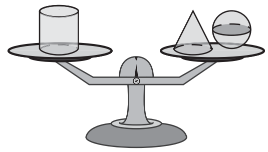
# • Volume and Surface Area of a Sphere

The relationship between the volume of a cylinder, the volume of a cone, and the volume of a sphere is a special one. If the heights and diameters of a cylinder and a cone equal the diameter of a sphere, then:

> The volume of a cone is $\frac{1}{3}$ the volume of a cylinder,
> and
> The volume of the sphere is twice the volume of the cone,
> and $\frac{2}{3}$ the volume of the cylinder.



Because of this special relationship, we are able to determine the formula for the volume of a sphere:

**Formula for the Volume of a Sphere**

$$\frac{4}{3}\pi r^3$$

The surface area of a sphere is 4 times the area of its greatest cross-sectional area. It's surface area also equals the lateral surface area of a cylinder with a diameter and height equal to the diameter of the sphere.

**Formula for the Surface Area of a Sphere**

$$A = 4\pi r^2$$

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**Practice:**

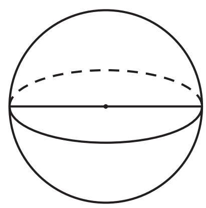
1. What fraction of the volume of a cylinder is the volume of a cone when both objects have the same height and base diameter? ________

2. What fraction of the volume of a cylinder is the volume of a sphere when the height and diameter of the cylinder equals the diameter of the sphere? ________

3. Find the volume and surface area of a 12-inch diameter playground ball. (Express in terms of $\pi$.)

   Volume ____________________

   Area ____________________



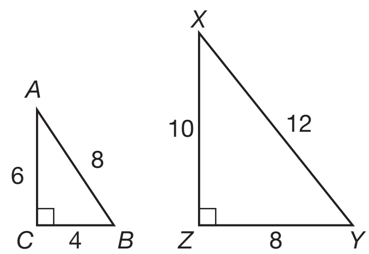
4. Find the volume of a sphere with a radius of 6 cm. Round to the nearest ten cubic centimeters. (You may use a calculator and use 3.14 for $\pi$.) ________

Name ________________________________



# • Ratios of Side Lengths of Right Triangles

- **Trigonometry** is a branch of mathematics that explores the relationships between the angles and sides of triangles. Some special math language is used in trigonometry.
- **Opposite Side and Adjacent Side**
  For each acute angle in a right triangle, one of the legs is the **opposite side** and the other leg is the **adjacent side.**
  From the perspective of $\angle A$, $\overline{BC}$ is the opposite side and $\overline{AC}$ is the adjacent side.
  From the perspective of $\angle B$, $\overline{AC}$ is the opposite side and $\overline{BC}$ is the adjacent side.
  We can find the ratios of side lengths.
  The ratio of the lengths of the opposite side of $\angle A$ to the hypotenuse is:




$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{8} = 0.5$$

The side opposite $\angle X$ is $\overline{YX}$. The hypotenuse is $\overline{XY}$. The ratio of the side opposite to the hypotenuse is:

$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{12} = 0.6$$

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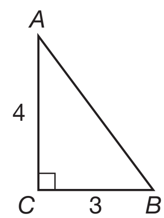
**Practice:**

**1.** Find each ratio for the illustrated right triangle. Express each ratio as a fraction and as a decimal rounded to three decimal places. With respect to $\angle A$, find the ratio of the lengths:




**a.** Opposite side to adjacent side ________________

**b.** Opposite side to hypotenuse ________________

**c.** Adjacent side to hypotenuse ________________

**2.** With respect to $\angle B$, find the ratio of the lengths:

**a.** Opposite side to adjacent side ________________

**b.** Opposite side to hypotenuse ________________

**c.** Adjacent side to hypotenuse ________________

Name ___________________________________



## • Using Scatterplots to Make Predictions

**Scatterplots** are graphs of points representing a relationship between two variables. Below are three scatterplots that display some common relationships.

