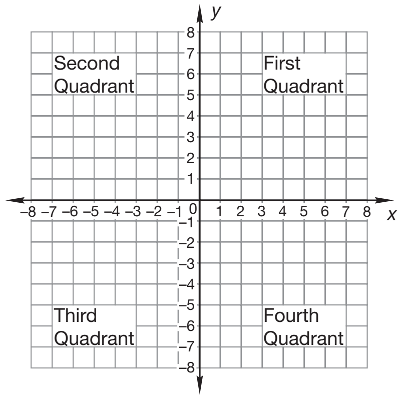
• The Coordinate Plane

Two perpendicular number lines form a **coordinate plane**.

***x*-axis**—the horizontal number line
***y*-axis**—the vertical number line
origin—the point at which the *x*-axis and the *y*-axis **intersect**
quadrants—the four regions that make up a coordinate plane.

The quadrants are numbered counterclockwise beginning with the upper right as the first.



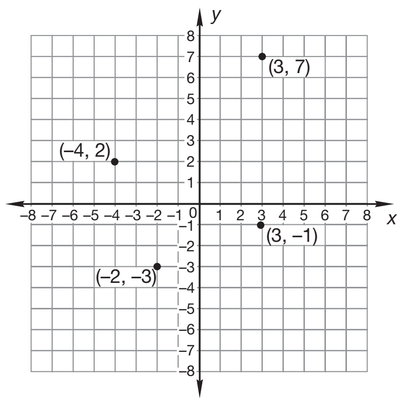
Any point on the coordinate plane can be identified with two numbers. These two numbers are called **coordinates** of the point.

Coordinates are written as a pair of numbers in parentheses, for example: (3, 7).

Reading and Plotting Coordinates

The first number shows the horizontal direction and distance from the origin.
The second number show the vertical direction and distance from the origin.
The sign of the number indicates the direction:

Positive coordinates are to the right or up.
Negative coordinates are to the left or down.



Practice:

Use graph paper for these activities.

1. Draw a coordinate plane. Graph and label the following points on the coordinate plane:
(4, 7) (–5, 2) (0, –3) (–2, 6) (–4, –1)

2. The vertices of rectangle *KLMN* are located at coordinates (3, 4), (4, –1), (–2, –1), (–2, 3). Plot the coordinates and find its perimeter and area.

P = ____________________

A = ____________________

3. Name the quadrant in which each point lies.

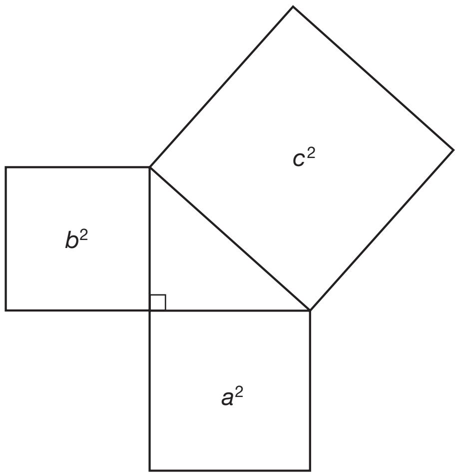
(–2, 7) __________ (3, 4) __________ (–4, –8) __________

• Pythagorean Theorem

Pythagorean Theorem

If a triangle is a right triangle, then the sum of the squares of the legs equals the square of the hypotenuse. $a^2 + b^2 = c^2$

Here is a picture that illustrates the Pythagorean Theorem. The areas of the two smaller squares together equal the area of the largest square.



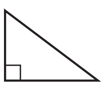
The Pythagorean Theorem applies to all right triangles and only to right triangles. A triangle with sides 3 cm, 4 cm, and 5 cm long is a right triangle because

$$3^2 + 4^2 = 5^2.$$

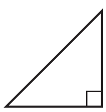
Practice:

Sketch squares on the sides of these triangles. Calculate the area of each square. Then use that information to find the length of the unknown side. If the side is irrational you may leave the answer in square root form or use a calculator and round the answer to one decimal place.

1.

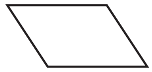
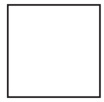
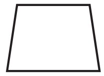
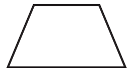
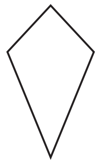
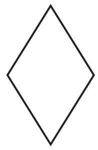
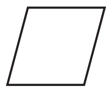


2.



