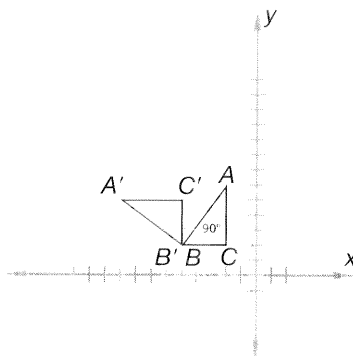


Math Language

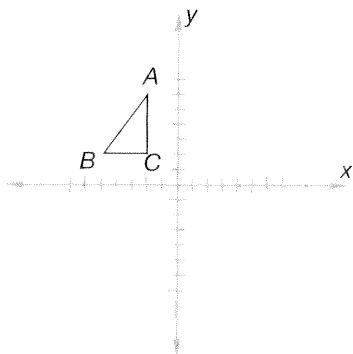
Recall that **counter-clockwise** means “in the opposite direction as the movement of the hands of a clock,” or “to the left.”

A positive **rotation** turns a figure counterclockwise about (around) a point. Here we show a 90° rotation about point B . Point B is the point of rotation; its location is fixed, and the figure spins around it. If we trace the path of any other point during this rotation, we find that it sweeps out an arc of 90° .

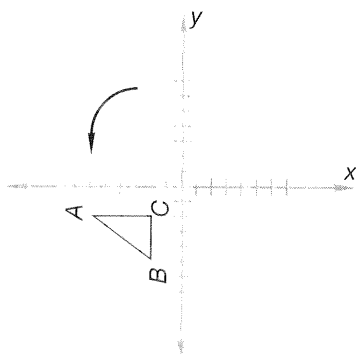


To graph a rotation it is helpful to have a piece of transparency film and a transparency marker. Otherwise use a piece of paper thin enough to use for tracing.

4. On a coordinate plane graph $\triangle ABC$ as illustrated above. We will rotate $\triangle ABC$ 90° about the origin. Place a piece of transparency film or tracing paper on the coordinate plane so that it covers $\triangle ABC$ and the origin. Trace $\triangle ABC$ and mark an inch or so of the x and y -axes through the origin as illustrated below.



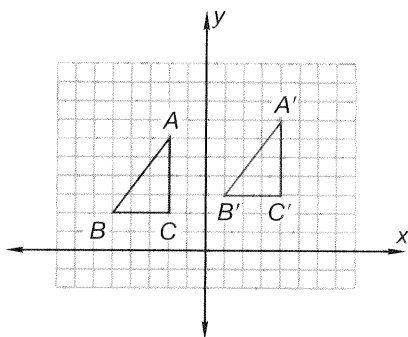
Press a pencil point on the tracing sheet at the origin and rotate the tracing sheet counterclockwise 90° so that the x -axis on the tracing paper aligns with the y -axis on the original coordinate plane below, and the y -axis on the tracing paper aligns with the x -axis below.



Use the position of the traced image to help you draw the rotated image on the coordinate plane. Name the rotated image $\triangle A'B'C'$ and label the coordinates of the vertices.

5. On the same coordinate plane rotate $\triangle ABC$ 180° . Name the image $\triangle A''B''C''$ and label the coordinates of the vertices.

A **translation** slides a figure a distance and direction without flipping or turning. Here we show a translation of $(6, 1)$. For any translation (a, b) , a describes the horizontal shift and b describes the vertical shift for each point of the figure.



6. Draw triangle ABC as illustrated on a coordinate plane. Then draw its image $\triangle A'B'C'$ translated 7 units to the right and 8 units down, or $(7, -8)$. Label the coordinates of $\triangle A'B'C'$.
7. If $\triangle ABC$ is translated so that the image of point A at A'' is located at the origin, then what would be the coordinates of points B'' and C'' ? Describe the translation.

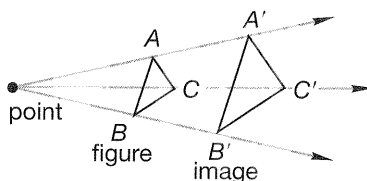


Visit www.SaxonPublishers.com/ActivitiesC3 for a graphing calculator activity.

Recall that reflections, rotations, and translations are called **isometries** (meaning “same measures”), or **congruence transformations**, because the original figure and its image are congruent. A **dilation** is a transformation in which the figure grows larger, while a **contraction** is a transformation in which the figure grows smaller. Dilations and contractions are **similarity transformations**, because the original figure and its image are similar and corresponding lengths are proportional.

Dilations of geometric figures occur away from a fixed point called the center of the dilation. On a coordinate plane the center of a dilation may be any point. In this book the center of dilation will be the origin.

Dilation of Triangle ABC



Note that corresponding vertices are on the same rays from the center of dilation and that the corresponding segments of $\triangle ABC$ and its image $\triangle A'B'C'$ are parallel.

Recall that to graph a dilation on a coordinate plane we prescribe a scale factor, such as 2. If the center of dilation is the origin, we may multiply the coordinates of the vertices of the figure by the scale factor. For example, using a scale factor of 3, a vertex at (1, 3) in the original figure would have a corresponding vertex at (3, 9) in its image after dilation.

8. On a coordinate plane draw rectangle $ABCD$ with A at (6, 4), B at (6, -2), C at (-2, -2), and D at (-2, 4). Then draw its image $\square A'B'C'D'$ using scale factor $\frac{1}{2}$, with the center of the contraction at the origin. What are the coordinates of the vertices of $\square A'B'C'D'$?
9. What fraction of the dimensions of $\square ABCD$ are the dimensions of $\square A'B'C'D'$? What fraction of the perimeter of $\square ABCD$ is the perimeter of $\square A'B'C'D'$?
10. What fraction of the area of $\square ABCD$ is the area of $\square A'B'C'D'$?

In the following table we summarize the transformations graphed in this investigation.

Transformations

Preserving Congruence	Preserving Similarity
Translation (slide)	Dilation (scale increased)
Rotation (turn)	Contraction (scale reduced)
Reflection (flip)	

For review, graph the following transformations on a coordinate plane.

11. Draw $\triangle XYZ$ with X at (-2, 5), Y at (-5, 2), and Z at (-2, 2). Then draw its reflection across the x -axis. Correctly label the corresponding vertices of $\triangle X'Y'Z'$.
12. Draw $\triangle PQR$ with P at (3, 0), Q at (3, -5), and R at (6, 0). Then draw its image $\triangle P'Q'R'$ after a 90° counterclockwise rotation about point P .
13. Draw $\triangle DEF$ with D at (-2, -2), E at (2, -2), and F at (0, 2). Then draw its image $\triangle D'E'F'$ after a translation of (5, 3).
14. Draw $\triangle JKL$ with J at (1, 2), K at (-1, -2), and L at (1, -2). Then draw its image $\triangle J'K'L'$ after a dilation of scale factor 2.

• **Negative Exponents**
• **Scientific Notation for Small Numbers**

Power Up

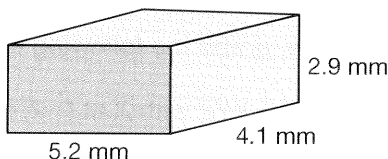
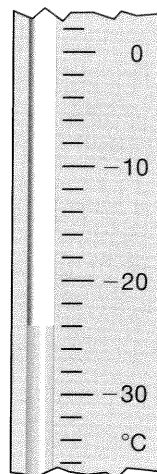
Building Power

facts

mental math

Power Up K

- a. **Number Sense:** 3.3×4
- b. **Measurement:** What temperature is indicated on this thermometer?
- c. **Algebra:** $25x = 250$
- d. **Proportions:** There are 2 oranges to every 3 apples in a fruit basket. If there are 8 oranges, how many apples are in the basket?
- e. **Percent:** How much is a 10% down payment on a \$20,000 car?
- f. **Scientific Notation:** Write 250 in scientific notation.
- g. **Geometry:** Approximate the volume of this object.



h. **Calculation:** $\sqrt{9} + \sqrt{16} - \sqrt{25}$

problem solving

The first four numbers Hexa said were 16, 32, 48, and 64. If she keeps counting this way, what is the 99th number Hexa will say?

New Concepts

Increasing Knowledge

negative exponents

Recall from Lesson 27 that we subtract the exponents when dividing exponential expressions that have the same base.

$$x^5 \div x^3 = x^2$$

We can understand this law by applying what we know about exponents and division.

$$\frac{x^5}{x^3} = \frac{\overset{1}{x} \cdot \overset{1}{x} \cdot \overset{1}{x} \cdot x \cdot x}{\underset{1}{x} \cdot \underset{1}{x} \cdot \underset{1}{x}} = x^2$$

Now consider the result when we reverse the numbers in the division. Following the Laws of Exponents, the exponent of the quotient is negative.

$$x^3 \div x^5 = x^{-2} \text{ (because } 3 - 5 = -2\text{)}$$

We will apply what we know about exponents and division to understand the meaning of x^{-2} .

$$\frac{x^3}{x^5} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot x \cdot x} = \frac{1}{x^2}$$

By performing the division we find that x^{-2} means $\frac{1}{x^2}$. We see that x^{-2} is the reciprocal of x^2 . This fact is another law of exponents.

Law of Exponents for Negative Exponents

$$x^{-n} = \frac{1}{x^n}$$

Applying this law to powers of 10, we see the following pattern. Note that $10^0 = 1$.

$$10^2 = 100$$

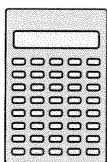
$$10^1 = 10$$

$$10^0 = 1$$

$$10^{-1} = \frac{1}{10} \text{ or } 0.1$$

$$10^{-2} = \frac{1}{100} \text{ or } 0.01$$

Evaluate Use the Laws of Exponents to find the product of 10^2 and 10^{-2} .



Very small numbers may exceed the display capabilities of a calculator. One millionth of one millionth is more than zero, but it is a very small number. On a calculator we enter



The product, one trillionth, contains more digits than can be displayed by many calculators. Instead of displaying one trillionth in standard form, the calculator displays the number in a modified form of scientific notation:

$$1^{-12} \text{ or perhaps } 1 \times 10^{-12}$$

Example 1

Which of the following does not equal 10^{-3} ?

