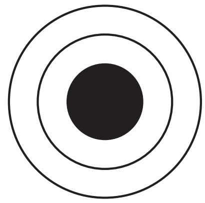
Name ______________________________

• Geometric Probabiltiy

Geometric Probability:
The probability based on the area of the regions
Formula:
Probability $= \dfrac{\text{area of included region}}{\text{area of known region}}$

You can measure the probability of hitting the bull's-eye on a target by applying the formula for geometric probability.

Example: The radius of the target is 10 inches and the radius of the bull's-eye is 4 inches. What is the probability that the bull's-eye is hit if the target is hit?



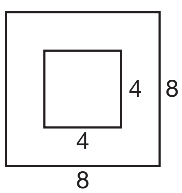
Area of target	**Area of bull's-eye**
$A = \pi r^2$	$A = \pi r^2$
$A = \pi(10 \text{ in.})^2$	$A = \pi(4 \text{ in.})^2$
$A = 100\pi \text{ in.}^2$	$A = 16\pi \text{ in.}^2$

$$\text{Probability} = \frac{\text{area of bull's-eye}}{\text{area of target}} = \frac{16\cancel{\pi}\ \cancel{\text{in.}^2}}{100\cancel{\pi}\ \cancel{\text{in.}^2}} = \frac{4}{25} \text{ or } 0.16$$

Practice:

1. For a–c, find the geometric probability that an object that lands in the shape will also land in the inner shape. Assume that it is equally likely that an object lands on any point of the shape.

a.



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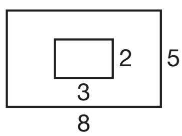
b.



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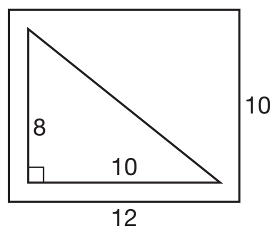
c.



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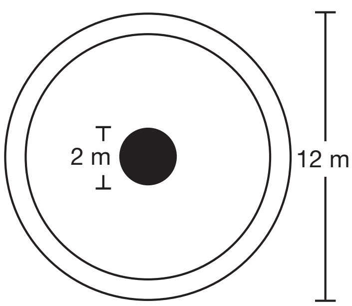
d.



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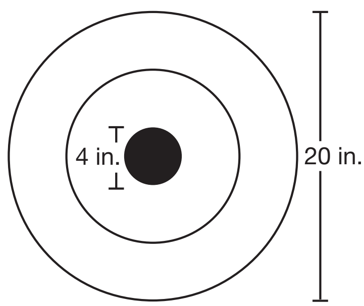
e.



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Name ______________________________

• Growth and Decay

In geometric sequences the ratio of consecutive terms is constant.

1, 4, 16, 64, 256, ...

The constant ratio is the multiplier from term to term.

$$1 \xrightarrow{\times 4} 4 \xrightarrow{\times 4} 16 \xrightarrow{\times 4} 64 \xrightarrow{\times 4} 256, \ldots$$

If the terms of a geometric sequence become smaller, then the constant ratio is a number between 0 and 1.

$$128 \xrightarrow{\times \frac{1}{2}} 64 \xrightarrow{\times \frac{1}{2}} 32 \xrightarrow{\times \frac{1}{2}} 16 \xrightarrow{\times \frac{1}{2}} 8, \ldots$$

Geometric sequences can be applied in the real world to study **growth and decay.** Growth is an increase, and decay is a decrease.

Practice:

1. Find the next term in this growth sequence. What is the constant ratio?

   2, 8, 32, 128, . . .

   ____

2. Find the next term in this decay sequence. What is the constant ratio?

   1024, 256, 64, 16, . . .

   ____

3. The booster club has experienced an increase in membership of 4% per year for the last three years. Their membership currently stands at 2,500. If this trend continues, about how many members will they have two years from now?

   ____

4. The number of stray animals brought into the animal shelter each year is declining. For the last 5 years the numbers have dropped 2% per year, with 1200 animals brought in this year. If this trend continues, about how many animals can the shelter expect two years from now?