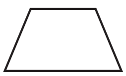
Name ______________________________

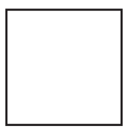
• Classifying Quadrilaterals

Quadrilateral—a four-sided polygon	
Parallelogram—a quadrilateral with two pairs of parallel sides	
Trapezoid—a quadrilateral with one pair of parallel sides	
Isosceles Trapezoid—a trapezoid in which the non-parallel sides are equal in length	
Trapezium—a quadrilateral with no pairs of parallel sides	
Kite—a trapezium that has two pairs of adjacent sides of equal length	
Rectangle—a quadrilateral with four right angles	
Rhombus—a quadrilateral with four equal-length sides	

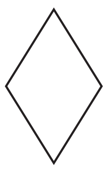
Practice:

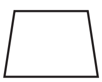
Identify each of the following quadrilaterals. If they qualify as more than one type, include each.

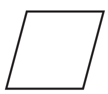
1.  ____________________

2.  ____________________

3.  ____________________

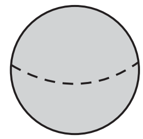
4.  ____________________

5.  ____________________

6.  ____________________

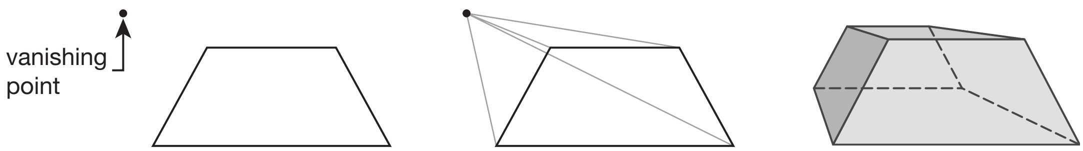
Name ______________________________

• Drawing Geometric Solids

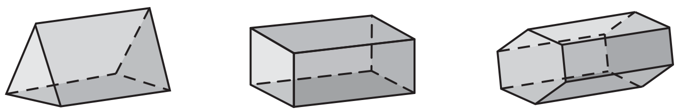


- Refer to Investigation 4 for descriptions and illustrations of geometric solids.
  The terms **face**, **edge**, and **vertex** refer to specific features of solids.

One-Point Perspective Drawing:

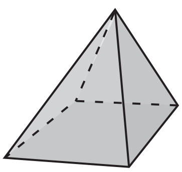
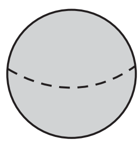
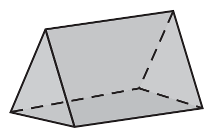


Practice:



1. Triangular Prism   Rectangular Prism   Hexagonal Prism   A rectangular prism has how many faces, edges, and vertices? ______________________________

2. Name each figure.

a.    **b.**    **c.** 

______________   ______________   ______________

d. Which of the above figures is not a polyhedron? ____________________

3. Draw a cone. First draw an ellipse as the base. Then draw a dot for the apex, or peak. Draw segments from the opposite points of the circular base to the apex.

4. Create a one-point perspective drawing for a prism. Begin by drawing a polygon in the foreground. Then pick a location for the vanishing point. Then lightly draw segments from the vertices of the polygon to the vanishing point. Finally, draw corresponding segments parallel to the sides of the polygon in the foreground.

