• Sequences

In an **arithmetic sequence** the same number **is added to** each term to find the next term.

4, 8, 12, 16, . . .

In the above arithmetic sequence, each term is 4 more than the previous term.

This table shows how the number and value of the terms are related.

Number (n)	1	2	3	4
Value (a)	4	8	12	16

To find the value for any term in this sequence, use this formula: $a_n = 4n$.

The 8th number in this arithmetic sequence is 32, because $4(8) = 32$.

In a **geometric sequence** the same number **multiplies** each term to find the next term.

2, 4, 8, 16, . . .

In the above geometric sequence, each term is two times the preceding term.

This table shows how the number and value of the terms are related.

Number (n)	1	2	3	4
Value (a)	2	4	8	16

To find the value for any term in this sequence, use this formula: $a_n = 2^n$.

The fifth term of the sequence is 32, because $2^5 = 32$.

Practice:

1. Describe each sequence as arithmetic, geometric, or neither.

a. 1, 5, 25, 125, 625, . . . ____________ **b.** 1, 7, 14, 21, 28, . . . ____________

c. 4, 16, 64, 256, . . . ____________ **d.** 1, 2, 4, 7, . . . ____________

2. What is the constant difference in this sequence? ____________

1, 6, 11,16, . . .

3. What is the constant ratio in this sequence? ____________

6, 36, 216, 1296, . . .

4. Write the first 4 terms described by these formulas.

a. $a_n = 5n$ ____________ **b.** $a_n = 4^n$ ____________

Name ________________________________

• Graphing Solutions to Inequalities on a Number Line

We solve inequalities the same way we solve equations, with one exception.
Here are the steps to solve the inequality $2x - 5x + 4 \le 10$.
Combine the like terms on the left side of the inequality.

$$-3x + 4 \le 10$$

Subtract 4 from both sides of the inequality.

$$-3x + 4 - 4 \le 10 - 4$$
$$-3x \le 6$$

Divide both sides of the inequality by –3, and here is the exception.
We reverse the direction of the inequality because we are dividing by a negative number.

$$\frac{-3x}{-3} \ge \frac{6}{-3}$$

Solve:

$$x \ge -2$$

Practice:

1. Solve each of these inequalities and graph their solutions on number lines.

a. $2(x - 1) > 4$ ______________________________

b. $3x + 4 > 7$ ______________________________

c. $\frac{-1}{2}x - 3 > 8$ ______________________________

d. $\frac{1}{3}x - 2 \le \frac{2}{3}x - 6$ ______________________________

2. Ed is thinking of a number. If he adds four to his number, the result is less than 6. Write and solve an inequality about Ed's number.

____

• Rational Numbers, Non-terminating Decimals, and Percents
• Fractions with Negative Exponents

- **Terminating and Non-terminating Decimals**

  Terminating decimal—a decimal that divides out exactly or terminates (stops).

$$\frac{3}{4} = 0.75$$

  Non-terminating, repeating decimal—a decimal that keeps repeating.

$$\frac{5}{6} = 0.8333\ldots$$

  Indicate the repeating digits by writing a bar over the repetend.

$$0.8\overline{3}$$

  To convert a fraction to a percent, multiply the fraction by 100%.

$$\frac{5}{6} \times 100\% = \frac{500\%}{6} = 83\frac{1}{3}\%$$

  Do not perform computations with a decimal number that has a bar over the repetend. Use the fraction equivalent, or round the decimal number to an appropriate number of decimal places.

- **Fractions with Negative Exponents**

  A negative exponent indicates a reciprocal. To make a negative exponent positive, move the base to its reciprocal and change the sign of the exponent.

$$x^{-n} = \frac{1}{x^n} \qquad 4^{-3} = \frac{1}{4^3}$$

  Remember that if the base of a negative exponent is a fraction, make the exponent positive by replacing the fraction with its reciprocal. $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2$ or $4^2 = 16$

Practice:

1. Convert each fraction to a decimal and a percent. Then write the decimal rounded to the nearest hundredth.

a. $\frac{5}{12}$ ______________________________

b $\frac{1}{11}$ ______________________________

c. $\frac{2}{3}$ ______________________________

d. $\frac{4}{9}$ ______________________________

2. Arrange in order from least to greatest.

$$4\frac{2}{23},\ 4\frac{1}{5},\ 4\frac{1}{12},\ 4.05,\ 4.083$$

• Using a Unit Multiplier to Convert a Rate

A unit multiplier can be used to convert a rate to a different unit of measure.

Example: Jan walks a mile in 20 minutes or $\frac{1 \text{ mile}}{20 \text{ min}}$.

You can use a unit multiplier to find her average rate in miles per hour.