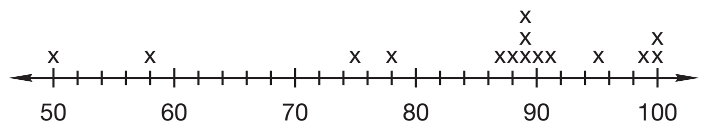
   ____

Name ________________________________

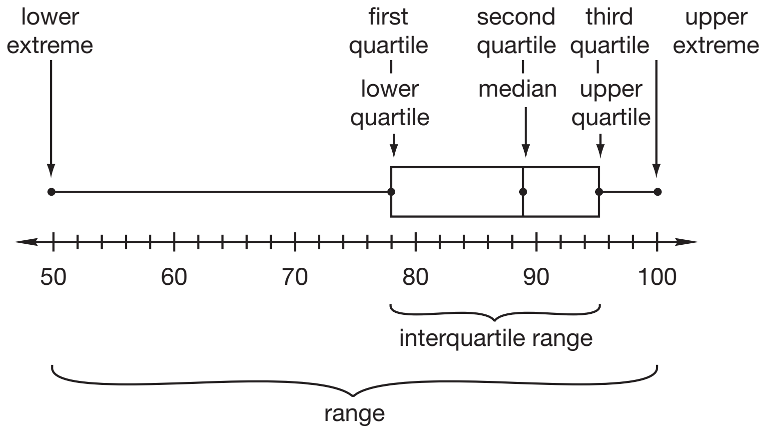
• Box-and-Whisker Plots

- A **line plot** is a visual display of data based on a number line.
  The following test scores appear on a line plot below:

78, 50, 99, 100, 75, 88, 89, 95, 58, 100, 87, 90, 91, 89, 89



- Another way to illustrate the spread of data is with a **box-and-whisker plot**, which identifies the median, quartiles, and the extremes.



A **line plot** and a **box-and-whisker plot** can be used to plot the same data.
The **line plot** is most helpful when you want to quickly convey the **mode** of data.
The **box-and-whisker plot** is better when trying to quickly convey the **median** of data.

Practice:

In a 5th grade class, the students kept a record of the number of books they read in a semester. The students' totals are recorded below.

15, 10, 12, 18, 22, 27, 12, 11, 23, 19, 18, 14, 20, 21, 23, 29, 26

1. Make a line plot of the data.

2. Create a box-and-whisker plot of these data.

3. Which of these graphs quickly conveys the mode? The median?

____

Name ________________________________

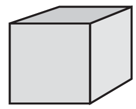
• Volume, Capacity, and Mass in the Metric System

In the metric system, units of volume, capacity, and mass are closely related. The relationships between these units are based on the physical characteristics of water under certain standard conditions.

Two commonly used relationships:

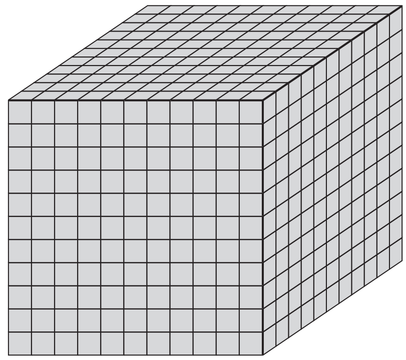
For Water Under Standard Conditions

Volume		Capacity		Mass
1 cm^3	=	1 mL	=	1 g



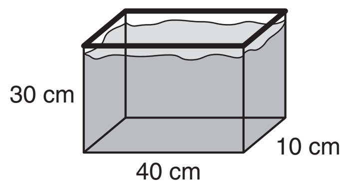
For Water Under Standard Conditions

Volume		Capacity		Mass
1000 cm^3	=	1 L	=	1 kg



One thousand cubic centimeters of water equals 1 liter of water, which has a mass of 1 kilogram.

Example: To determine how many liters of water are in this tank and the mass of the water, we find the volume of the tank.



(40 cm)(10 cm)(30 cm) = 12,000 cm^3

Each cubic centimeter of water is 1 milliliter. Twelve thousand milliliters is **12 liters.** Each liter of water has a mass of 1 kilogram, so the mass of the water in the tank is **12 kilograms**.

Practice:

a. What is the mass of 5 liters of water in kg? ____________________

b. What is the volume of 4 liters of water in cm^3? ____________________

c. An aquarium that is 50 cm long, 20 cm wide, and 30 cm deep can hold how many liters of water? ____________________

d. If the mass of the aquarium in Exercise c is 5 kilograms when it is empty, then what would be its mass when it is $\frac{1}{2}$ full of water? ________________

Name ____________________________________

• Compound Average and Rate Problems

Look at these examples to learn how to solve problems that include more than one average or rate.

Tamara earned \$20/hour tutoring a student for 8 hours last month. She also earned \$25/hour giving piano lessons for 10 hours during the same month.