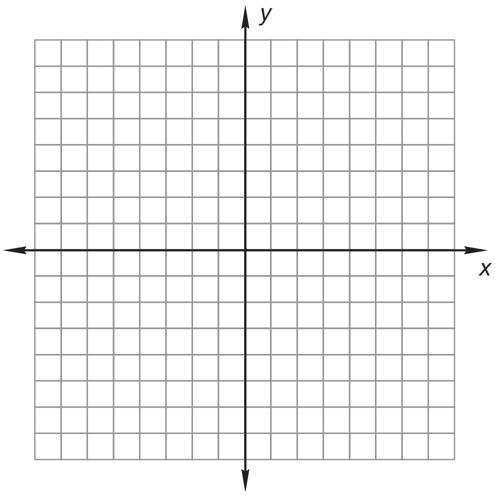
Name ________________________________

• Graphing Transformations

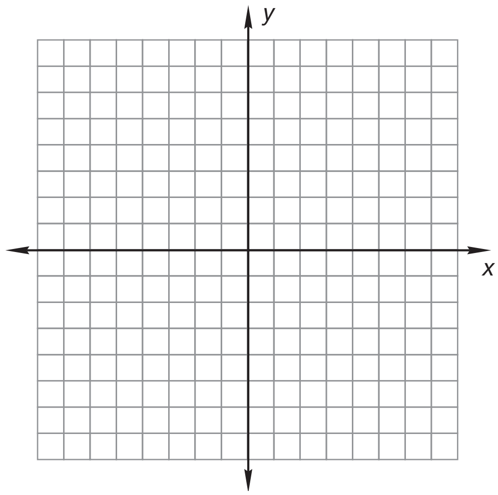
- Refer to Investigation 5 for descriptions and illustartions of transformations.
- Perform the following transformations in the space provided.

Practice:

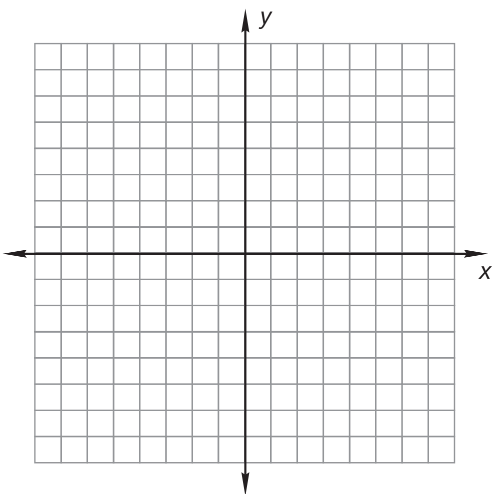
1. Draw △*ABC* with *A* at (4, 5) *B* at (2, 1) and *C* (6, 1). Then draw its reflection across the x-axis. Correctly label the corresponding vertices of △′*A*′*B*′*C*′.



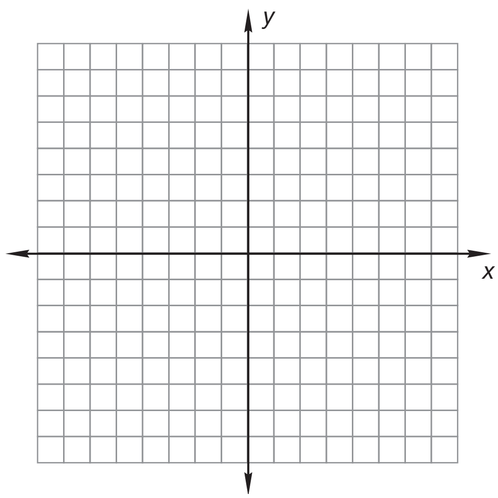
2. Draw △*KLM* with *K* at (–2, 0) *L* at (7, 0) and *M* (–2, 3). Then draw its image △*K*′*L*′*M*′ after a 90° rotation about point *K*.



3. Draw △*PQR* with *P* at (–6, –7) *Q* at (0, –7) and *R* (–3, –1). Then draw its image △*P*′*Q*′*R*′ after a translation of (5, 3).



4. Draw △*XYZ* with *X* at (–2, 4) *Y* at (–2, –6) and *R* (4, –6). Then draw its image △*X*′*Y*′*Z*′ after a contraction of scale factor $\frac{1}{2}$.



Name ______________________

• Collect, Display, and Interpret Data

- Refer to Investigation 6 to review the collection, display, and interpretation of data.

Practice:

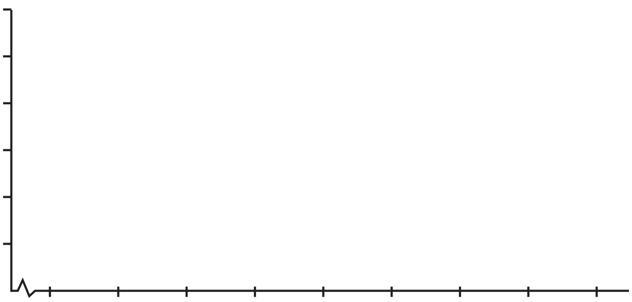
1. A researcher surveyed a sampling of 8th grade students to determine which type of music they preferred. Students were given five choices: rock, blues, country, pop, and rap.

In the space below, display the following data in a bar graph. Rock: 9; Blues: 2; Country: 5; Pop: 5; Rap: 7.