A $\frac{1}{10^3}$

B $\frac{1}{1000}$

C 0.001

D -1000

Solution

An exponential expression with a negative exponent is the reciprocal of the expression with the opposite exponent, as shown in A. Since 10^3 equals 1000, B is just an alternate form of A. Choice C is the decimal equivalent of B. The only number that does not equal 10^{-3} is choice **D**, -1000.

A negative exponent does not imply a negative number. Actually, 10^{-3} is a positive number.

Example 2

Find each missing exponent.

a. $10^{-2} \cdot 10^{-4} = 10^{\square}$

b. $\frac{10^{-2}}{10^{-4}} = 10^{\square}$

Solution

- a. To multiply exponential expressions with the same base we can add the exponents. We add -2 and -4 .

$$(-2) + (-4) = -6$$

The missing exponent is -6 .

- b. To divide exponential expressions with the same base we can subtract the exponents. In this case we subtract -4 from -2 . Instead of subtracting a negative, we add its opposite.

$$(-2) - (-4)$$

$$(-2) + (+4) = 2$$

The missing exponent is 2 .

Verify Find the answers to **a** and **b** by simplifying each exponential expression and performing the multiplication and division.

Example 3

Simplify:

a. 2^{-3}

b. $(-2)^3$

Solution

We distinguish between negative powers and powers of negative numbers.

- a. To simplify 2^{-3} we rewrite the expression with a positive exponent.

$$2^{-3} = \frac{1}{2^3}$$

Then we apply the positive exponent to the base.

$$\frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$$

- b. The expression $(-2)^3$ is written with a positive exponent. Three negative factors produce a negative product.

$$(-2)^3 = (-2)(-2)(-2) = -8$$

Example 4Simplify: $3^2 \cdot 3^{-2}$ **Solution**

To simplify $3^2 \cdot 3^{-2}$ we add the exponents. Adding the exponents 2 and -2 results in the exponent zero.

$$3^2 \cdot 3^{-2} = 3^0$$

Recall that $x^0 = 1$ if x is not zero. Therefore $3^0 = 1$. We can confirm this by applying what we have learned about negative exponents. We will rewrite the multiplication by expressing 3^{-2} with a positive exponent.

Step:	Justification:
$3^2 \cdot 3^{-2}$	Given expression
$3^2 \cdot \frac{1}{3^2}$	Wrote 3^{-2} with positive exponent
$9 \cdot \frac{1}{9}$	Applied exponents
1	Simplified

We confirm that the expression 3^0 equals **1**.

Example 5Express with positive exponents and simplify: $2x^{-1}yx^2y^{-2}z$ **Solution**

A negative exponent indicates a reciprocal, so x^{-1} is $\frac{1}{x}$ and y^{-2} is $\frac{1}{y^2}$. A quick way to change the sign of the exponent is to draw a division bar and shift the exponential expression to the opposite side of the division bar.

$$\frac{2x^{-1}yx^2y^{-2}z}{1} = \frac{2yx^2z}{xy^2}$$

All exponents are positive. Now we reduce.

$$\frac{2yx^2z}{xy^2} = \frac{2\cancel{y}xxz}{x\cancel{y}y} = \frac{2xz}{y}$$

Connect How can we apply the Multiplication Law of Exponents to simplify the original expression?

We use negative powers of 10 to write small numbers in scientific notation. By small numbers we mean numbers between 0 and 1.



Positive numbers named with negative powers of 10.

scientific notation for small numbers

**Example 6**

Write this number in standard form.

$$1.5 \times 10^{-3}$$

Solution

The power 10^{-3} equals 0.001. If we perform the multiplication we find the product is 0.0015.

$$\begin{aligned} &1.5 \times 10^{-3} \\ &1.5 \times 0.001 = \mathbf{0.0015} \end{aligned}$$

Notice that we can find the product simply by shifting the decimal point in the coefficient three places to the left. Thus the power of ten indicates both the direction and the number of places the decimal shifts.

Visit www.SaxonPublishers.com/ActivitiesC3 for a graphing calculator activity.

**Example 7**

The diameter of a red blood cell is about 0.000007 meters. Write that number in scientific notation.

Solution

The coefficient is 7. In standard form the decimal point is six places to the left, so the power of 10 is -6 .

$$0.000007 = 7 \times 10^{-6}$$

Visit www.SaxonPublishers.com/ActivitiesC3 for a graphing calculator activity.

Example 8

Compare: 1×10^{-6} \bigcirc -10

Solution

The expression 1×10^{-6} is a positive number, though a small one (0.000001). Any positive number is greater than a negative number.

$$1 \times 10^{-6} > -10$$

Practice Set

Simplify:

a. 3^{-2}

b. 2^{-3}

c. 5^0

d. $2^{-3} \cdot 2^3$

e. **Analyze** Arrange in order from least to greatest:

$$\frac{1}{2}, 0, 1, 2^{-2}, -1, 0.1$$

Evaluate Find the missing exponent in problems **f–g**. Check your answers by substituting the exponent answer into each equation and solving.

f. $10^{-3} \cdot 10^{-4} = 10^{\square}$

g. $\frac{1}{1} = 10^{\square}$

Simplify:

h. $x^{-3}y^2xy^{-1}$

i. $\frac{6x^{-2}y^3z^{-1}}{2xy}$

j. Write 10^{-4} as a decimal number.

k. Write 5×10^{-5} in standard form.

l. Write 2.5×10^{-2} in standard form.

m. Write 0.008 in scientific notation.

n. Write 0.000125 in scientific notation.

o. If lightning strikes a mile away the sound reaches us in about 5 seconds, but its light reaches us in about 5 millionths of a second. Write 5 millionths in scientific notation.

p. A nanometer is 10^{-9} meters. Write that number in standard form.

Written Practice

Strengthening Concepts

Evaluate For problems 1–2, record the information in a ratio table. Estimate an answer and then solve by writing and solving a proportion.

* 1. Ruben wrote 2 pages in 3 hours. At that rate, how long will it take him to write 9 pages?
(49)

* 2. If 9 gallons of gas cost \$20.25, how much would it cost to fill a 25 gallon tank?
(49)

* 3. On a piano, the ratio of black keys to white keys is 9 to 13. If there are 88 keys on a piano, how many are black?
(45)

* 4. Kerry mixes apple slices and raisins at a 3 to 1 ratio to make her fruit salad. If she wants 16 cups of fruit salad, how many cups of apple slices will she need?
(45)

5. The shipping charges totaled \$9.26. Included in the charge was a \$3.50 flat fee. The rest of the cost was for weight, at \$0.64 per pound. How much did the package weigh?
(3, 4)

Solve:

* 6. $12x - 3 = 69$
(50)

* 7. $\frac{x}{4} + 1 = 12$
(50)

* 8. $\frac{x}{3} - 4 = 5$
(50)

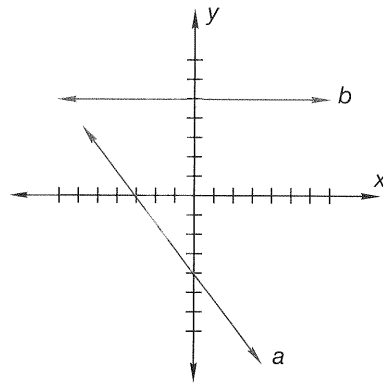
* 9. $-x + 1 = 6$
(50)

* 10. $1 - m = -1$
(50)

* 11. $3x + 2x - 1 = 99$
(50)

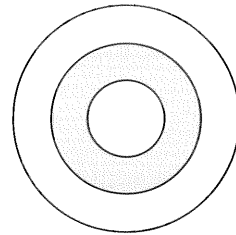
- * 12. Refer to the graph of lines a and b to answer the following questions.
(Inv. 1)

- Which line intersects the y -axis at -4 ?
- Which line is perpendicular to the y -axis?
- What is the slope of line a ?
- In which quadrant do lines a and b intersect?



- * 13. Graph $y = 4x + 1$. Is $(3, 13)$ on the line?
(47)

- * 14. The figure shows three concentric (meaning "same center") circles. The diameters of the three circles are 4 cm, 8 cm, and 12 cm. Find the area of the shaded region. Use 3.14 for π and round the answer to the nearest square centimeter.



- * 15. Sketch a model of a book that measures 2 in. by 10 in. by 8 in. What is the volume of the book?
(Inv. 4, 42)

- * 16. Write 3.4×10^{-5} in decimal notation.
(51)

Combine like terms to simplify:

17. $9(x - 3) + 5(x + 5)$
(31, 36)

18. $3x^2 - 3x + x - 1$
(31)

Simplify:

* 19. $\frac{(2xy)(3x^2y)}{6xy^2}$
(36)

* 20. $-6 - (-5)$
(33)

21. $2\frac{1}{2} \cdot 1\frac{3}{5} - 2\frac{3}{8}$
(13, 23)

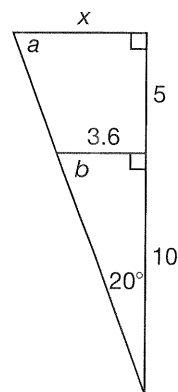
* 22. $(2.3 \times 10^4)(1.5 \times 10^3)$
(46)

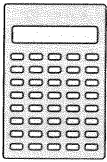
23. Compare 0.62 \bigcirc $\frac{5}{8}$
(12)

24. Write $\frac{7}{10}$ **a** as a decimal and **b** as a percent.
(12)

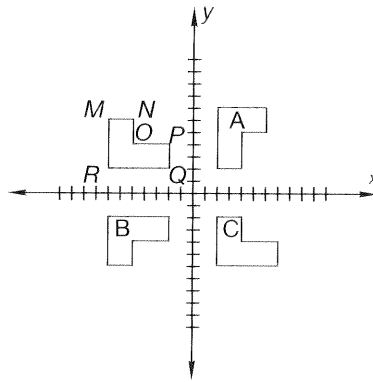
25. a. Redraw the triangles separately and find a and b .
(35)

- Explain how you know that the two triangles are similar.
- Find the scale factor from the smaller to the larger triangle, then find x .