To find the average she earned last month, you cannot simply find the average of \$20 and \$25 dollars. Instead, follow these steps:

Step 1: Find the amount Tamara earned for the tutoring and the piano lessons:
$\$20 \times 8$ hours $=$ \$160 for tutoring
$\$25 \times 10$ hours $=$ \$250 for piano lessons

Step 2: Find the total amount Tamara earned for both jobs:
18 hours at $x = \$410.00$

Step 3: Find the average earned (x) per hour by dividing:
$x = \$410/18 = \$22.78$

Example: Many **rate problems** can be solved like average problems as seen in the next example.

It took Mia 4 hours to drive the 240 miles to Atlanta. On the return trip, she was tired and took many breaks. It took her 6 hours to get home. Here's how to determine her average speed.

The average speed on the trip to Atlanta was $\frac{240 \text{ mi}}{4 \text{ hr}} = 60$ mph.

The average speed on the return trip was $\frac{240 \text{ mi}}{6} = 40$ mph.

Divide the total distance by the total time: $\frac{240 \text{ mi} + 240 \text{ mi}}{4 \text{ hr} + 6 \text{ hr}} = \frac{480 \text{ mi}}{10 \text{ hr}} = 48$ mph.

Practice:

1. Yuri earned \$8 per hour babysitting for 12 hours over the weekend. The next weekend she earned \$10 per hour for 8 hours of wrapping holiday gifts. What is Yuri's average hourly rate for the two weekends?____________________

2. Brittany earned \$6 per hour working in the yard for 5 hours one day. She earned \$7.50 per hour babysitting for 4 hours that night. She then earned \$10 per hour for working 6 hours in her parent's store the next day. What is Brittany's average hourly rate for the two day's work?____________________

3. John went on a 100 km bike ride. He covered the first 75 km in 3 hours. The last 25 km took him 2 hours. Find John's average speed for the full 100 km.____________________

• Reviewing the Effects of Scaling on Volume

When the dimensions of an object are scaled by a factor, the volume of the object increases (or decreases) by the cube of that factor.

Scale Factor

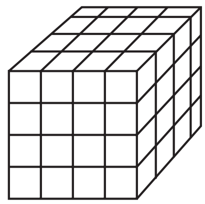
Ratio of lengths = scale factor

Ratio of areas = $(\text{scale factor})^2$

Ratio of volumes = $(\text{scale factor})^3$

Practice:

1. This cube is constructed of 64 blocks. How many blocks are needed to construct a cube with twice the dimensions of this cube?



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2. How many 3-inch cubes can fit in a 12-inch cubic container?

____

3. An artist created a $\frac{1}{60}$ scale model of a building for a model city. The volume of the model is what fraction of the volume of the real building?

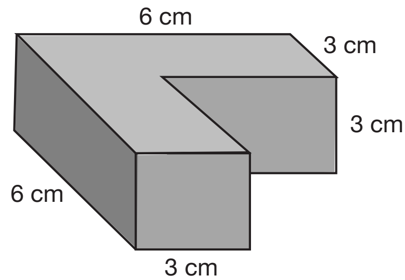
____

4. A crate has the following dimensions: 5 ft × 4 ft × 3 ft. What is the volume of a second crate that is twice the size of the original crate?

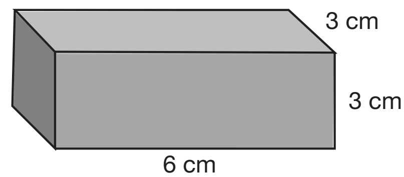
____

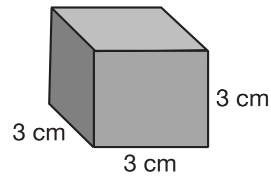
• Volume and Surface Area of Compound Solids

Breaking a complex figure into combinations of simpler forms helps us approximate or calculate the volume and surface area.



This solid can be broken down into two rectangular prisms:



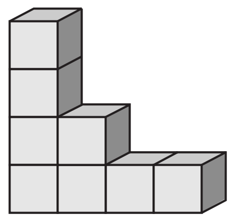


We find the volume of the parts and add to find the total volume.

We can also combine the surface area of the parts, but we are careful not to count surfaces that are not part of the surface of the original solid.

Practice:

1. This figure is composed of 1-inch cubes. Find its volume and exposed surface area (excluding the base).



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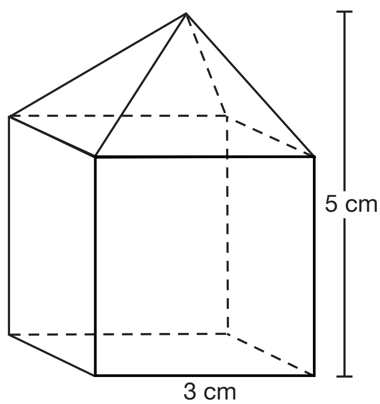
2. This figure is composed of a square pyramid and a cube. Find its volume.



