• Distributive Property
• Order of Operations

Distributive Property
$a(b + c) = a \cdot b + a \cdot c$
$a(b + c + d) = a \cdot b + a \cdot c + a \cdot d$
$a(b - c) = a \cdot b - a \cdot c$

- We **expand** $2(a + b)$ and get $2a + 2b$.
- We **factor** $2a + 2b$ and get $2(a + b)$.

Examples: Expand: $3(x + 2) = 3x + 6$

Factor: $6x + 9$ ⟵ divide to remove a common factor from each term

$3(2x + 3)$

- If there is more than one operation, follow this order:
  1. Simplify within **parentheses.**
  2. Simplify **exponent** expressions.
  3. **Multiply** and **divide** in order from left to right.
  4. **Add** and **subtract** in order from left to right.

- Remember this order as: "**P**lease/**e**xcuse/**m**y **d**ear/**A**unt **S**ally."
- Follow the order of operations to simplify within the parentheses.
- Simplify expressions with multiple grouping symbols beginning from the innermost symbol.

Practice:

Expand.

1. $4(2 + y) =$ ______________

2. $5(x - 3) =$ ______________

Factor.

3. $16w + 12 =$ ______________

4. $6x - 15 =$ ______________

Simplify.

5. $3 + 4 \cdot 5 - 6 =$ ______________

6. $(36 \div 6) \cdot (12 - 6) =$ ______________

7. $2^3 \cdot (14 - 9) + 8 =$ ______________

8. $20 - [3 \cdot (14 \div 7)] =$ ______________

Name ______________________________

• Multiplying and Dividing Fractions

- To multiply fractions, multiply numerators, and then multiply denominators.

  Example: $\frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$

- To find $\frac{3}{4}$ of $\frac{8}{9}$, multiply the fractions. We may reduce (cancel) before we multiply.

$$\frac{{}^{1}3}{{}_{1}4} + \frac{8^{2}}{9_{3}} = \frac{2}{3}$$

- If the product of two fractions is 1, the fractions are **reciprocals**.

  Example: $\frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1 \longleftarrow$ $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals

- Form the reciprocal of a fraction by reversing the numbers in the numerator and denominator, or inverting the fraction.
- Use reciprocals to help divide fractions.

  Example: $2 \div \frac{3}{4}$

  $$\downarrow \quad \downarrow \quad \downarrow$$

  $$\frac{2}{1} \times \frac{4}{3} = \frac{8}{3} = 2\frac{2}{3}$$

  Example: How many $\frac{1}{8}$s are in $\frac{1}{4}$?

  $$\frac{1}{4} \div \frac{1}{8}$$

  $$\frac{1}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{1} = \frac{2}{1} = 2$$

Practice:

1. $\frac{1}{3} \times \frac{2}{5} =$ ____________________

2. $\frac{3}{8} \cdot \frac{4}{5} =$ ____________________

3. $1 \div \frac{2}{3} =$ ____________________

4. $\frac{1}{2} \div \frac{3}{4} =$ ____________________

5. What number is $\frac{1}{2}$ of $\frac{3}{8}$? ____________________

6. How many $\frac{5}{8}$ are in $\frac{3}{4}$? ____________________

Name ________________________________

• Multiplying and Dividing Mixed Numbers

Three steps for multiplying and dividing mixed numbers.

1. Write whole numbers or mixed numbers as fractions.	
2. Multiplying: Multiply numerators. Multiply denominator.	2. Dividing: Instead of dividing, multiply by the reciprocal of the divisor.
3. Simplify the answer by reducing if possible. Improper fractions may be expressed as whole numbers or mixed numbers.	

Examples:

$$2\frac{1}{3} \times 1\frac{1}{2}$$

$$\frac{7}{3} \times \frac{3}{2} = \frac{21}{6} = 3\frac{3}{6} = 3\frac{1}{2}$$

$$1\frac{2}{3} \div 2\frac{1}{2}$$

$$\frac{5}{3} \div \frac{5}{2}$$

$$\frac{5}{3} \times \frac{2}{5} = \frac{10}{15} = \frac{2}{3}$$

Practice:

1. $1\frac{1}{3} \times 2\frac{1}{2} =$ ____________________

2. $1\frac{1}{4} \times 1\frac{1}{2} =$ ____________________

3. $2\frac{1}{3} \div 1\frac{3}{4} =$ ____________________

4. $2\frac{3}{4} \div 1\frac{1}{2} =$ ____________________

5. What is the area of a rectangle $2\frac{2}{3}$ inches long and $1\frac{1}{8}$ inches wide? __________

6. A recipe calls for $1\frac{2}{3}$ cups of sugar. How many cups of sugar are needed if the recipe is doubled? ______________________________

• Adding and Subtracting Decimal Numbers

- When adding or subtracting decimal numbers:
  1. Align decimal points, placing point on right of whole number.
  2. You may add zeros as placeholders.
  3. Add columns. Place a decimal point in the sum aligned with the other decimal points.