2. The 9th graders all had run a mile at the end of the school year. Below are their times:

Mile Times

5:27	12:10	6:45
7:44	5:59	7:31
5:44	7:35	7:14
9:31	6:55	5:20
6:52	7:51	8:28
8:21	9:52	9:11



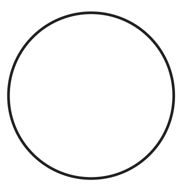
Display this data in a histogram. Sort the data into one minute intervals.

3. In a poll taken to predict the final results for class president, the following results were collected:

Out of 80 students randomly polled:
40 said they would vote for Julia Martinez.
28 said they would vote for Beth Caldwell.
12 were undecided.

In the space below, create a table and circle graph with the data.

Response	Number	%



• Probability Simulation

Suppose a fruit juice company states on its bottles, "One in four get their next bottle of juice free! Just look inside the cap to see if you're a winner." If you decide to purchase a bottle of their juice, what is the probability that you will win a free bottle of juice?

The **theoretical probability** is $\frac{1}{4}$.

This means that, out of all the juice bottles produced for the contest, one-fourth contain a cap with "winner" printed on it. Thus, on average, one in four bottles is a winner. However, this does not mean that if you buy four bottles of juice you are guaranteed a winner. To determine the probability of finding at least one winner in four purchases, conduct this simulation:

Step One: Place three blue marbles and one yellow marble in a bag.
Step Two: For each trial pick and replace four marbles one at a time. Keep track of your results. Make a chart that looks like this:

	Pick 1	**Pick 2**	**Pick 3**	**Pick 4**	**At least 1 winner**
Trial 1					
Trial 2					

Practice:

1. Answer the following questions about the above simulation.

a. What did one pick from the bag represent?

____

b. What did each four-pick trial represent?

____

c. What did the picking of a yellow marble represent?

____

d. What did the picking of a blue marble represent?

____

e. Explain: If one in four bottles contains a winning cap, does buying four bottles guarantee a winning cap? Why or why not?

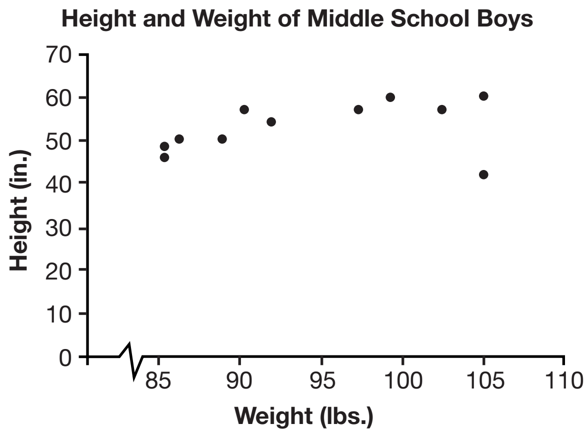
____

Name ________________________________

• Scatterplots

Scatterplots can show if there is a relationship between two sets of data. Below is a chart of the height and weight of a group of middle school boys and a scatterplot of the same data.

Height (in.)	Weight (lbs)
60	98
58	102
63	105
57	90
55	92
50	88
50	86
48	85
57	97
58	102
45	85
45	105



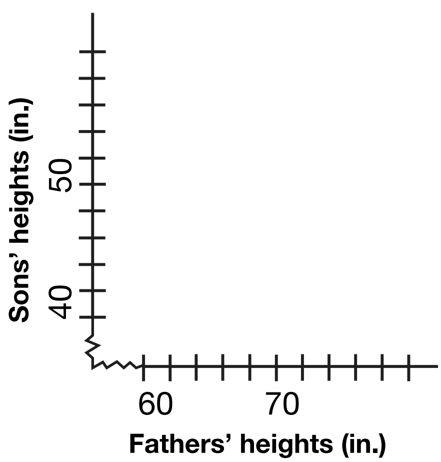
Although the points do not lie on a straight line, we see that the points have a somewhat linear relationship. A **line of best fit** can be drawn on the graph that passes near all of the points. This means that a **correlation** exists between the height and weight of this group of middle school boys. Quantities can be positively correlated, like the above correlation; negatively correlated; or not correlated.

Practice:

1. Carlandra wants to determine if there is a correlation between the heights of fathers and sons at a certain age. Make a scatterplot of the data. Are the quantities positively or negatively correlated? Sketch a line of best fit.

