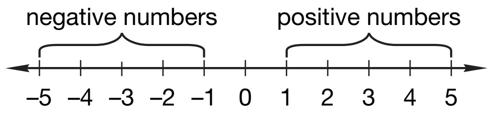
Name ____________________________________

• Number Line: Comparing and Ordering Integers

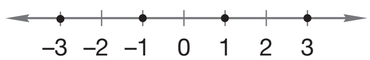


- **Zero** is neither positive nor negative. It is the origin.
- **Integers** include all the counting numbers {1, 2, 3, . . .}, their opposites {. . . –3, –2, –1}, and zero.
- **Whole numbers** are the counting numbers and zero.
- A **set** is a collection of items.
  Example: The set of even numbers {. . . –4, –2, 0, 2, 4, . . .}

- **Absolute value** of a number is the distance between the number and the origin (zero). It is a positive number. Two vertical bars indicate absolute value.
  Example: The absolute value of negative five is five since –5 is 5 units from zero. $|-5| = 5$

- We **compare** numbers using the equal sign (=), the greater than (>) and the less than (<) symbols. The small end of the symbol points to the lesser number. ($2 < 4$)

We graph a number on a number line by drawing a dot at the point that represents the number. Below we graph –3, –1, 1, and 3.



Practice:

1. Arrange these integers in order from least to greater.

   –2, 5, 1, 0, –3

   ____

2. Which number below is an odd number but not a whole number?

   –2   –1   0   3

   ____

3. Compare: –5 ◯ –2

4. Compare: 3 ◯ –4

5. Write two numbers that are six units from zero. ____________________

6. Simplify: $|-1| =$ ____________________

7. Sketch a number line and graph –3, 0, and 3.

• Operations of Arithmetic

Terminology

Addition	addend $+$ addend $=$ sum
Subtraction	minuend $-$ subtrahend $=$ difference
Multiplication	factor $\times$ factor $=$ product
Division	dividend $\div$ divisor $=$ quotient

- Check subtraction by adding.
- Check division by multiplying.

Some Properties of Addition and Multiplication

Name of Property	Representation	Example
Commutative Property of Addition	$a + b = b + a$	$3 + 4 = 4 + 3$
Commutative Property of Multiplication	$a \cdot b = b \cdot a$	$3 \cdot 4 = 4 \cdot 3$
Associative Property of Addition	$(a + b) + c = a + (b + c)$	$(3 + 4) + 5 = 3 + (4 + 5)$
Associative Property of Multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$
Identity Property of Addition	$a + 0 = a$	$3 + 0 = 3$
Identity Property of Multiplication	$a \cdot 1 = a$	$3 \cdot 1 = 3$
Zero Property of Multiplication	$a \cdot 0 = 0$	$3 \cdot 0 = 0$

- Note that the properties do not apply to subtraction and division.

Practice:

Name each property illustrated in Problems 1 and 2.

1. $2 \cdot 5 = 5 \cdot 2$ ______________________________

2. $(3 \cdot 5) \cdot 4 = 3 \cdot (5 \cdot 4)$ ______________________________

3. What is the product when the difference of 8 and 6 is multiplied by the sum of 8 and 6? ____________________

4. Simplify: $42 - 66 =$ ____________________

5. Find the unknown: $12y = 36$. ____________________

Name ______________________________

• Addition and Subtraction Word Problems

To solve word problems:

1. Identify the pattern or plot.
2. Write an equation for the given information.
3. Solve for the unknown number.
4. Check to see if the answer makes sense.

Combining (some, some more) $s + m = t$	some ⟵ + more ⟵ total ⟵	Find by subtracting Find by subtracting Find by adding
Separating (some went away) $s - a = l$	starting amount ⟵ − some went away ⟵ what's left ⟵	Find by adding Find by subtracting Find by subtracting
Comparing (greater/less) $g - l = d$	greater ⟵ − lesser ⟵ difference ⟵	Find by adding Find by subtracting Find by subtracting
Elapsed Time $l - e = d$	later ⟵ − earlier ⟵ difference ⟵	Find by adding Find by subtracting Find by subtracting

Practice:

1. Amy went to the store with $10.00 and came home with $4.65. How much money did Amy spend at the store? ____________________