This scatterplot indicates a strong relationship between the variables plotted. It has a **positive correlation**, which means that as one variable increases, the other variable increases.

This scatterplot also indicates a strong relationship; however, it has a **negative correlation**. As one variable increases, the other decreases.

This scatterplot shows **no meaningful relationship**. Researchers use these terms to describe relationships between variables. They also describe these relationships algebraically to determine the **predictability** of the relationship.

In this scatterplot we see that the **line of best fit** passes through the origin and point (10.0, 27.0), so it's slope is 2.7. Therefore, the algebraic equation is: $\boldsymbol{y = 2.7x}$
In this equation, $x$ represents volume and $y$ represents mass. This equation can be used to predict the mass if the volume is 100 cm$^3$: $y = 2.7x$; $y = 2.7 \cdot 100$; $\boldsymbol{y = 270}$ **g**

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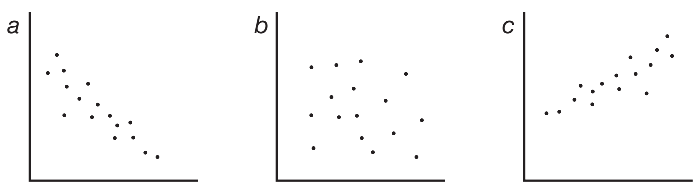
**Practice:**

1. Identify each graph as having a positive correlation, a negative correlation, or no meaningful relationship.




2. A company increased its advertising minutes over the course of a year. The advertising minutes and the sales in are recorded in the scatterplot. Draw a line of best fit. Then predict the dollars in sales when the advertising minutes

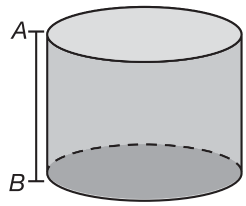
   reach 120. ________________

## • Calculating Area as a Sweep

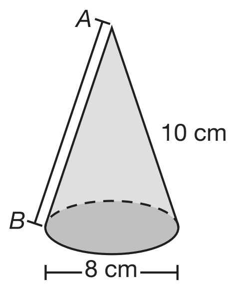
Refer to Lesson 114 for guidance to solve the following problems.

**Practice:**

1. Segment *AB* is 10 cm and the diameter of the cylinder is 12 cm. Describe how to calculate the lateral surface area of the cylinder. Find the lateral surface area expressed in terms of $\pi$.




2. Imagine slant height segment *AB* sweeping once around the cone. The points on $\overline{AB}$ move an average of half the circumference. Find the lateral surface area of the cone in terms of $\pi$.



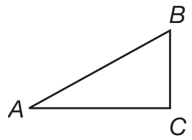

3. Imagine a broom steadily narrowing from 14 in. to 10 in. as it sweeps 8 inches to form a trapezoid. Multiply the average length of "broom" times the distance swept, to find the area of the trapezoid.

4. A sector of a circle is used to create the lateral surface of a cone with a slant height of 6 inches and a diameter of 4 inches. Using 3 for $\pi$, estimate the lateral surface area of the cone.

# • Relative Sizes of Sides and Angles of a Triangle

Notice the relationships between angle size and sides of these triangles:




$$m\angle A < m\angle B < m\angle C$$
$$BC < CA < AB$$

- The side opposite the smallest angle is the shortest side.
- The side opposite the largest angle is the longest side.

The order of side lengths of a triangle, when written from shortest to longest, corresponds to the order of the sides' opposite angles when written from smallest to largest.

$$m\angle A < m\angle B < m\angle C$$
$$BC < CA < AB$$

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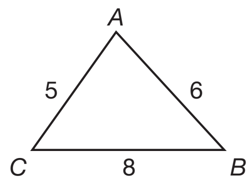
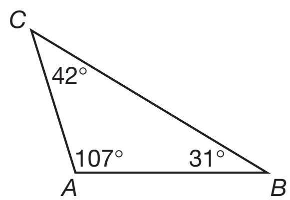
**Practice:**

**1.** Place the angles in order from smallest to largest. ____________________




**2.** Place the sides in order from shortest to longest. ____________________




**3.** Two sides of a triangle are equal in measure and are greater than 60°. Sketch a triangle that fits this description.