26. Find $\sqrt{b^2 - 4ac}$ when $a = 6$, $b = 5$, and $c = -1$.
(14, 36)
- * 27. James is building a 10-inch by 12-inch rectangular picture frame. To assure the frame has right angles, James measures the two diagonals to see if they are equal. When the diagonals are equal, how long is each diagonal? (Express your answer to the nearest tenth of an inch.)
(Inv. 2)
28. Arnold flipped a coin twice and it landed heads up both times. If he flips the coin again, what is the probability the coin will land heads up?
(32)
29. Find all possible values of x for the equation $x^2 - 16 = 0$.
(14, 36)
- * 30. a. Which figure—A, B, or C—is a translation of figure $MNOPQR$?
(Inv. 5)
- b. For the figure you chose, give the coordinates that locate M' , N' , O' , P' , Q' , and R' .



Early Finishers
Real-World
Application

In a vacuum, the speed of light is constant at approximately 300,000,000 meters/second. If the distance from the Earth to the Sun is about 149,000,000,000 meters, about how many seconds does it take light to reach the Earth? Express your answer in scientific notation.

- Using Unit Multipliers to Convert Measures
- Converting Mixed-Unit to Single-Unit Measures

Power Up*Building Power***facts**

Power Up K

mental math

- Number Sense:** 7.5×20
- Statistics:** The mean of two numbers is the number halfway between. What is the mean of 10 and 20?
- Fractional Parts:** How much do you save if a \$90 item is 10% off?
- Scientific Notation:** Write 0.0051 in scientific notation.
- Rate:** Jackie finishes 50 problems per hour. Ray finishes 40 problems per hour. How many more problems will Jackie finish than Ray if they work for 30 minutes?
- Geometry:** Two angles of a triangle are 60° and 60° . Find the measure of the third angle. What type of triangle is the triangle?
- Powers/Roots:** List the perfect squares from 1 to 169.
- Calculation:** $22, + 2, \times 2, + 2, \times 2, \sqrt{\quad}$

problem solving

A number is divided by 2 and then added to 11. The result is 19. What is the number? Explain your thinking.

New Concepts*Increasing Knowledge***using unit multipliers to convert measures**

A **unit multiplier** is a ratio in which the numerator and denominator are equivalent measures but different units. Since 12 inches equal one foot, we can write these two unit multipliers for that relationship.

$$\frac{12 \text{ in.}}{1 \text{ ft}} \qquad \frac{1 \text{ ft}}{12 \text{ in.}}$$

Each of these ratios equals 1 because the measures above and below the division bar are equivalent. Since multiplying by 1 (or a form of 1) does not change a quantity, we can multiply by a unit multiplier to convert a measure from one unit to another unit.

Example 1

The sapling apple tree is 64 inches tall. Convert 64 inches to feet by multiplying 64 inches by $\frac{1 \text{ ft}}{12 \text{ in.}}$.

Solution

The unit multiplier cancels inches and leaves feet as the unit.

$$64 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{64}{12} \text{ ft} = 5\frac{1}{3} \text{ ft}$$

Example 2

A certain double feature at a theater is 270 minutes long. Use a unit multiplier to convert 270 minutes to hours.

Solution

Since 60 minutes equal one hour, we have a choice of two unit multipliers.

$$\frac{60 \text{ min}}{1 \text{ hr}} \qquad \frac{1 \text{ hr}}{60 \text{ min}}$$

To cancel minutes we choose the unit multiplier with minutes below the division bar.

$$270 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} = \frac{270}{60} \text{ hr} = 4\frac{1}{2} \text{ hr}$$

Connect In addition to canceling units in examples 1 and 2, could we also cancel numbers? Explain.

We can use unit multipliers to convert measures to desired or appropriate units. Appropriate units are correct and meaningful to the intended reader.

Example 3

Daniel is buying refreshments for a meeting. He wonders if four quarts of juice is enough for 30 students. Is four quarts a reasonable amount of juice for the meeting? If not, suggest a more reasonable quantity.

Solution

If each student has the same amount of juice, then one serving is 4 quarts divided by 30.

$$\frac{4 \text{ quarts}}{30 \text{ students}} = \frac{4}{30} \text{ quarts per student}$$

The answer is not very meaningful. A more appropriate unit is ounces. We convert 4 quarts to ounces and then we divide by 30.

$$4 \text{ qt} \cdot \frac{32 \text{ oz}}{1 \text{ qt}} = 128 \text{ oz}$$

$$\frac{128 \text{ oz}}{30 \text{ students}} = 4 \text{ ounces per student}$$

Since four ounces is half a cup, **four quarts is probably not enough juice.** If Daniel buys **eight quarts**, then each student could have a full cup (8 ounces) of juice.

Thinking Skill**Explain**

Is canceling units of measure like canceling numbers? Explain.

**converting
mixed-unit
to single-unit
measures**

Some units like feet and inches are used together to express a measure.

Mr. Seymour is 6 ft 3 in. tall.

To express Mr. Seymour's height with a single unit we could give his height in inches (75 in.) or in feet. Below we show his height in feet. Three inches is $\frac{3}{12}$ of a foot, which equals $\frac{1}{4}$ ft or 0.25 ft. We can combine 6 ft with $\frac{1}{4}$ ft or 0.25 ft.

$$6 \text{ ft } 3 \text{ in.} = 6\frac{1}{4} \text{ ft} = 6.25 \text{ ft}$$

Example 4

Samantha drove 100 miles in an hour and 45 minutes. How many hours did it take for Samantha to drive 100 miles?

Solution

We express 45 minutes as $\frac{45}{60}$ of an hour.

$$45 \text{ min} = \frac{45}{60} \text{ hr} = \frac{3}{4} \text{ hr} = 0.75 \text{ hr}$$

Samantha drove for 1 hr 45 min, which is $1\frac{3}{4}$ hr or 1.75 hr.

Example 5

Noel ran one mile in 7 minutes and 30 seconds. Find Noel's time in minutes. Express the result as a mixed number and as a decimal number.

Solution

We convert 30 sec to $\frac{30}{60}$ minutes which is $\frac{1}{2}$ or 0.5 min.

$$7 \text{ min } 30 \text{ sec} = 7\frac{1}{2} \text{ min} = 7.5 \text{ min}$$

Example 6

To measure a room for carpeting, Juan converts the length and width of the room from feet and inches to yards. A room that is 13 ft 6 in. long is how many yards long?

Solution

First we convert 13 ft 6 in. to feet.

$$13 \text{ ft } 6 \text{ in.} = 13\frac{6}{12} \text{ ft} = 13.5 \text{ ft}$$

Then we convert feet to yards.

$$13.5 \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{13.5 \text{ yd}}{3} = 4.5 \text{ yd}$$

The room is **4.5 yd** long.

Practice Set

- a. **Justify** One day is 24 hours. Write two unit multipliers that have days and hours as the units. Identify the unit multiplier you would use to convert days to hours. Explain your choice.
- b. **Represent** Write two unit multipliers for this equivalence:
$$16 \text{ oz} = 1 \text{ pt}$$
- c. A gallon is 128 oz. Convert 128 oz. to pints using a unit multiplier.
- d. An inch is 2.54 cm. A bookcase that is 50 inches high is how many centimeters high? Use a unit multiplier to perform the conversion.
- e. Convert 24 quarts to gallons using a unit multiplier. (1 gal = 4 qt)
- f. Carter claims he can run 10,000 centimeters in 14 seconds. Express his claim in more appropriate terms.
- g. The newborn child was 21 inches long and weighed 8 pounds 4 ounces. Convert these measures to feet and to pounds respectively, expressed as mixed numbers and as decimal numbers.
- h. Marsha swam 400 meters in 6 minutes and 12 seconds. Convert that time to minutes.
- i. A room is 11 ft 6 in. long and 11 ft 3 in. wide. Find the perimeter of the room in feet.
- j. Convert 11 ft 3 in. to feet. Then use a unit multiplier to convert that measure to yards.

Written Practice

Strengthening Concepts

1. ⁽⁴⁵⁾ The hen to rooster ratio in the barnyard is 11 to 2. If there are 26 hens and roosters in all, how many are roosters?
2. ^(3, 4) Sally sells popcorn for \$1 and beverages for \$2. What will be her total sales if 19 people each buy one bag of popcorn and one beverage?
3. ^(3, 4) For a collect call, a phone company charges a \$1 connection fee, then 4 cents per minute for the duration of a call. How much will it cost for an 11 minute call?

For problems 4–6, record the information in a ratio table. Estimate an answer and then solve by writing and solving a proportion.

- * 4. ⁽⁴⁹⁾ Cynthia sews four dresses in 3 hours. How many dresses could she sew in 9 hours?
- * 5. ⁽⁵²⁾ Sergio estimates that his trip will take 3 hours and 30 minutes. Express Sergio's estimate as a mixed number of hours and as a decimal number of hours.

- * 6. ⁽⁵²⁾ A light is left on for 40 days. Convert 40 days to hours using a unit multiplier. If the light bulb has an estimated life of 1400 hours, is the light likely to still be on after 40 days?

Analyze Solve.

* 7. ⁽⁵⁰⁾ $20x + 50 = 250$

* 8. ⁽⁵⁰⁾ $\frac{x}{3} + 5 = 7$

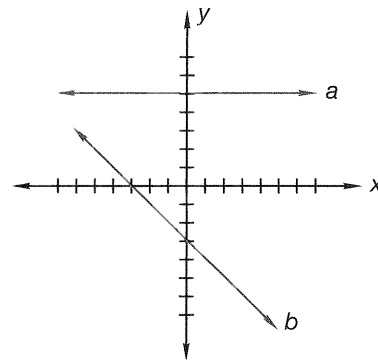
* 9. ⁽⁵⁰⁾ $x + 5 + x = 25$

* 10. ⁽⁵⁰⁾ $14 - m = 24$

11. ⁽³²⁾ Victoria is playing a board game with two number cubes. She rolls both cubes once.

- List the possible totals (sample space).
- Predict: Which totals do you predict are least likely and which totals are most likely. Justify your prediction.

- * 12. ^(Inv. 1, 44) **Analyze** Refer to the graph of lines a and b to answer the following questions.