Decide whether to use $\frac{60 \text{ min}}{1 \text{ hour}}$ or $\frac{1 \text{ hour}}{60 \text{ min}}$

We choose $\frac{60 \text{ min}}{1 \text{ hour}}$ because we can cancel the minutes.

$$\frac{1 \text{ mile}}{20 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hour}}$$

Cancel the minutes and simplify.

$$\frac{1 \text{ mile}}{20 \cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{1 \text{ hour}} = \frac{60 \text{ miles}}{20 \text{ hours}}$$

$$\frac{60 \text{ miles}}{20 \text{ hours}} = \frac{3 \text{ miles}}{1 \text{ hour}}$$

So, Jan walks 3 miles per hour or 3 mph.

Practice:

Analyze and use a unit multiplier to perform the following rate conversions.

1. 36 miles per gallon to miles per quart. (1 gal = 4 qt)

   ____

2. Ms. Roberts used 110 gallons of water in an hour while working in the garden. Find the amount of water in quarts per hour.

   ____

3. Alex practices her harp 16 hours per week. Find the amount of time she practices in minutes per week.

   ____

4. 300 yards per minute to yards per second

   ____

5. Jeb used 3 quarts of paint in 3 hours. Find Jeb's rate in minutes.

   ____

6. Kethia ran 1200 yards in 4 minutes. Find her rate in feet per minute.

   ____

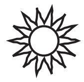
Name ______________________________

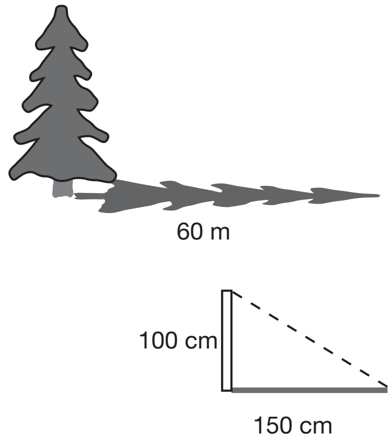
• Applications Using Similar Triangles

Similar triangles
Similar triangles have matching angles.
The side lengths of similar triangles are proportional.

You can use similar triangles to measure some objects indirectly.

Example: This tree is too tall to measure. Follow these steps to find the height of the tree.





1. Stand a meter stick in a vertical position.
2. Measure the lengths of both shadows.
3. Think of the tree and its shadow as a triangle. Think of the meter stick and its shadow as a similar triangle. The corresponding angles are congruent, so the triangles are similar and the corresponding sides are proportional.
4. Write a proportion.

	△1	△2
height	*t*	100 cm (meter stick)
shadow	60 m	150 cm

5. Cross multiply to solve:

$$\frac{t}{60} = \frac{100}{150} \qquad 150t = 60 \cdot 100 \qquad t = 60 \cdot \frac{100}{150} \qquad \frac{6000}{150} = 40 \text{ meters.}$$

Practice:

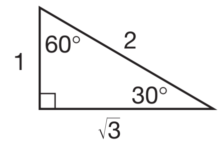
1. Justine wanted to find the height of a new downtown building. She used a meter stick to create a triangle similar to the one created by the building and its shadow. The building shadow was 50 meters long, and the meter stick shadow was 80 centimeters long. How tall is the new building? ____________________

2. Jana noticed a man from the cable company working at the top of the telephone pole. She wondered how high up the man was. She "stepped off" the length of the shadow of the pole. It was 16 steps. Each step was about 1 foot. She then marked the length of her own shadow. It measured 4 steps. Jana is about 5 feet tall. About how high up is the man on the pole? ____________________

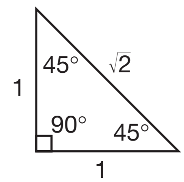
• Special Right Triangles

Two Special Right Triangles

A triangle with angles measuring 30, 60, and 90 degrees is a special right triangle. All 30-60-90 triangles have side lengths in the ratio of $1:\sqrt{3}:2$.



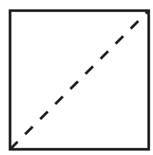
A triangle with angles measuring 45, 45, and 90 degrees is a special right triangle. All 45-45-90 triangles have side lengths in the ratio of $1:1:\sqrt{2}$.



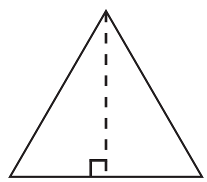
If you know the length of one side of a special triangle, you can use the ratio to find the length of the other two sides.

Example: One leg on a 45-45-90 triangle is 2 cm.
Multiply each number in the ratio by 2.
Ratio: $1:1:\sqrt{2}$ becomes $2:2:2\sqrt{2}$.
Two sides of the triangle are 2 cm. The third side is $2 \times \sqrt{2}$ or about $2 \times 1.41 = 2.82$ cm.

Using what you know about these triangles also helps you find information about a square and an equilateral triangle sides and the area of a triangle.



A square is two 45-45-90 triangles.



An equilateral triangle is two 30-60-90 triangles.