## • Division by Zero

Using a calculator, try to divide $\frac{0}{0}$. Try dividing any number by 0.
Notice that the calculator displays an error message when division by zero is entered. The display is frozen and other calculations cannot be performed until the erroneous entry is cleared.

We say that division by zero is **undefined.**
When performing algebraic operations, we guard against dividing by zero.

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**Practice:**

1. Using a calculator, divide each of these numbers by 0: 7, 45, 104, −4, −222. Remember to clear the calculator between calculations. What answers are displayed?

   ______________________

2. If we attempt to form two division facts for the multiplication fact $9 \times 0 = 0$, one of the arrangements is not a fact. Which arrangement is not a fact and why?

   ______________________

3. Under what circumstances does the following expression not equal 1? Why?

$$\frac{x - 7}{x - 7}$$

   ______________________

4. Find the number or numbers that may not be used in place of the variables in these expressions:

**a.** $\frac{12}{x}$ ______________________ **b.** $\frac{5}{3x}$ ______________________

**c.** $\frac{3}{x - 4}$ ______________________ **d.** $\frac{y + 7}{3y - 9}$ ______________________

## • Significant Digits

Because measures are not exact, a measure can be expressed only to a degree of precision. The number of digits we use to express a measure with confidence is called the number of **significant digits**.

Below are rules for counting significant digits:

1. Count all non-zero digits. (The number 4.73 has 3 significant digits.)
2. Count zeros between non-zero digits. (The number 20.3 has 3 significant digits.)
3. Count trailing zeros in a decimal number. (The number 2.40 has 3 significant digits.)
4. Do not count zeros if there is not a non-zero digit to the left. (The number 0.045 has 2 significant digits.)
5. Trailing zeros of whole numbers are ambiguous. The number of significant digits in 1300 might be 2, 3 or 4. Expressing a number in scientific notation removes the ambiguity. (The number $1.30 \times 10^3$ has 3 significant digits.)
6. Exact numbers have an unlimited number of significant figures, so we do not count any digits when dealing with exact numbers. Exact numbers include counts and exact conversion rates like 12 in./ft and 2.54 cm/in.

Follow these two rules when performing calculations with measures:

**A product or quotient of two measurements may only have as many significant digits as the measurement with the least number of significant digits.**

**A sum or difference may show no more decimal places than the measurement with the fewest number of decimal places.**

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### Practice:

**1.** Write the significant digits in the following numbers.

**a.** 20.98 min ______________

**b.** $3.503 \times 10^3$ cm ______________

**c.** 0.0084 mL ______________

**d.** 76.50 g ______________

**2.** Perform each calculation and express each answer with the correct number of significant digits.

**a.** $\frac{12.5 \text{ hrs}}{1.5 \text{ min}}$ ______________

**b.** 2.75 lbs + 3.9 lbs + 23.8 lbs ______________

Name ____________________



## • Sine, Cosine, Tangent

**Trigonometry Ratios**

| Name | Abbreviation | Ratio |
|---|---|---|
| sine | sin | $\frac{\text{opposite}}{\text{hypotenuse}}$ |
| cosine | cos | $\frac{\text{adjacent}}{\text{hypotenuse}}$ |
| tangent | tan | $\frac{\text{opposite}}{\text{adjacent}}$ |

You can use a calculator to find the decimal values of trig ratios. On a graphing calculator the trig functions are often abbreviated.