- Which line intersects the y -axis at 5?
- What is the slope of line b ?
- In which quadrant do lines a and b intersect?

13. ^(Inv. 5) Triangle ABC with vertices at $(2, 1)$, $(1, 4)$, and $(2, 2)$ is reflected in the x -axis. What are the coordinates of the vertices of $\triangle A'B'C'$?

14. ^(39, 40) A 14-inch diameter circular porthole (window) is framed in a brass ring and mounted in the side of a ship.

- Find the area of the glass. Round your answer to the nearest square inch.
- What is the inner circumference of the brass mounting ring? (Use $\frac{22}{7}$ for π .)

15. ⁽⁴²⁾ Find the volume of a box with dimensions 5 in. by 5 in. by 8 in. Sketch the box resting on a square face and indicate its dimensions.

16. ⁽⁴³⁾ Find the surface area of the box in exercise 15.

Generalize Simplify:

17. ^(31, 36) $4(x - 4) - 2(x - 6)$

18. ⁽³¹⁾ $b + h + b + h$

* 19. ⁽⁵¹⁾ 5^{-2}
 (4.2×10^7)

* 20. ⁽³³⁾ $(-3) - (-7)$

21. ⁽⁴⁶⁾ $\frac{(4.2 \times 10^7)}{(1.4 \times 10^2)}$

22. ^(13, 22) $\left(\frac{2}{3}\right)^2 + \frac{2}{3}$

* 23. **Connect** a. Write $\frac{19}{20}$ as a decimal and as a percent.

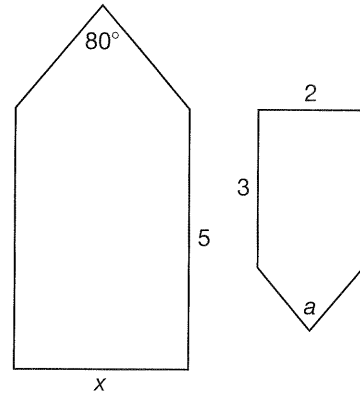
(12)

b. How would you report your score on a test for which you answered 19 out of 20 questions correctly?

24. The pentagons are similar.

(35)

- a. Find a .
- b. What is the scale factor from the smaller to the larger pentagon?
- c. Find x .



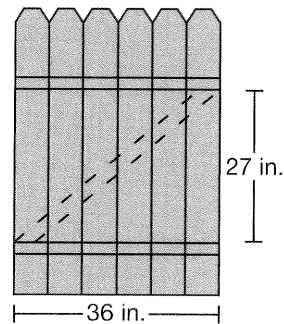
25. Simplify and express with positive exponents.

(51)

$$\frac{x^{-1}y^2}{xy}$$

* 26. Chester needs a length of 2-by-4 to make a diagonal brace for a wooden gate that has the dimensions shown. The 2-by-4 must be at least how many inches long?

(Inv. 2)



* 27. An ant weighs about 10^{-5} kg. Write this as a decimal number.

(51)

28. Sketch a quadrilateral with just one pair of parallel sides. What type of quadrilateral did you sketch?

(Inv. 3)

Find all values of x .

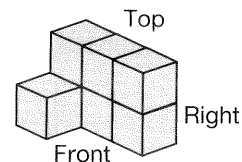
29. a. $7x^2 = 7$

(14, 36)

b. $|x| - 15 = 5$

30. Sketch the front, right side, and top views of this figure.

(Inv. 4)



• Solving Problems Using Measures of Central Tendency

Power Up

Building Power

facts

Power Up K

mental math

a. **Number Sense:** 7.5×20

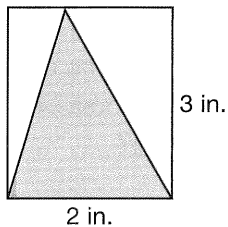
b. **Proportions:** $\frac{25}{100} = \frac{4}{x}$

c. **Probability:** A spinner has 8 equal sections numbered 1–8. What is the probability that the spinner will stop on a number greater than 5?

d. **Measurement:** The odometer read 2901 mi at the beginning of the trip. At the end of the trip, it read 3247 mi. How long was the trip?

e. **Scientific Notation:** Write 2.08×10^7 in standard notation.

f. **Geometry:** Find the area of the rectangle, then find the area of the shaded triangle.



g. **Estimation:** Estimate the total cost of items priced at: \$2.45 plus \$7.45 plus 3 items at \$9.95.

h. **Calculation:** $\sqrt{121}, \times 9, + 1, \sqrt{\quad}, - 1, \times 7, + 1, \sqrt{\quad}, \times 6, + 1, \sqrt{\quad}$

problem solving

About how many sheets of paper the thickness of this page would make a stack 1 cm high? About how many centimeters thick is this page? What should be the product of your two answers?

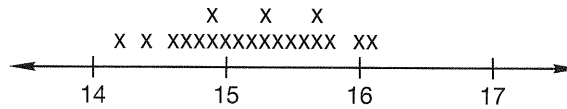
New Concept

Increasing Knowledge

To summarize data, we often report an average of some kind, like the **mean**, **median**, or **mode**.

Suppose a popular television show received these ratings over its season:

16.0	15.2	15.3	15.5
15.7	15.1	14.8	15.6
15.0	14.6	14.4	15.8
15.3	14.7	14.2	15.7
14.9	15.4	14.9	16.1



The data above are displayed on a **line plot**. Each data point is represented with an X above its value on a number line.

The **mean** TV rating is:

$$\text{mean} = \frac{16.0 + 15.7 + \dots + 16.1}{20}$$

$$\text{mean} = 15.21$$

Recall that the **median** of an ordered list of numbers is the middle number or the mean of the two central numbers. There are an even number of data, so the numbers 15.2 and 15.3 share the central location. We compute the mean of these two numbers to find the median.

$$\text{median} = \frac{15.2 + 15.3}{2} = 15.25$$

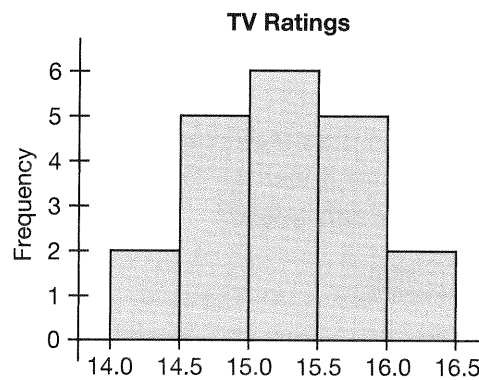
The **mode** is the most frequently occurring number in a set. In this example, there are three modes: 14.9, 15.3, and 15.7.

The **range** of this data is the difference between the highest and lowest ratings.

$$\text{range} = 16.1 - 14.2$$

$$\text{range} = 1.9$$

We may also consider separating the data into intervals such as: 14.0–14.4, 14.5–14.9, 15.0–15.4, 15.5–15.9, 16.0–16.4. Data organized in this way can be displayed in a **histogram**, as shown below.



Data which fall within specified ranges are counted, and the tallies are represented with a bar. For example, the tallest bar tells us that 6 data points fall within the range 15.0–15.4.

The highest peak of a histogram will always indicate the mode of the data ranges.

At the end of a season, Nielsen Ratings reports the mean season rating as a measure of a show's success. For various types of data, the mean is a frequently reported statistic. It is commonly referred to as the "average," even though the median and mode are other averages.

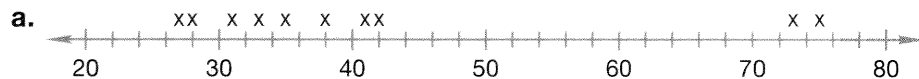
Example 1

The owner of a cafe was interested in the age of her customers in order to plan marketing. One day she collected these data on customers' ages.

28	33	31	41	42
27	75	73	38	35

- a. Make a line plot of the data, then find the mean, median, mode, and range.
- b. Which measure (mean, median, mode, or range) best represents the typical age of the customers?

Solution



$$\begin{aligned} \text{mean} &= 28 + 33 + 31 + 41 + 42 + 27 + 75 + 73 + 38 + 35 \\ &= 42.3 \end{aligned}$$

$$\text{median} = \frac{35 + 38}{2} = 36.5$$

$$\text{range} = 75 - 27 = 48$$

There is no number that occurs more frequently than the others, so the mode is **not reported**.

- b. The **median** gives the best description of the typical age of the customers. Half of the customers surveyed were younger than 36.5, and half were older. The mean, on the other hand, is much higher than the median because of a few customers that were older than the rest.

It is common to see medians used to report median age of residents by state, or median home prices by state.

Example 2

Suppose the median age of residents of a certain state was 30. Twenty years later it was 37. What changes in the population may have occurred in the twenty years?

Solution

When the median age was 30, half of the residents were younger than 30 and half were older. Twenty years later the median age had raised to 37. **This might happen if fewer children are born, if older people live longer, or if people younger than 37 move out of state or people older than 37 move into the state.** Any of these factors would contribute to an "older" population.

Example 3

Thinking Skill

Analyze

Which measure of central tendency would most accurately convey the salary a prospective employee could expect from the company? Why?

Consider the salaries of the employees of a small business. In an interview, a prospective employee was told that the average salary of employees is \$50,000. How might this information be misleading?

Yearly Salary (in thousands \$)

32	43
34	45
32	67
30	85
37	95

mean: 50

median: 40

mode: 32

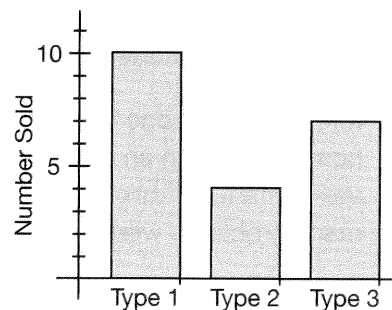
range: 65

Solution

The interviewer reported the mean salary, which is much higher than the median because of a few salaries which are greater than most of the others. A prospective employee who heard that the average salary is \$50,000 would be disappointed if his or her starting salary is near \$30,000.

Example 4

Josiah sells three different hats to test their popularity. His sales are reported in a bar graph below. The height of each bar corresponds to the number of hats of that type Josiah sold.