  Example: 12.5 + 3.75 + 2 ⟶

$$\begin{array}{r} 12.50 \\ +\ \ 3.75 \\ \underline{+\ \ 2.00} \\ 18.25 \end{array}$$

Practice:

1. 6.5 + 2.12 = ____________________

2. 3.84 + 14 = ____________________

3. 9.3 − 7.88 = ____________________

4. 5 − 1.25 = ____________________

5. The weekend snowfall was 9.5 inches. That brings the monthly snowfall total to 15.25 inches. What was the snowfall total prior to the weekend?

____

6. Sarah's temperature went up 1.5 degrees. If her body temperature was 98.6°F to start, what is her temperature now?

____

• Multiplying and Dividing Decimal Numbers

- To **multiply decimal numbers**:
  1. Multiply to find the product.
  2. Count all the decimal places in the factors to locate the position for the decimal point in the product. Add zeros as placeholders when needed.

Keyword: count

$$\begin{array}{r l} \left.\begin{array}{r} 0.12 \\ \times\ 0.3 \end{array}\right\} & \text{3 places in factors} \\ \hline .036 & \text{3 places in product} \end{array}$$

- To **divide by a whole number**:
  1. Place the decimal point in the quotient above the decimal point in the dividend.
  2. Divide.
  3. Fill empty places with zeros if necessary.

Keyword: up

$$\begin{array}{r} .07 \\ 6\overline{)0.42} \end{array}$$

- To **divide by a decimal number**:
  1. Make the divisor a whole number by moving both decimal points the same number of places. Add zeros as placeholders when needed.
  2. Place the decimal point in the quotient above the decimal point in the dividend.
  3. Divide.
  4. Fill empty places with zeros if necessary.

Keywords: over, over, up

$$\frac{0.42}{0.6}$$

$$\begin{array}{r l} 0.7 & \text{align decimal points} \\ 06.\overline{)\,04.2} & \text{move decimal points} \end{array}$$

Practice:

1. $0.3 \times 0.46 =$ ____________________

2. $8.1 \times 0.7 =$ ____________________

3. $6.75 \div 3 -$ ____________________

4. $6.15 \div 0.05 =$ ____________________

5. What is the cost of 12.5 gallons of gas at \$2.489 per gallon? Round to the nearest cent.

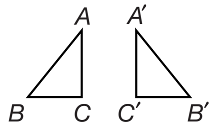
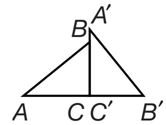
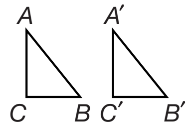
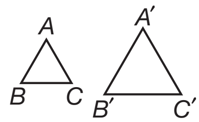
____

6. If 2.4 pounds of oranges cost \$3.00, then what is the cost per pound?

____

Name ____

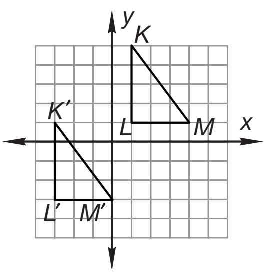
• Transformations

Reflection (flip)	reflecting a figure as in a mirror or "flipping" a figure over a certain line	
Rotation (turn)	turning a figure about a certain point	
Translation (slide)	sliding a figure in one direction without turning the figure	
Dilation (enlargement)	enlarging a figure with a scale factor without changing the shape	

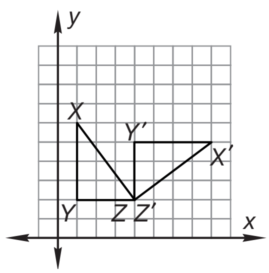
Practice:

Identify the transformation in each exercise.

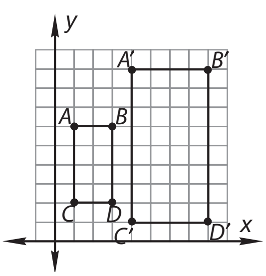
1. ______________________________



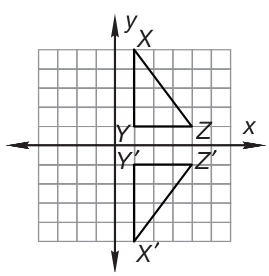
2. ______________________________



3. ______________________________



4. ______________________________



Name ____________________

• Laws of Exponents

- A counting-number exponent indicates how many times a base is a factor.

  Example: $x^4 = x \cdot x \cdot x \cdot x$

- Laws of Exponents describe relationships between exponents for certain operations.