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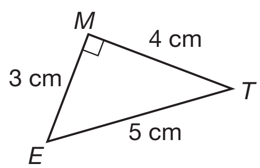
**Practice:**

**1.** Name the trig function for each ratio.

**a.** $\frac{\text{opposite}}{\text{hypotenuse}}$ **b.** $\frac{\text{adjacent}}{\text{hypotenuse}}$ **c.** $\frac{\text{opposite}}{\text{adjacent}}$

________ ________ ________

**2.** Find the sine, the cosine, and the tangent for ∠*T*. Express each ratio as a fraction and as a decimal.

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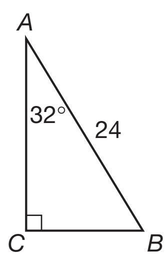
**3.** Use a calculator with trig functions to find to the nearest thousandth:




**a.** the sine of ∠*A*. **b.** the tangent of ∠*A*. **c.** the cosine of ∠*A*.

________ ________ ________

Name ______________________________



## • Complex Fractions

$$\frac{\frac{1}{4}}{\frac{2}{5}} \qquad \frac{15\frac{1}{3}}{90} \qquad \frac{12}{4\frac{1}{3}}$$

All of the above fractions are examples of complex fractions.
A **complex fraction** is a fraction containing one or more fractions in the numerator or denominator. It is customary to express fractions with integers when possible.

Below are two methods for simplifying a complex fraction:

- **Method One**

Simplify $\frac{\frac{1}{4}}{\frac{2}{5}}$

**Step One**: Find a common denominator: 20.
**Step Two:** Use the Identity Property for Multiplication.
Multiply the complex fraction by $\frac{20}{20}$:

$$\frac{{}^{5}\cancel{20}}{{}_{4}\cancel{20}} \cdot \frac{\frac{1}{\cancel{4}_1}}{\frac{2}{\cancel{5}_1}} = \frac{5}{8}$$

- **Method Two**

Simplify $\frac{\frac{1}{4}}{\frac{2}{5}}$

**Step One:** Treat the complex fraction as a fraction division problem:

Dividend
Divisor $\frac{\frac{1}{4}}{\frac{2}{5}} \longrightarrow \frac{1}{4} \div \frac{2}{5}$

**Step Two:** Solve. $\frac{1}{4} \div \frac{2}{5} = \frac{1}{4} \cdot \frac{5}{2} = \frac{5}{8}$

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**Practice:**

**1.** Simplify.

**a.** $\frac{\frac{2}{3}}{8}$ ______________________ **b.** $\frac{5\frac{1}{2}}{3\frac{3}{4}}$ ______________________

**c.** $\frac{6\frac{2}{5}}{3\frac{5}{9}}$ ______________________ **d.** $\frac{\frac{1}{6}}{\frac{7}{9}}$ ______________________

## • Rationalizing a Denominator

$$\frac{2}{\sqrt{5}}$$

When a fraction's denominator contains a radical, we take steps to form an equivalent fraction that has a rational number denominator.
Follow these steps to rationalize the denominator in the fraction $\frac{2}{\sqrt{5}}$.

| | |
|---|---|
| $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ | Use the identify property of multiplication to multiply the fraction by a fraction equivalent to 1. $\frac{\sqrt{5}}{\sqrt{5}} = 1$ |
| $\mathbf{\frac{2\sqrt{5}}{5}}$ | Multiplied<br>Rationalized denominator |

Be sure you reduce the fraction after you rationalize the denominator if possible.

| | |
|---|---|
| $\frac{4}{\sqrt{2}}$ | Fraction with a radical |
| $\frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ | Identity property of multiplication |
| $\frac{4\sqrt{2}}{2}$ | Rationalized denominator |
| $\mathbf{2\sqrt{2}}$ | Reduced |

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**Practice:**

**1.** Rationalize the denominator of each fraction.

**a.** $\frac{7}{\sqrt{2}}$ ____________________ **b.** $\frac{\sqrt{3}}{\sqrt{8}}$ ____________________

**c.** $\frac{4}{\sqrt{10}}$ ____________________ **d.** $\frac{6}{\sqrt{6}}$ ____________________

**e.** $\frac{2\sqrt{2}}{\sqrt{3}}$ ____________________ **f.** $\frac{1}{\sqrt{2}}$ ____________________