Find the mode of the data. Why are we not interested in the mean or median?

Solution

The mode of the types of hats is Type 1, the most popular choice. In this case, the data Josiah collected was qualitative data (hat type), not quantitative data (numbers), so mean and median are not important.

Practice Set

The amount of electricity (in kWh) used by one household each month of a year is listed below.

420 450 480 440 420 490 580 590 510 450 430 480

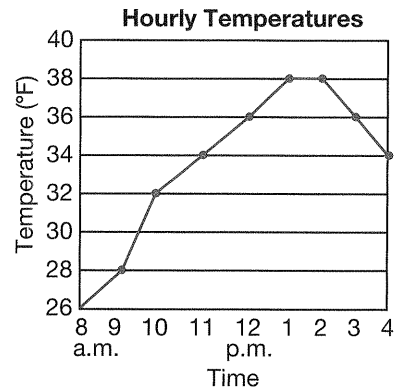
- a. Make a line plot of the data.
- b. **Analyze** Find the mean, median, mode, and range of the data.
- c. Which measure would you report to describe the difference between the most electricity used in a month and the least?
- d. Which measure would you report to someone who wanted to compute the total amount of electricity used in the year? Explain your answer.
- e. If the greatest data point (590) were changed to 500, which measure of central tendency (mean, median, or mode) would change the most?
- f. **Evaluate** Make a list of data values that fits the statement "Half of the days of February were colder than 30° F." Find the mean, median, mode and range of the data.

Written Practice

Strengthening Concepts

1. ⁽⁴⁵⁾ The ratio of adults to children attending the concert was 2 to 3. If there were 54 children, how many adults were there?
- * 2. ⁽⁴⁹⁾ **Evaluate** Each team in the league has five starters and two alternates. If there are 30 starters in all, how many alternates are there?
3. ⁽³⁾ It takes Robert one minute to gather his materials to begin his homework, then an additional minute for each problem on his assignment. If it takes him 20 minutes to complete the assignment, how many problems were on his assignment?
- * 4. ⁽²⁰⁾ **Conclude** Each angle of an equilateral triangle measures how many degrees?
- * 5. ⁽⁵²⁾ **Connect** The pole vaulter cleared the bar at 490 cm. Convert 490 cm to meters using a unit multiplier.

- * 6. **Analyze** Find the mean, median, and mode of the five temperature readings at each hour from 10 a.m. to 2 p.m. According to the graph, for half the time between 10 a.m. and 2 p.m., it was warmer than what temperature?



Analyze Solve.

* 7. $-2x = -16$
(50)

* 8. $\frac{w}{2} = 1.5$
(38)

* 9. $-z + 3 = 7$
(50)

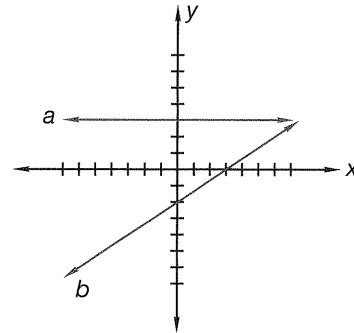
* 10. $23 - p = 73$
(50)

11. A meter is about 3.3 ft, so 4.9 m is about
(52)

A 2 ft B 8 ft C 16 ft D 20 ft

12. Refer to the graphed lines to answer the following questions.
(Inv. 1, 44)

- a. What is the slope of line b ?
b. Line b intersects the y -axis at what point?
c. In which quadrant do lines a and b intersect?



13. Graph $y = -x$ and the point $(4, -5)$. Is the point on the line?
(41)

14. Find the **a** area and **b** circumference of a circle with a diameter of 18 inches. Express your answer in terms of π .
(39, 40)

15. Find the volume of a crate with dimensions 4 ft by 3 ft by 2 ft.
(42)

16. A type of spider weighs about 1×10^{-4} lb. Write this as a decimal number.
(51)

Simplify.

17. $3(x - y) - 2(x + y)$
(31, 36)

* 18. $5x^2 - 3x - 2x^2 + 4x$
(31)

* 19. $\frac{2^3 \cdot 2^0}{2^1 \cdot 2^2}$
(27)

20. $\frac{x^3 y^3 z}{xy^3 z^3}$
(27)

* 21. $-5 - (-7)$
(31)

22. $(1.5 \times 10^3)(1.5 \times 10^{-2})$
(46)

23. a. Write 98% as a decimal and reduced fraction.
(12)

- b. For a \$50,000 fundraising campaign, 98% of the funds have been raised. Which of the three forms would you choose to mentally calculate this amount?

24. Find $\frac{x}{2m}$ when $x = 5$ and $m = -5$.
(14, 36)

* 25. Express with all positive exponents: $6x^{-1}yz^{-2}$
(51)

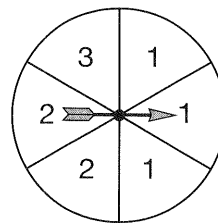
* 26. Sergio claims he can lift 10,000 grams over his head. Is his claim reasonable? Justify your answer.
(52)

* 27. A coin is flipped and the spinner is spun.
(32) Which sample space is most useful? Why?

A Sample Space {H1, H2, H3, T1, T2, T3}

B Sample Space {H1, H1, H1, H2, H2, H3, T1, T1, T1, T2, T2, T3}

Find $P(H \text{ and } 1)$.



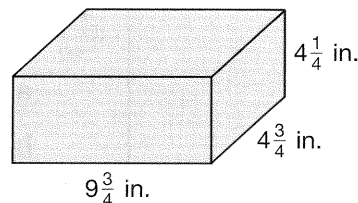
28. Kellie high jumped 5 ft 3 in. Express that height as a mixed number of feet and as a decimal number of feet.
(52)

29. Find all values of x which are solutions.
(14, 36)

a. $x^2 + 2 = 27$

b. $-5|x| = -15$

30. Estimate the **a** volume and **b** surface area of this tissue box.
(42, 43)



Early Finishers

Real-World Application

While traveling on vacation, Mr. and Mrs. Rodriguez decide to spend an afternoon biking. There are two bike-rental companies nearby to choose from. Buck's Bikes charges \$10 per hour per bike. Cycles Galore charges \$4 per hour per bike plus an initial fixed fee of \$15 per bike. Let x represent the time (in hours) that the bikes are rented and y represent the total cost.

- Write an equation to represent the total amount it would cost Mr. and Mrs. Rodriguez to rent two bikes from Buck's Bikes.
- Write an equation to represent the total amount it would cost Mr. and Mrs. Rodriguez to rent two bikes from Cycles Galore.
- Graph both equations on the same coordinate plane.
- Use your graph to determine which company would be cheaper if Mr. and Mrs. Rodriguez want to ride for three hours.

• Angle Relationships

Power Up

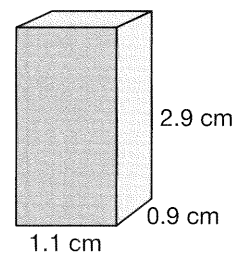
Building Power

facts

Power Up K

mental math

- a. **Number Sense:** 3.5×8
- b. **Fractional Parts:** 20% is $\frac{1}{5}$. How much is a 20% tip on a bill of \$45?
- c. **Algebra:** $x^2 = 81$
- d. **Scientific Notation:** Write 5 trillion in scientific notation.
- e. **Rate:** Vu and Tim began running at the same time and from the same place. Vu ran west at 8 miles per hour. Tim ran east at 9 miles per hour. After 30 minutes, how far had each run and how far were they from each other?
- f. **Geometry:** Estimate the volume of this box.
- g. **Proportions:** A survey of 50 students at a college found that 30 could name the Vice President. If the survey was representative of all 5000 students, then about how many of all the students could name the Vice President?
- h. **Calculation:** $\sqrt{49}, \times 5, + 1, \sqrt{\quad}, \times 4, + 1, \sqrt{\quad}, \times 3, + 1, \sqrt{\quad}, \times 2, + 1, \sqrt{\quad}$



problem solving

At first, one-third of the class were boys. Then another boy joined the class. Now $\frac{3}{8}$ of the class are boys. How many students are in the class now? (Hint: There are fewer than 40 students in the class.)

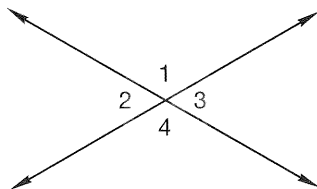
New Concept

Increasing Knowledge

Intersecting lines form pairs of adjacent angles and pairs of opposite angles.

Adjacent angles:

- $\angle 1$ and $\angle 3$
- $\angle 3$ and $\angle 4$
- $\angle 4$ and $\angle 2$
- $\angle 2$ and $\angle 1$



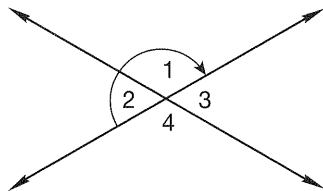
Opposite angles:

- $\angle 1$ and $\angle 4$
- $\angle 2$ and $\angle 3$

Adjacent angles share a common vertex and a common side but do not overlap.

Opposite angles are formed by two intersecting lines and share the same vertex but do not share a side. Opposite angles are also called **vertical angles**. Vertical angles are congruent.

Angles formed by two intersecting lines are related. If we know the measure of one of the angles, then we can find the measure of the other angles.

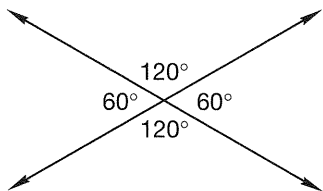


Reading Math

We can use symbols to abbreviate an angle's measure. We read the expression " $m\angle 1 = 120^\circ$ " as "the measure of angle 1 is 120 degrees."

Together $\angle 1$ and $\angle 2$ form a straight angle measuring 180° . If $\angle 1$ measures 120° , then $\angle 2$ measures 60° . Likewise, $\angle 1$ and $\angle 3$ form a straight angle, so $\angle 3$ also measures 60° and vertical angles 2 and 3 both measure 60° .

