3}} = 2^x$

70. $4^x = 2^{\frac{1}{2}}$

71. $3^{2x} = 9^4$

72. $2^x = \frac{1}{32}$

- Reasoning** 73. How many different ways are there to rewrite the expression $16x^4$ using only the property of raising a product to a power? Show the ways.

Apply What You've Learned



Look back at the information on page 417 about the two CDs that Emilio is considering for the investment of his prize money.

When interest is compounded quarterly, interest is added to the account's principal every three months (one-quarter of a year). The bank calculates the interest using a rate that is $\frac{1}{4}$ of the annual interest rate.

- a. How many times per year is the interest from the Bank West investment compounded? What is the interest rate used to calculate the interest?
- b. Suppose Emilio chooses the Bank West CD. Confirm the values in the table below, and then complete the table. What is the value of the CD after one year?

Bank West CD

Quarter	Starting Principal	Interest Earned	Ending Principal
1	\$15,820.00	\$150.29	\$15,970.29
2	\$15,970.29	■	■
3	■	■	■
4	■	■	■

- c. If the Bank West CD earned simple interest instead of compound interest, what would the value of Emilio's CD be after one year?