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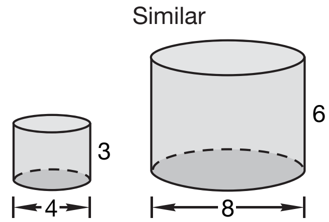
3. The slant height of the pyramid in Excercise 2 is 2.5 cm. What is the total surface area of the figure? ____________________

4. Use geometric solids to create a figure that is a combination of simple shapes. Then estimate the volume of the figure.

• Similar Solids

These two cylinders are similar:
Similar figures have the same shape, which means that their corresponding dimensions are **proportional.**
Dimensions of similar figures are related by a **scale factor**.



Relationship of Scale Factor to Area and Volume

Ratio of dimensions	=	scale factor
Ratio of surface areas	=	(scale factor)^2
Ratio of volumes	=	(scale factor)^3

Example: An architect is building a model of a building and each cm of her model is equal to 1 meter on the building.

The scale of the model is 1 cm = 100 cm (1 meter). The scale factor is 100.

The surface area of the building is 100^2, or 10,000 times the surface area of the model.

The volume of the building is 100^3, or 1,000,000 times the volume of the model.

Practice:

1. The volume of a model building built to a 1:12 scale is what fraction of the volume of the building?

____

2. The surface area of a miniature table built to a 1:8 scale is what fraction of the surface area of the table?

____

3. Joseph is wrapping a large box. The surface area is 880 sq in. His little sister is wrapping a similar box that is $\frac{1}{4}$ the size. What is the surface area of her box?

____

4. Brittany wants to create a small solid sculpture modeled after a larger one. She decided to use a 1:5 scale. The amount of clay needed to create the new sculpture is about what fraction of the volume of the larger sculpture?

____

Name ____________________

• Consumer Interest

- **Interest** is an amount of money added as a charge to money that is borrowed—which is called a loan. Interest is usually calculated as a **percentage** of the amount loaned, known as the **principal**.
- Interest is commonly added to real-estate mortgages, auto loans, and unpaid credit-card balances.
- We can find the interest added to a principal if we know the **interest rate**. We can find the interest rate if we compare the current amount due to the amount due before interest was added.

Practice:

1. Magda has a student loan balance of $6000. The bank charges 4% interest on the balance each year. What is the monthly interest added to Magda's current balance?

____

2. Jorge's credit card charges 1.24% monthly on unpaid balances. If Jorge pays $180 on a balance of $1790, what will his new balance be after one month's interest has been added?

____

3. a. Sylvia borrowed $7800 to buy a car. When her first payment is due, she notices that the current balance with interest is $7856.88. What annual interest rate is being added to her loan? Round your calculation to the nearest hundredth place.

____

b. Sylvia pays $1500 on a loan balance of $7856.88. Then interest is added and her new balance is $6392.64. What annual interest rate is Sylvia paying?

____

• Converting Repeating Decimals to Fractions

Some fractions, such as $\frac{5}{11}$ convert to repeating decimals.

$$\frac{5}{11} = 5 \div 11 = .454545 \text{ or } \frac{5}{11} = 0.\overline{45}$$

To convert a repeating decimal to a fraction we write three equations using algebra to remove the repeating part of the decimal number.

Follow these steps:

Step 1—First Equation Write your first equation with a variable (f) equal to the decimal number.

$$f = 0.\overline{45}$$

Step 2—Second Equation Count the number of digits that repeat. Then multiply both sides of the first equation by a power of 10 using the number of repeating digits as the exponent. The decimal $0.\overline{45}$ has two digits that repeat, so multiply by 10^2, or 100.

$100f = 100(0.\overline{45})$	Multiplied both sides of first equation by 100
$100f = 45.\overline{45}$	Simplified

Step 3—Third Equation Subtract the first equation from the second equation, and solve the third equation.

Second equation	$100f = 45.\overline{45}$
− First equation	$f = 0.\overline{45}$
Third equation	$99f = 45.0$
	$99f = 45$
	$f = \frac{45}{99} = \frac{15}{33}$

Practice:

1. Convert each repeating decimal to a reduced fraction.

   a. $0.\overline{3}$ ____________________

   b. $0.\overline{36}$ ____________________

   c. $0.\overline{90}$ ____________________

2. Write $15.\overline{15}\%$ as a repeating decimal, then as a reduced fraction.

   ____

3. Write $66.\overline{6}\%$ as a repeating decimal, then as a reduced fraction.

   ____