Since $\angle 3$ and $\angle 4$ together form a straight angle (as do $\angle 2$ and $\angle 4$), we find that $m\angle 4 = 120^\circ$. Thus, vertical angles 1 and 4 both measure 120° .



Two angles whose measures total 180° are called **supplementary angles**. We say that $\angle 2$ is the supplement of $\angle 1$ and that $\angle 1$ is the supplement of $\angle 2$. Supplementary angles may be adjacent angles, like $\angle 1$ and $\angle 2$, but it is not necessary that they be adjacent angles.

Angle Pairs Formed by Two Intersecting Lines

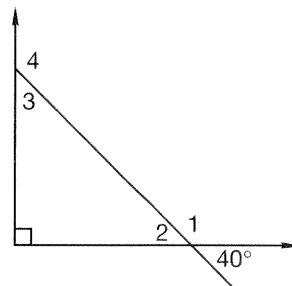
Adjacent angles are supplementary.
Opposite angles (vertical angles) are congruent.

Example 1

Refer to this figure to find the measures of angles 1, 2, 3, and 4.

Solution

- Angle 1 is the supplement of a 40° angle, so $m\angle 1 = 140^\circ$.
- Angle 2 is the supplement of $\angle 1$. Since $\angle 1$ measures 140° , $m\angle 2 = 40^\circ$.
- Angle 3 and $\angle 2$ are the acute angles of a right triangle. The sum of the angle measures of a triangle is 180° , so $m\angle 3 = 50^\circ$.
- Angle 4 is the supplement of $\angle 3$, so $m\angle 4 = 130^\circ$.



Analyze When you know the measure of one of the acute angles of a right triangle, how do you find the measure of the other acute angle?

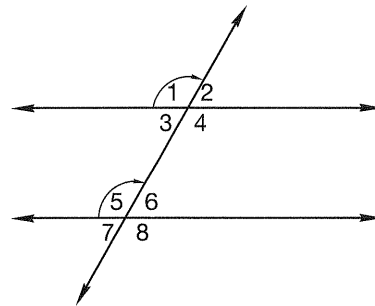
Notice that the measures of angles 2 and 3 in example 1 total 90° . Two angles whose measures total 90° are **complementary angles**, so $\angle 3$ is the complement of $\angle 2$, and $\angle 2$ is the complement of $\angle 3$.

Angles Paired by Combined Measures

Supplementary: Two angles totaling 180°
Complementary: Two angles totaling 90°

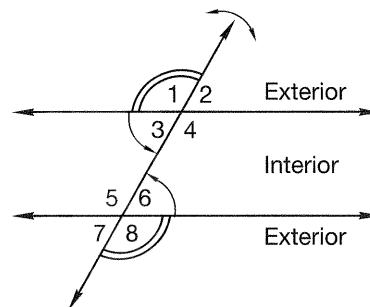
In the following figures we name several pairs of angles formed by parallel lines cut by a third line called a **transversal**. If the transversal were perpendicular to the parallel lines, then all angles formed would be right angles. The transversal below is not perpendicular, so there are four obtuse angles that are the same measure, and four acute angles that are the same measure. We will provide justification for these conclusions in a later lesson.

Corresponding angles are on the same side of the transversal and on the same side of each of the parallel lines. Corresponding angles of parallel lines are congruent. One pair of corresponding angles is $\angle 1$ and $\angle 5$. Name three more pairs of corresponding angles.



Alternate interior angles are on opposite sides of the transversal and between the parallel lines. Alternate interior angles of parallel lines are congruent. One pair of alternate interior angles is $\angle 3$ and $\angle 6$. Name another pair of alternate interior angles.

Alternate Sides of Transversal



Alternate exterior angles are on opposite sides of the transversal and outside the parallel lines. Alternate exterior angles of parallel lines are congruent. One pair of alternate exterior angles is $\angle 1$ and $\angle 8$. Name another pair of alternate exterior angles.

Congruent Angle Pairs Formed by a Transversal Cutting Two Parallel Lines

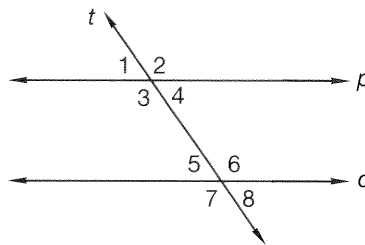
Corresponding angles: Same side of transversal, same side of lines
Alternate interior angles: Opposite sides of transversal, between lines
Alternate exterior angles: Opposite sides of transversal, outside of lines

In summary, if parallel lines are cut by a non-perpendicular transversal, the following relationships exist.

- All the obtuse angles that are formed are congruent.
- All the acute angles that are formed are congruent.
- Any acute angle formed is supplementary to any obtuse angle formed.

Example 2

In the figure parallel lines p and q are cut by transversal t . The measure of $\angle 8$ is 55° . What are the measures of angles 1 through 7?



Solution

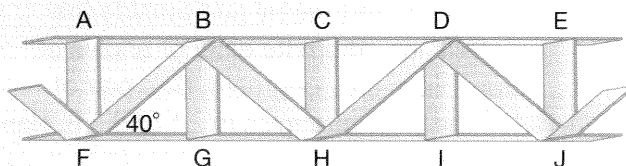
We are told that p and q are parallel. Angle 1 and $\angle 8$ are alternate exterior angles and are congruent.

Statement:	Justification:
$m\angle 1 = 55^\circ$	Alternate exterior angle to $\angle 8$
$m\angle 2 = 125^\circ$	$\angle 2$ and $\angle 1$ are supplementary
$m\angle 3 = 125^\circ$	$\angle 3$ and $\angle 1$ are supplementary
$m\angle 4 = 55^\circ$	$\angle 4$ and $\angle 1$ are vertical and congruent
$m\angle 5 = 55^\circ$	$\angle 5$ and $\angle 8$ are vertical and congruent
$m\angle 6 = 125^\circ$	$\angle 6$ and $\angle 8$ are supplementary
$m\angle 7 = 125^\circ$	$\angle 7$ and $\angle 8$ are supplementary

Example 3

Structural engineers design triangles into buildings, bridges, and towers because triangles are rigid. The sides of triangles do not shift when force is applied like the sides of quadrilaterals do.

A railroad bridge built with a steel truss is strengthened by triangles that keep the bridge straight under the weight of a train.



Knowing one acute angle in this truss is sufficient to find the measures of all the angles. Find the measures of angles ABF , FBC , and FGB .

Segments that look parallel are parallel, and segments that look perpendicular are perpendicular.

Solution

Angle ABF and the 40° angle are alternate interior angles between parallel lines. So $m\angle ABF = 40^\circ$. Together angles ABF and FBC form a straight angle (180°), so $m\angle FBC = 140^\circ$. Angle FGB is a right angle, so $m\angle FGB = 90^\circ$.

Practice Set

Conclude Refer to Figure 1 for exercises a–c.

- Name two pairs of vertical angles.
- Name four pairs of supplementary angles.
- If $m\angle a$ is 110° , then what are the measures of angles b , c , and d ?

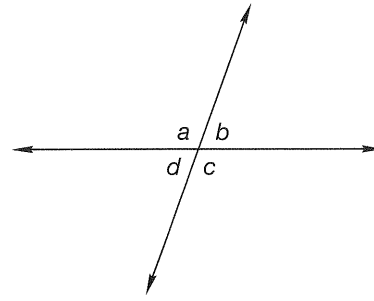


Figure 1

Refer to Figure 2 for exercises d–f.

- Which angle is the complement of $\angle g$?
- Which angle is the supplement of $\angle g$?
- If $m\angle h$ is 130° , then what are the measures of $\angle g$ and $\angle f$?

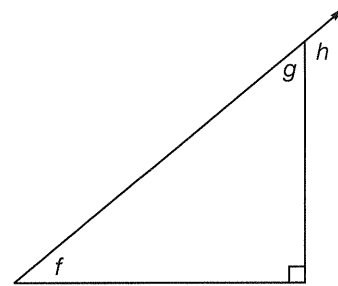


Figure 2

In Figure 3 parallel lines R and S are cut by transversal T . Refer to this figure for exercises g–j.

- Which angle corresponds to $\angle f$?
- Name two pairs of alternate interior angles.
- Name two pairs of alternate exterior angles.
- If $m\angle a$ is 105° , what is the measure of $\angle f$?

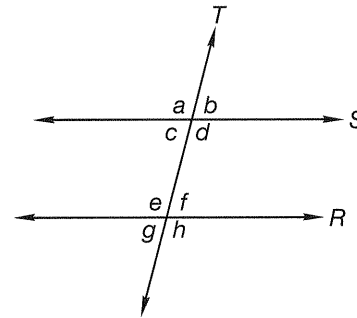


Figure 3

Written Practice

Strengthening Concepts

- ⁽⁴⁵⁾ The ratio of stars to stripes is 50 to 13. If there were 400 stars in the parade, how many stripes were there?
- * ⁽⁵²⁾ **Connect** From the front to the back of the property Wally stepped off 40 paces. He estimated that each pace was a yard. Convert 40 yards to feet using a unit multiplier to estimate the depth of the property in feet.

3. Solve by writing and solving a proportion: The recipe calls for 2 cups of flour and 7 cranberries. Cathy is multiplying the recipe so that it feeds more people. If the new recipe calls for 49 cranberries, how many cups of flour are needed?

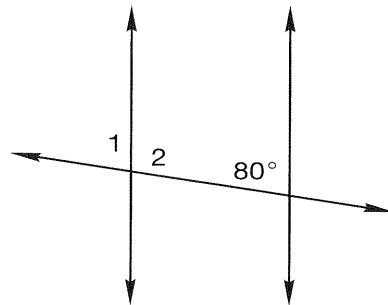
*** 4.** *Evaluate* For 5 minutes every day at 10 a.m., James counted the number of birds that came to the bird feeder outside his window. Over 10 days, he collected this data: 3, 5, 4, 5, 5, 4, 6, 5, 6, 6.

- Display the data with a line plot.
- Find the range and the mean, median, and mode of the data.
- James said, "Most often, there were 5 birds that came to the bird feeder." Which measure of central tendency did he use?

For problems **5** and **6**, use the figure of the parallel lines cut by a transversal.