____

Fathers' heights (in.)	Sons' heights (in.)
72	46
72	54
66	44
70	48
68	42
64	40
64	44
66	42
72	50
74	52
74	54



Name ______________________________

• Sampling Methods

When conducting a study, it is necessary to follow certain procedures to make sure the results are as accurate as possible. Here are some tips:

1. When conducting a study that addresses a large number of people, it is often necessary to study a **sample**, or portion, of the population.
2. To reduce sampling error the sample should be **representative**, or very similar to the larger population, large enough, and choosen at **random**.
3. To select your random sample, you can use:
   a. **random sampling**: Assign a number to each person and use a random number generator to select a sample of people.
   b. **stratified sampling**: Identify groups within the population and randomly choose a sample within each group.
4. Be careful to gather information in a way that is not **biased**. For a study involving a **survey**, choose questions carefully, avoiding bias. Your questions should never sway the persons answering them.

Practice:

Identify a problem with each survey below.

1. A survey was conducted at the high school to determine the amount of time people in Cederville spend watching television.

____

2. A survey was conducted to determine the support for a new tax measure. 25 names were selected from a list of registered voters to answer the survey.

____

3. In a district-wide school survey to determine how many hours students study each night, researchers took a random sampling of 7th and 11th graders in all schools.

____

4. To determine the future of an old shopping mall, the city conducted a survey to determine whether citizens of their town would support renovating the mall. On a Saturday, researchers asked every 10^{th} shopper the following question: "Our mall is in such bad shape. We need to renovate the mall so people would shop here more often. Would you support renovating the mall?"

____

• Compound Interest

When you deposit money into a bank, you often earn interest on your money. The **interest** is the amount of money the bank pays. The amount of money you deposit is called the **principal**.

The bank may pay you **simple interest** or **compound interest**.

Simple interest is paid on the principal only. For example, if you deposit $200 into your account that pays 5% simple interest, you would be paid $10 on your account each year. If you take your money out of the bank after three years, you would have a total of $230.

Compound interest is paid on the accumulated interest **as well** as on the principal. If you deposit $200 in an account with 5% annual percentage rate, the amount of interest you would be paid after each year increases. If the earned interest is left in the account, after three years you would have a total of $231.53.

After time, compound interest can amount to quite a lot as this table shows:

Number of years	Simple Interest	Compound Interest
3	$230	$ 231.53
10	$300	$ 325.78
30	$500	$ 864.39
50	$700	$2293.48

We can express periodic compound interest using recursive rules for sequences. For example, the first term in the sequence is the original deposit of $200. Each succeeding term is the product of 1.05 and the previous term.

$$\begin{cases} a_1 = 200 \\ a_n = 1.05a_{n-1} \end{cases}$$

Most calculators can compute compound interest. For example, find the compound interest on your calculator for year number three using these keys. Does it match the amount in the table above?

[1] [0] [0] [0] [×] [(] [1] [+] [.] [1] [)] [y^x] [5] [=]

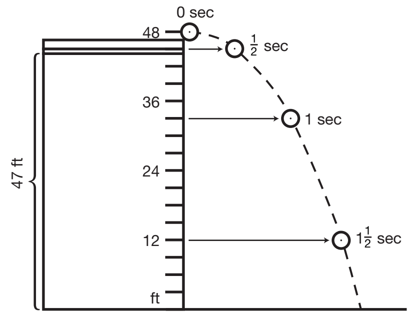
or [1] [.] [1]

Practice:

1. Find the simple interest and the compound interest on a deposit of $500 with a 7% annual interest rate. Put your results in a table that illustrates interest after 3, 5, 10, 20, 40, and 50 years.

Name ______________________________

• Non-Linear Functions

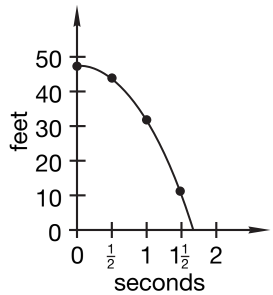


If a ball is thrown from a high ledge, it will not follow a straight-line path to the ground. The path is curved. This kind of curve is called a **parabola.** We can write a function that models the height of the ball at any given time. The graph of the function is a parabola. The following is a **quadratic function**:

$$y = -16x^2 + 48$$

It models the height (y) of the ball at any time (x). This quadratic function comes from the study of physics and the force of gravity. When we only consider the force of gravity and ignore other forces such as wind and friction, the height of any falling object on Earth can be described by a quadratic equation.