2. Danny collected 38 cans at the playground. He then collected 27 cans at the ball field. How many cans did Danny collect in all? ____________________

3. From 2000 to 2005, the population of Red Valley School increased from 487 students to 732 students. How many more students were in Red Valley School in 2005 than in 2000? ____________________

4. The soccer team sold 187 flags on Saturday. For the whole weekend, they sold 265 flags. How many flags did the soccer team sell on Sunday? _________

5. Which equation shows how to find what year someone was born when you know that they were 81 in 2005? Circle the correct answer.

a. $81 - b = 2005$ **b.** $2005 + 81 = b$ **c.** $b - 2005 = 81$ **d.** $2005 - b = 81$

Name ______________________________

• Multiplication and Division Word Problems

To solve word problems:

1. Identify the pattern or plot.
2. Write an equation for the given information.
3. Solve for the unknown number.
4. Check to see if the answer makes sense.

Equal Groups	number of groups ⟵	Find by dividing
n* × *g* = *t	× number in group ⟵	Find by dividing
	total ⟵	Find by multiplying

Example: Cory sorted 375 quarters into groups of 40 so that he could put them in rolls. How many rolls can Cory fill with quarters? Explain why your answer is reasonable.

Write the equation: $n \times 40 = 375$
Find the unknown: $375 \div 40 = 9$ R 15
Cory can fill 9 rolls with 15 quarters remaining.
Our answer is reasonable because 9 rolls of 40 quarters would be 360, whereas 10 rolls of 40 quarters would equal 400 quarters. So 375 quarters is not enough to fill 10 rolls but is enough to fill 9 rolls, leaving 15 quarters unrolled.

Practice:

1. Emma arranged the cards in 9 rows with 18 cards in each row. How many cards did Emma arrange? ____________

2. Jordan had 876 pennies. How many rolls of 50 pennies can he fill? Explain why your answer is reasonable. ______________________________

____

3. The muffins were arranged in rows with 12 muffins in each row. If there were 96 muffins, how many rows of muffins were there? ____________

4. Three student tickets to the water park cost $77.25. The cost of each ticket can be found using which of the following equations? Circle your answer.

a. $\$77.25 \times 3 = t$ **b.** $3 \times t = \$77.25$

c. $\$77.25 \times t = 3$ **d.** $\frac{t}{3} = \$77.25$

• Fractional Parts

- Fractions describe part of a group.

numerator	1	number of parts described
denominator	3	number of equal parts

- To find fractional parts of a group, we divide by the denominator and multiply by the numerator.

  Example: There were 30 questions on the test. Two fifths were multiple-choice. How many questions were multiple choice?

  $30 \div 5 = 6$ questions in each fifth

  $6 \times 2 = 12$ questions in two fifths

  So, $\frac{2}{5}$ of 30 is **12 multiple-choice questions.**

- To compare fractions, we can compare the numerator to the denominator to find out if the fraction is greater than or less than $\frac{1}{2}$. For example, $\frac{3}{10}$ is less than $\frac{1}{2}$ because 3 is less than half of 10. However, $\frac{3}{4}$ is greater than $\frac{1}{2}$ because 3 is more than half of 4. Thus, $\frac{3}{4}$ is greater than $\frac{1}{2}$.

  Example: Arrange these fractions from least to greatest: $\frac{3}{6}, \frac{3}{5}, \frac{3}{8}$

  Ask: "Is the fraction less than $\frac{1}{2}$, equal to $\frac{1}{2}$, or greater than $\frac{1}{2}$?"

  $\frac{3}{6}$ is equal to $\frac{1}{2}$, $\frac{3}{5}$ is greater than $\frac{1}{2}$, $\frac{3}{8}$ is less than $\frac{1}{2}$.

  Therefore, from least to greatest, the fractions are $\mathbf{\frac{3}{8}, \frac{3}{6}, \frac{3}{5}}$.