*** 5.** Find $m\angle 1$. Justify your answer.

*** 6.** Find $m\angle 2$. Justify your answer.



Analyze Solve. Then select one equation and write a story for it.

*** 7.** $3 = 2x + 3$

*** 8.** $3x + 7 - x = 21$

*** 9.** $20 - x = 1$

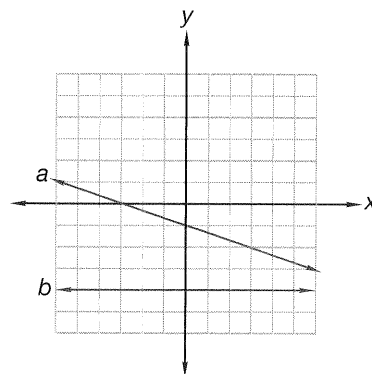
*** 10.** $-2w = -3$

11. Simplify and express with all positive exponents.

$$xyx^0y^{-2}x^{-1}$$

12. Refer to the graphed lines to answer the following questions.

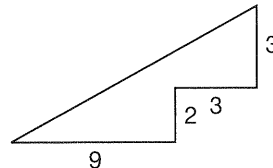
- Which line intersects the y -axis at -1 ?
- What is the slope of line b ?
- Which line is horizontal?
- In which quadrant do the lines intersect?



13. Graph $y = x - 2$. Is the point $(9, 7)$ on the line?

- * 14. **Generalize** Naomi cuts circles from squares with sides 20 cm long.
(40)
- What is the diameter of the largest circle she can cut from the square?
 - What is the area of the largest circle? (Use 3.14 for π .)
 - What is the area of the waste to the nearest sq. cm?

15. Find the **a** area and **b** perimeter of the figure
(37, Inv. 2) to the right. (Units are inches.)



Simplify:

16. $-(x - y) - (x + y)$
(31, 36)

17. $2x^2 - 3x - x^2 + 5$
(31)

18. $\frac{2.9 \times 10^{12}}{2.9 \times 10^3}$
(46)

19. $-2 - (-2)$
(33)

20. $\frac{x^9 y}{x^8 y}$
(27)

21. $\left(\frac{-1}{2}\right)^3 + \frac{7}{8}$
(22, 36)

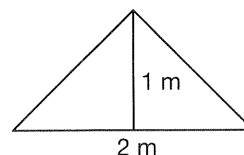
22. The desktop measured 76.2 cm wide. Convert this measure to meters and write in scientific notation.
(51)

23. Write 100% **a** as a decimal and **b** as a reduced fraction.
(12)

- * 24. **Analyze** Raul planned a painting of his mountain cabin. The cabin stands 20 ft tall and the trees around it are 30 ft tall. The height of the cabin in his painting will be 6 inches.
(35)

- What is the scale from the actual cabin to the painting?
- How tall should Raul paint the trees?

- * 25. Kyla constructed a simple tent with stakes, ropes, and a length of plastic sheeting. She used the plastic sheeting for the roof and floor of the tent, leaving the ends open. The tent stands 1 m tall and 2 m wide at the base. What is the minimum length of plastic used? (Round up to the next whole meter.)
(Inv. 2)



Find all solutions.

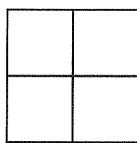
26. $\frac{x}{12} = \frac{12}{9}$
(34)

27. $\frac{x^2}{2} = 18$
(14, 36)

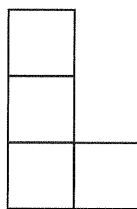
28. $-3|x| = -3$
(1, 14)

29. $\frac{m}{5} = 0.2$
(38)

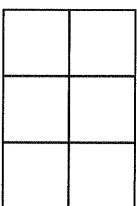
30. Use the views shown to draw a three-dimensional view of the figure.
(Inv. 4)



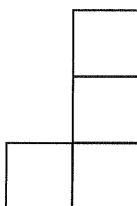
Top View,
Bottom View



Left-side
View



Front View,
Back View



Right-side
View

Early Finishers

*Real-World
Application*

Mike can swim the 100-yard freestyle in two minutes. If he swims at the same rate, how many seconds will it take him to swim 50 meters (1 meter \approx 1.09 yard)?

• Nets of Prisms, Cylinders, Pyramids, and Cones

Power Up

Building Power

facts

Power Up K

mental math

- a. **Number Sense:** 1.5×8
- b. **Statistics:** Find the mean of 16 and 20.
- c. **Fractional Parts:** $66\frac{2}{3}\%$ of \$81
- d. **Probability:** What is the probability of rolling a number less than 3 on a number cube?
- e. **Geometry:** Two angles of a parallelogram measure 80° and 100° . Find the measure of the other two angles.
- f. **Measurement:** The odometer read 2388 mi at the end of the trip. At the beginning, it had read 1208 mi. How long was the trip?
- g. **Rate:** Ronnie ran $\frac{1}{4}$ mile in 1 minute. At that rate, how long would it take him to run a mile?
- h. **Calculation:** $200 \div 2, \sqrt{\quad}, \times 5, - 1, \sqrt{\quad}, \times 2, + 2, \sqrt{\quad}, \times 2, + 1, \sqrt{\quad}$

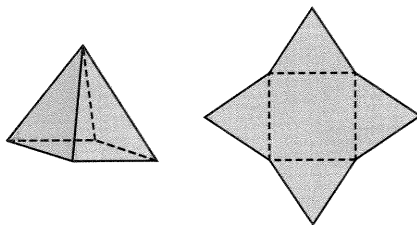
problem solving

Two brothers are 5 years apart, and the sum of their ages is 41. What are the ages of the brothers?

New Concept

Increasing Knowledge

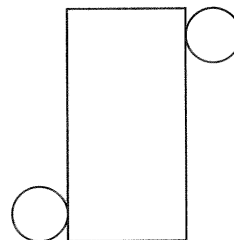
If we think of the surface of a solid as a hollow cardboard shell, then cutting open and spreading out the cardboard creates a net of the solid. For example, here we show a net for a pyramid with a square base.



Example 1

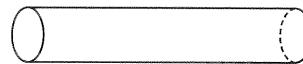
This net represents the surfaces of what geometric solid? Sketch the solid and describe how this surface area formula relates to each part of the net:

$$s = 2\pi rh + 2\pi r^2$$



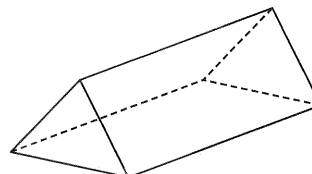
Solution

A geometric solid with two circular bases and a lateral surface that unwraps to form a rectangle is a **cylinder**. The first part of the formula, $2\pi rh$, applies to the lateral surface area (circumference, $2\pi r$, times height, h) and the second part, $2\pi r^2$, applies to the two circular bases.



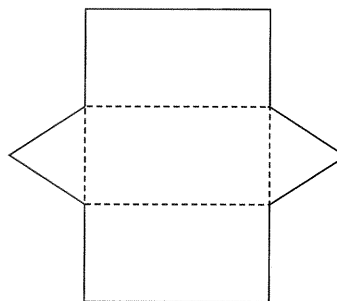
Example 2

Sketch a net for this triangular prism.



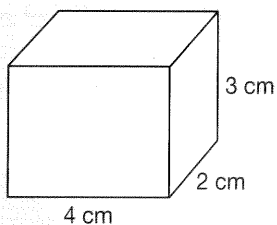
Solution

The prism has two congruent triangular bases, and three rectangular lateral faces.

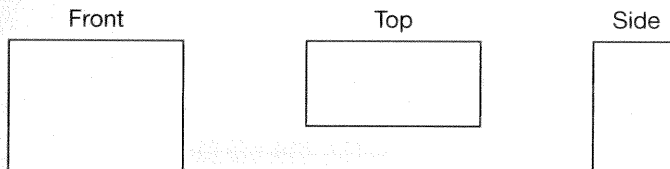


Example 3

Sketch the front, top, and right-side views of this figure. Then sketch a net for this figure. Describe how the two sets of sketches are related.

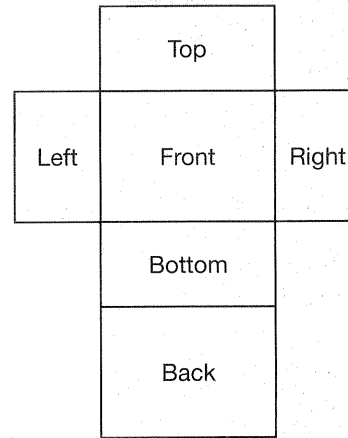


Solution



One example of a net:

The front, top and right-side views are the three different rectangles that appear in the net. Each rectangle appears twice, because the front and back faces are congruent, as are the top and bottom faces and the left and right faces.

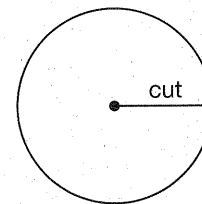


Activity

Net of a Cone

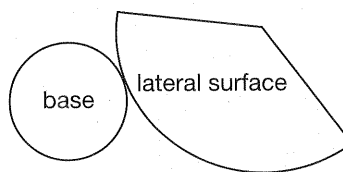
Materials needed: unlined paper, compass, scissors, glue or tape, ruler.

Using a compass, draw a circle with a radius of at least two inches. Cut out the circle and make one cut from the edge to the center of the circle. Form the lateral surface of a cone by overlapping the two sides of the cut. The greater the overlap, the narrower the cone. Glue or tape the overlapped paper so that the cone holds its shape.

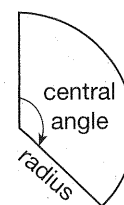


To make the circular base of the cone, measure the diameter of the open end of the cone and use a compass to draw a circle with the same diameter. (Remember, the radius is half the diameter.) Cut out the circle and tape it in place using two pieces of tape.

Now disassemble the cone to form a net. Cut open the cone by cutting the circular base free on one side. Unroll the lateral surface by making a straight cut to the point (apex) of the cone. The net of a cone has two parts, its circular base and a sector of a circle that forms the lateral surface of the cone.