Laws of Exponents for Multiplication and Division

Law	Description	Example
$x^a \cdot x^b = x^{a+b}$	When the bases are the same, and you multiply, you add the exponents.	$x^5 \cdot x^3 = x^8$
$\frac{x^a}{x^b} = x^{a-b}$	When you divide, you subtract the exponents.	$\frac{x^5}{x^3} = x^2$
$(x^a)^b = x^{ab}$	When you raise a term with an exponent to a power, you multiply the exponents.	$(x^3)^2 = x^6$

Practice:

1. $x^6 \cdot x^2 =$ ____________________

2. $\frac{x^4}{x^2} =$ ____________________

3. $(x^4)^3 =$ ____________________

4. $2^4 \cdot 2^2 =$ ____________________

5. $(2^2)^3 =$ ____________________

6. Write this expression as a power of 10.

$\frac{10^6}{10^3} =$ ____________________

• Scientific Notation for Large Numbers

- Numbers used in science are often very large or very small.
- **Scientific notation** is a way of writing a number as a decimal number times a power of 10.

  Example: $93{,}000{,}000 = 9.3 \times 10^7$

  "Nine point three times ten to the seventh."

- To write a large number in standard form:
  1. Shift the decimal point to the right the number of places shown by the positive exponent.
  2. Use zero as a placeholder.

  Example: Write 4.26×10^6 in standard form.

  $4.26 \times 10^6 \longrightarrow 4260000 \longrightarrow 4{,}260{,}000$

- To write a large number in scientific notation:
  1. Place the decimal point to the right of the first non-zero digit.
  2. Use the power of 10 to show the real location of the decimal point.
  3. Drop any trailing zeros.

  Example: Write 405,700,000 in scientific notation.

  $405{,}700{,}000 \longrightarrow 405{,}700{,}000 \longrightarrow 4.057 \times 10^8$

Practice:

1. Write "three point two times ten to the fifth" with digits. ____________

2. Use words to write 5.7×10^4. ____________

Write each number in standard form.

3. $4.68 \times 10^6 =$ ____________

4. $9.01 \times 10^3 =$ ____________

Write each number in scientific notation.

5. $20{,}500{,}000 =$ ____________

6. $7{,}900{,}000 =$ ____________

Name ________________________________

• Ratio

- A **ratio** is a comparison of two numbers by division.

  For example, if there are 16 girls and 12 boys in the class, the ratio of girls to boys can be expressed as a reduced fraction.

$$\frac{\text{Girls}}{\text{Boys}} = \frac{16}{12} = \frac{4}{3}$$

- Write the ratio in the order stated
- Write the ratio as a reduced fraction, but not as a mixed number.
- The ratio of "3 to 4"can be written four ways:

with the word "to"	3 to 4
as a fraction	$\frac{3}{4}$
as a decimal number	0.75
with a colon	3:4

- We can round large numbers and reduce the ratio.

  Example: $\frac{\text{1217 home fans}}{\text{897 visiting fans}} \approx \frac{1200}{900} = \frac{4}{3}$

- A rate is a ratio of two measures with different units. The units do not cancel, but remain part of the rate.

  Example: $\frac{\text{35 miles}}{\text{gallon}} \longrightarrow$ 35 miles per gallon

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Practice:

Use for Problems 1 and 2: There are 32 students in the math class. Fourteen students are boys and eighteen students are girls.

1. What is the ratio of boys to girls? ________

2. What is the ratio of girls to boys? ________

Use for Problems 3 and 4: A 25-foot tree casts a shadow 35 feet long.

3. What is the ratio of the height of the tree to the length of the shadow? ________

4. What is the ratio of the length of the shadow to the height of the tree? ________

5. Mrs. Shaw corrected 25 tests in 5 minutes. Find the rate of test corrections per minute. ____________________

6. If 1487 adults and 912 children attended the concert, what was the approximate ratio of adults to children? ____________________

Name ______________________________

• Repeating Decimals

- Some fractions convert to decimal numbers that have repeating digits.

  For example: $\frac{3}{11} = 0.27272727\ldots$

- The repeating digits are called the **repetend**.
- Repeating digits are shown with a bar over the repetend.

  For example, write 0.27272727 . . . as $0.\overline{27}$

- To round a repetend:
  1. Remove the bar and write the repeating digits.
  2. Round as usual.

  Example: Round $2.\overline{45}$ to the nearest thousandth.

  $2.45\mathbf{4}545\ldots \longrightarrow 2.455$

- To make comparing numbers with a repetend easier, show each number with the same number of places. Then compare.

  Example: Order 0.3, $0.\overline{3}$, and 0.33 from least to greatest.

  Think: $0.3 = 0.300$

  $0.\overline{3} = 0.33\overline{3}$

  $0.33 = 0.330$

  In order: 0.3, 0.33, $0.\overline{3}$

Practice:

Write each fraction as a decimal number.

1. $\frac{2}{11} =$ ____________________ **2.** $\frac{5}{6} =$ ____________________

Round each decimal number to the nearest thousandth.

3. $0.\overline{7} =$ ____________________ **4.** $1.\overline{21} =$ ____________________

Arrange in order from least to greatest.

5. $0.\overline{2}$, 0.2, 0.22 $=$ ______________________________

6. $\frac{1}{4}$, $0.\overline{25}$, 0.2 $=$ ______________________________