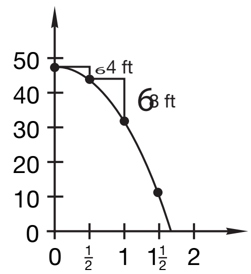
t	*h*
0	48
$\frac{1}{2}$	44
1	32
$1\frac{1}{2}$	12



x	*y*
0	48
$\frac{1}{2}$	44
1	32
$1\frac{1}{2}$	12

We can chart the quadratic function and graph the points. We see that the graph of the height of the falling ball versus the time and the graph of the function are the same:

The graphed functions are **non-linear functions**; their points **do not** lie on a straight line. The height of the ball is not subject to a constant rate of change. The ball moves faster as time goes by.



Practice

1. George held a rubber ball 4 ft above the ground and hit it with a paddle straight up at 40 ft/sec. He has studied physics and predicted that considering only gravity, the height (h) in feet of the ball at (t) seconds could be described by this function: $h = -16t^2 + 40t + 4$

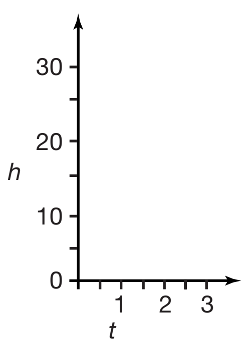
a. Make a table to find the height of the ball at 0, $\frac{1}{2}$, 1, 2, and $2\frac{1}{2}$ seconds.

b. Plot the points (t, h). **c.** What is the maximum height of the ball? **d.** When does it reach this height? **e.** Approximately when does the ball hit the ground?

a.

t	*h*

b.



c. ______________________

d. ______________________

e. ______________________

Name ________________________________

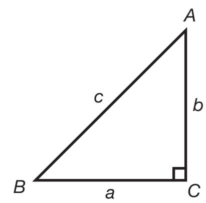
• Proof of the Pythagorean Theorem

When mathematicians want to demonstrate that a particular idea is true, they construct a **proof.** A proof describes how certain given information leads to a certain conclusion. The Pythagorean Theorem states:

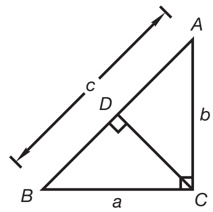
$$a^2 + b^2 = c^2.$$

The following proof for the Pythagorean Theorem is based upon the characteristics of similar triangles.

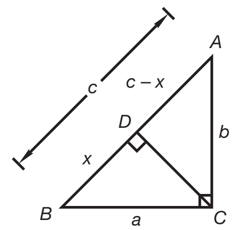
1. Look at the right triangle. $\angle C$ is a right angle.



2. A segment is drawn from vertex *C* across the triangle to $\overline{AB}$ so that the segment is perpendicular to $\overline{AB}$. Point *D* divides $\overline{AB}$ into two short segments.



3. We can label the distance from *B* to *D* as *x*. The distance from *D* to *A* is $c - x$.



4. We now have a right triangle *ABC* which is divided into two smaller right triangles named $\triangle BCD$ and $\triangle ACD$. These three triangles are similar.
5. We can write a proportion that relates the hypotenuse of $\triangle ABC$ and $\triangle BCD$ to corresponding legs of $\triangle ABC$ and $\triangle ACD$:

	hyp	*leg*
$\frac{\triangle ABC}{\triangle BCD}$	$\frac{c}{a}$	$= \frac{a}{x}$

6. We then write a proportion that relates the hypotenuse of $\triangle ABC$ and $\triangle ACD$ to corresponding legs of $\triangle ABC$ and $\triangle ACD$:

	hyp	*leg*
$\frac{\triangle ABC}{\triangle BCD}$	$\frac{c}{a}$	$= b(c - x)$

7. Cross multiply the proportions. The term *cx* appears in both cross products. In the first cross product we see that *cx* equals a^2. This means that we can replace *cx* with a^2 in the second cross product.

$$c^2 - a^2 = b^2 \quad \text{or} \quad b^2 = c^2 + a^2$$

8. Now we transform the equation by solving for c^2.

$$c^2 = a^2 + b^2 \quad \text{or} \quad a^2 + b^2 = c^2$$