Extend An alternate method for calculating the area of the lateral surface of a cone is to calculate the area of the portion of a circle represented by the net of the lateral surface. Use a protractor to measure the central angle of the lateral surface of the cone you created. The measure of that angle is the fraction of a 360° circle represented by the lateral surface. Use a ruler to measure the radius. Find the area of a whole circle with that radius. Then find the area of the sector by multiplying the area of the whole circle by the fraction $\frac{\text{central angle}}{360}$.



Thinking Skill

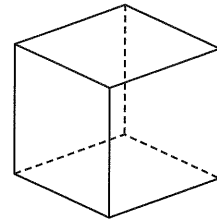
Connect

The formula for the surface area of a cone is $s = \pi r l + \pi r^2$. Which parts of the formula apply to which parts of the net?

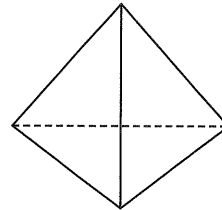
Practice Set

a. The net created in this lesson's activity represents the surfaces of what geometric solid? Sketch the solid.

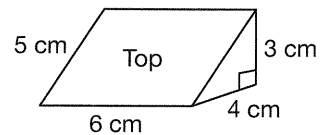
b. Sketch a net for this cube.



c. This pyramid is called a tetrahedron. All of its faces are congruent equilateral triangles. Draw a net of this pyramid.



d. Sketch the back, top, and right side views of this triangular prism. Then sketch a net for the figure and label the dimensions.



e. **Model** Build a model of the figure in exercise c by cutting, folding, and taping the net.

Written Practice

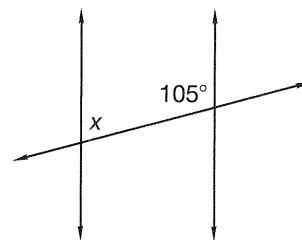
Strengthening Concepts

1. ⁽³⁴⁾ A manufacturing company has a debt to equity ratio of 3 to 2. If the company has a debt of \$12 million, how much does it have in equity?
2. ^(3, 4) It takes Jack 5 minutes to drive from home to the nearest ATM machine. Then it takes 2 minutes in line at the ATM for each customer ahead of him. How long will it be before Jack can use the ATM if he leaves from home and then waits behind 3 customers?
- * 3. ⁽⁵³⁾ **Justify** Yueling read 5 books of these lengths: 105, 97, 96, 99, 103. Write a statement using a measure of central tendency to communicate how many pages Yueling read. Explain your choice.

4. ^(31, 36) $(-2)^2 + (-2)^3$

5. ⁽⁴⁴⁾ $\frac{4}{3} = \frac{x}{1.5}$

- *6. ⁽⁵⁴⁾ **Conclude** A transversal cuts parallel lines. Find x .



Solve. Then select one of the equations and write a word problem for it.

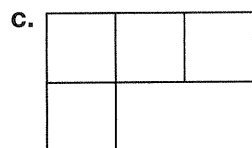
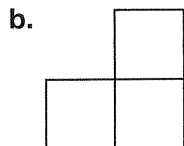
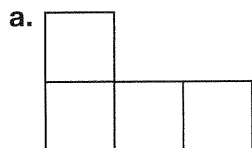
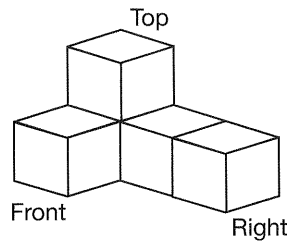
7. $4 = -m + 11$
(50)

8. $9x + 9 = 90$
(50)

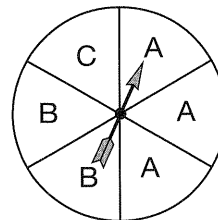
9. $-5 - x = -9$
(50)

10. $3y - y - 1 = 9$
(36)

11. Describe each of the following views of this figure as top, right, or front.
(Inv. 4)



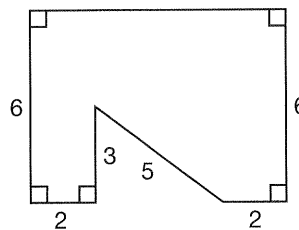
12. If the spinner is spun once, Roger says the sample space is $\{A, A, A, B, B, C\}$. Simon says the sample space is $\{A, B, C\}$. Who is correct? Explain your answer.
(32)



13. Garrett does an Internet search for the nearest location of a particular store. He requests a search of a location within a 60 mi radius of his home. About how many square miles does the search cover?
(40)

14. Graph $y = \frac{4}{5}x - 2$. Is the point $(5, 2)$ a solution?
(47)

15. Find the perimeter and area of this figure.
(37)
(Units are in ft.)



- * 16. **Analyze** Moving at a uniform rate the train traveled 200 miles in five hours.
(49)
- Express the average speed of the train as a unit rate.
 - How long did it take the train to travel 100 miles?

* 17. **Analyze** Convert 440 ft to yards. Use a unit multiplier.
(52)

18. The dimensions of an air mattress are 80 inches by 33 inches by 5 inches. What is the volume of the air mattress? If the average set of human lungs can hold about 244 in.^3 , about how many breaths will it take to inflate the mattress?
(42)

Generalize Simplify.

* 19. $9^{-2}x^{-1}y^0x$
(27, 51)

20. $\frac{x^5m^2}{mx}$
(27)

21. $\left(\frac{1}{3}\right)^2 \div \frac{2}{3}$
(22)

22. $1.2 \div 0.05$
(25)

23. a. Write $\frac{1}{100}$ as a decimal and percent.
(12)

b. How does $\frac{1}{100}$ compare to 0.009?

24. Factor:
(21)

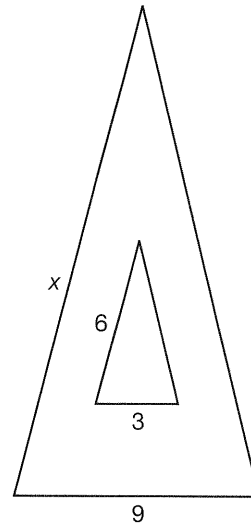
a. $6x - 15$

b. $x^2 - x$

25. The triangles are similar.
(35)

a. What is the scale factor from the small to large triangle?

b. Find x .



26. Brooke finished the facts practice test in 1 minute and 18 seconds. Express her time as a mixed number of minutes and as a decimal number of minutes.
(52)

27. Rahm is playing his favorite video game. Every time his player grabs a wrong object, Rahm loses 5 points. If Rahm accidentally grabs a wrong object 23 times, what is the effect on his score?
(36)

28. A human hair measures 50 millionths (50×10^{-6}) of a meter. Which is thicker, 100 human hairs or a nickel that is $\frac{2}{10^3}$ meter thick?
(51)

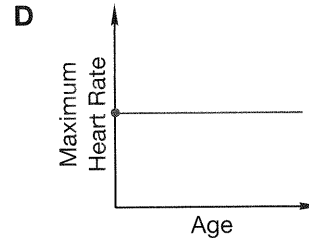
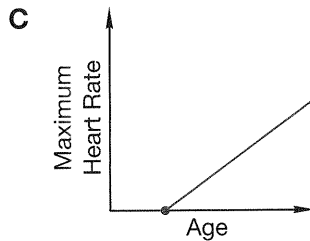
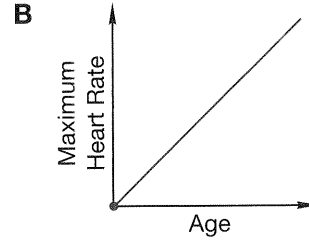
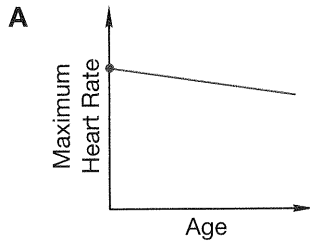
* 29. Combine like terms to simplify.

(31, 36)

a. $2(x + b) - (-x - b)$

b. $2x + 1 - 3x + 4$

30. As humans get older, their maximum heart rate decreases. Which of the following graphs illustrates this relationship? Is it a proportional relationship?
(41)



Early Finishers

Real-World Application

A manager at the local movie theater wants to increase popcorn profits. The cost of a bucket of popcorn is \$0.05 for popcorn kernels, \$0.02 for butter, and \$0.25 for the bucket. If the manager wants to sell 115 buckets of popcorn a night and make a profit of \$365.00, how much should the manager charge for each bucket of popcorn? Note: The profit is the amount of money made after subtracting the cost.

The Slope-Intercept Equation of a Line

Power Up

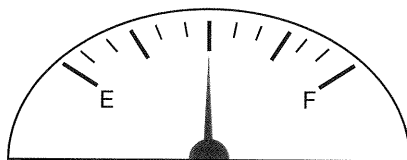
Building Power

facts

Power Up L

mental math

- a. **Number Sense:** 6×3.1
- b. **Powers/Roots:** $\sqrt{2 \cdot 2} \cdot \sqrt{3 \cdot 3}$
- c. **Scientific Notation:** Write 4.013×10^{-4} in standard form.
- d. **Percent:** 75% of 84
- e. **Geometry:** Can the sides of a triangle measure 7 ft, 3 ft, and 3 ft?
- f. **Fractional Parts:** The gas tank holds 20 gallons when full. About how much gas is in the tank now?



- g. **Rate:** Tommy rode his bike 20 miles per hour north. Christina rode 15 miles per hour south. If they started at the same place and time, how far apart are they after 1 hour? After 2 hours?
- h. **Calculation:** $6 \times 7, -2 \div 4$, square it, $-1 \div 9, +5 \div 2, \div 2, \div 2$

problem solving

A hiker estimates his hiking time by using the following rule: $\frac{1}{2}$ hour for every 1 mile, plus $\frac{1}{2}$ hour for each 1000 foot rise in elevation. A pair of hikers is planning a 12-mile hike to the summit of a mountain with a 5000 ft rise in elevation. They want to reach the summit by 3:00 p.m. At what time should they begin hiking?

New Concept

Increasing Knowledge

If the equation of a line is written in slope-intercept form, we can read the slope and y-intercept directly from the equation.

$$y = (\text{slope})x + (\text{y-intercept})$$