Practice:

1. How many minutes is $\frac{1}{3}$ of an hour? ______________________

2. Mrs. Brown drove 375 miles and used 15 gallons of fuel. Mrs. Brown's car traveled an average of how many miles per gallon of fuel? ____________

3. Arrange these fractions from least to greatest. ______________

$\frac{5}{9}, \frac{2}{7}, \frac{2}{4}$

4. Arrange these fractions from least to greatest. ______________

$\frac{3}{4}, \frac{4}{9}, \frac{6}{12}$

5. Three fourths of the 16 questions on the quiz were multiple-choice. How many multiple-choice questions were there? Explain why your answer is reasonable.

____

• Converting Measures

To convert from one unit of measure to another, multiply if converting to a smaller unit and divide to convert to a larger unit.

Example: Convert 15 ft to yards.
Yards is a larger unit, so
$15 \div 3 = 5$; 15 ft = 5 yds.

Example: Convert 8 gallons to quarts.
Quarts is a smaller unit,
so $8 \times 4 = 32$; 8 gal = 32 qt.

Practice:

1. The blacktop area at school is 75 yards long and 40 yards wide. What is the length and width of the blacktop area in feet? ____________________
2. Darrin is 5 ft 6 in. tall. How many inches tall is Darrin? ____________________
3. Mrs. Chang has a ribbon 5 meters long. How many centimeters long is the ribbon? ____________________
4. Maya bought 12 quarts of water. How many gallons of water did Maya buy?

   ____
5. Ramon was 7 pounds 6 ounces at birth. How many ounces did Ramon weigh?

   ____
6. The race is 5000 meters long. How many kilometers long is the race? __________

Name ____________________________

• Rates and Averages
• Measures of Central Tendency

- A **rate** is a division relationship between two measures.

  Examples: 65 miles per hour (65 mph)
  24 miles per gallon (24 mpg)

  Rate problems involve the two units that form the rate.

  Example: number of **hours** $\times$ 65 **miles per hour** $=$ number of **miles**

- The **average** of a set of numbers is a central number that is found by dividing the sum of the elements of a set by the number of elements. The average is also known as the **mean**.

  To find an average:
  1. Add the elements.
  2. Divide by the number of elements.

  Example: In the four art classes there are 32, 28, 29, and 31 students. What is the average number of students in an art class?

  $$\text{Average} = 32 + 28 + 29 + 31 = \frac{120}{4} = 30$$

  The average number of students in an art class is 30 students.

- The **median** is the central number in a set of data.
  To find the median, arrange the numbers in order of size and select the middle number, or the average of the two middle numbers.
- The **mode** is the most frequently occurring number.
  To find the mode, select the number that appears most often.
- The **range** of a set of data is the difference between the greatest and the least numbers in the set.

Practice:

Molly's math test scores for this marking period are 96, 95, 91, 95, and 88. Use this data to answer Problems 1–4.

1. What is the average of Molly's math test scores? ________

2. What is the median of the scores? ________

3. What is the mode of the test scores? ________

4. What is the range of scores? ________

5. It took Peter 2 hours to get to the city. He drove an average of 60 miles per hour. About how many miles did Peter drive? ________________

Name ______________________________

• Perimeter and Area

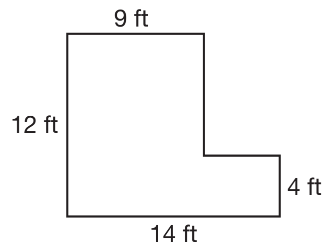
- **Perimeter** is the distance around a shape.
- **Area** is the measure of a surface.

Formulas for the Perimeter and Area of a Rectangle

$P = 2l + 2w$

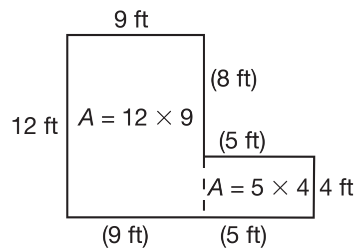
$A = lw$

- To find the **perimeter of a complex shape**:
  - Add the lengths of all the sides



- To find the **area of a complex shape**:
  1. Divide the shape into rectangular parts.
  2. Find the area of each part.
  3. Add the areas to find the total area.

- Use the measurements you know to find the measurements you don't know.