Practice:

1. Sketch a 30-60-90 triangle and indicate the angle measures and side lengths if the shortest side is 4 cm. ____________
2. Sketch a 45-45-90 triangle. One leg is 3. Indicate the angle measurements and side lengths. ____________
3. How long is the diagonal of a square with sides 10 in. long? ____________
4. What is the height of an equilateral triangle with sides 10 in. long? ____________

• Percent of Change

Percents can be used to describe a change. The change may be:

an **increase** (her salary increased by 15%), or

a **decrease** (the computer was marked down 15%).

Use a ratio table and make the orignal 100%.

Example: Maggie bought a purse for $40, 25% off the original price.

Step 1: Fill in the information you know, including 100%.

	%	Actual Count
Original	100	x
Change (–)	25	
New		40

Step 2: We subtract 25 from 100 because the change is a decrease. (Be sure you add during this step if the change is an increase.) We write a variable for the number we want to know.

	%	Actual Count
Original	100	x
Change (–)	25	
New	75	40

Step 3: Use the row you know and the row you want to know to write a proportion. Then calculate.

$\frac{100}{75} = \frac{x}{40}$ $75x = 100 \times 40$ $x = 100 \times \frac{40}{75}$ $x = \$53.33$

Practice:

1. The number of students at Jefferson High increased by 16% from 650. How may students now attend the high school? ________________

2. Fred invests $630 in his savings account each year. He earns interest at a rate of 4.5% per year. After the first year, how much is in Fred's account? ________

3. Mr. Martinez received a raise of 14%. His monthly salary was $5600. What is his new monthly salary? ________________

4. Yuri sold her 35 mm camera. The original price was $550. She took a 15% loss. What did she sell her camera for? ________________

5. Robin bought a $225 bike on sale for $180. What percent is the bike marked down? ________________

Name ____________________

• Probability Multiplication Rule

Multiplication Counting Principle

If an experiment has two parts, the first part with m possible outcomes and the second with n possible outcomes; then the total number of possible outcomes for the experiment is the product $m \cdot n$.

Multiplication Rule for Probability

If events A and B are independent, then $P(A \text{ and } B) = P(A) \cdot P(B)$

Practice:

1. There are eight counters in a bag. Three of the counters are blue and 5 of the counters are yellow. A counter is picked and then returned to the bag. A second counter is picked.

a. What is the probability that you picked a blue and then a yellow counter?

____

b. What is the probability both counters were blue?

____

c. What is the probability that neither counter was blue?

____

2. You pick a card from a standard deck of 52 playing cards. You replace it and pick a second card.

a. What is the probability of picking two black cards?

____

b. What is the probability of picking two hearts?

____

c. What is the probability of picking first a club and then a queen?

____

• Direct Variation

Direct Variation Equation
$y = kx$

Characteristics of Direct Variation
1. When one variable is zero, the other is zero, so its graph intersects the origin. 2. As one variable increases, the other increases by the same factor, so the graph lies on a rising line.

In the equation $y = kx$, we find the constant of variation by dividing each side by x.

$$\frac{y}{x} = k$$

A table can be used to illustrate an example of direct variation:

$y = 3x$

x	y	$\frac{y}{x}$
1	3	$\frac{3}{1} = 3$
2	6	$\frac{6}{2} = 3$
3	9	$\frac{9}{3} = 3$

Practice:

1. Joseph earns $10.00 dollars an hour.

a. Write a direct variation equation to show the relationship. ________

b. Illustrate this relationship on a table and a graph.

2. State whether each equation represents direct variation.

a. $y = \frac{3}{2}x$ ________ **b.** $y = 3x$ ________ **c.** $y = 4x + 3$ ________

Name ______________________________

• Solving Direct Variation Problems

Remember: Quantities that vary directly are proportional.

Look at the ratio between the number of tickets sold and the price.

# of movie tickets	Price ($)
1	8
2	16
3	24
4	32

Price = 8 × number

Every price/number ratio in the table equals the constant, which is 8.

$$\frac{32}{4} = \frac{24}{3} = \frac{16}{2} = \frac{8}{1} = 8$$

One variable (the price) is determined by multiplying the other variable (the number of movie tickets) by the constant.

To find the cost of any number of movie tickets we multiply by the constant. For example 11 tickets would cost $8 \times 11 = \$88.00$.

Practice:

1. Yuri drives at a constant speed. She goes 192 miles in 3 hours.

a. What distance will she travel in 5 hours? ____________________

b. How long does it take her to cover 128 miles? ____________________

2. A recipe for 8 people requires 4 cups of sugar and 10 cups of flour. How much would be required for a recipe for

a. 10 people? **b.** 16 people? **c.** 3 people?

________________ ________________ ________________

3. Big Mo's is having a sale on holiday decorations. He began the sale with 5000 boxes of decorations. He noticed that at the end of the first 2 hours, 4500 boxes were left. If the rate of purchase remains steady throughout the day, how many will be sold by the end of an 8 hour day? ________________