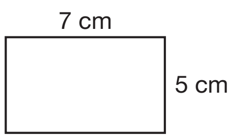
$P = 12 + 9 + 8 + 5 + 4 + 5 + 9 = 52$ ft

$A = (12 \times 9) + (5 \times 4)$
$108 + 20 = 128$ sq. ft

Practice:

Find the perimeter and area of each rectangle.

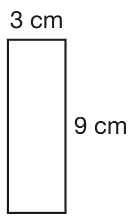
1.



$P =$ ____________________

$A =$ ____________________

2.



$P =$ ____________________

$A =$ ____________________

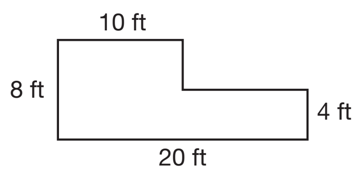
3. Amanda is placing one-foot square floor tiles and baseboard in a 10 ft-by-8 ft room. Ignoring doorways, how many feet of baseboard does she need?

____

How many tiles does she need?

____

4. Find the perimeter and area of a room with these dimensions.



$P =$ ____________________

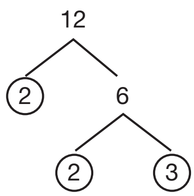
$A =$ ____________________

Name ______________________________

• Prime Numbers

- **Prime numbers** are counting numbers greater than 1 that have exactly two different counting number factors: the number itself and 1.
  The numbers 2, 3, 5, 7, 11, and 13 are prime.
- **Composite numbers** have more than two factors.
  The numbers 4, 6, 8, 9, 10, and 12 are composite numbers.
- **Prime factorization** is writing a composite number as a product of prime numbers.

Use a factor tree.



$12 = 2 \cdot 2 \cdot 3$

Use division by primes.

$$\begin{array}{r} 3 \\ 3\overline{)9} \\ 5\overline{)45} \end{array}$$

$45 = 3 \cdot 3 \cdot 5$

Divisibility Tests

Condition	Number is Divisible by	Example Using 3420
the number is even (ends with 0, 2, 4, 6, or 8)	2	342<u>0</u>
the sum of the digits is divisible by 3	3	$3 + 4 + 2 + 0 = \underline{9}$ and 9 is divisible by 3
the number ends in 0 or 5	5	342<u>0</u>

Practice:

1. Draw a factor tree for 54. Make the first two branches 6 and 9.

____

2. Find the prime factors of 48 by dividing by prime numbers.

____

Write the prime factorization for each number in Problems 3 and 4.

3. 18

____

4. 27

____

5. Is 52,611 a prime or composite number? Tell how you know.

____

Name ________________________________

• Rational Numbers
• Equivalent Fractions

- **Whole numbers** are the counting numbers and zero.
- **Integers** are all the counting numbers, their opposites, and zero.
- **Rational numbers** are numbers that can be expressed as a ratio of two integers.

  Examples: $-2, -\frac{1}{2}, \frac{3}{2}, 3$

  The number 3 is a whole number, an integer, and a rational number.

- **Equivalent fractions** have the same value.

  $\frac{4}{8}, \frac{3}{6}, \frac{2}{4}$, and $\frac{1}{2}$ are equivalent fractions.
- To reduce fractions with large terms:
  1. Write the prime factorization of the terms of the fraction.
  2. Remove pairs of like terms from the numerator and denominator.
  3. Simplify.

  Example: $\frac{144}{180} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}} \cdot 5} = \frac{4}{5}$
- To form an equivalent fraction, multiply the fraction by a fraction equal to 1.

  Example: $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$
- To compare fractions, use equivalent fractions with a common denominator.

Practice:

Describe each number in Problems 1 and 2 as a whole number, an integer, or a rational number. Use every term that applies.

1. 4 ____________________ **2.** $-\frac{2}{3}$ ____________________

Complete each equivalent fraction in Problems 3 and 4. Show the multiplication.

3. $\frac{1}{6} = \frac{\quad}{18}$ ____________________

4. $\frac{2}{3} = \frac{\quad}{15}$ ____________________

5. Using prime factorization, reduce $\frac{24}{40}$. ______________________________

6. Compare: $\frac{2}{3} \bigcirc \frac{3}{4}$ **7.** Compare: $\frac{2}{3} \bigcirc \frac{3}{5}$