Thinking Skill

Explain

If one in three boxes contains a free pass, does buying three boxes guarantee a free pass? Why or why not?

Simulation

1. For each trial, simulate the "purchase" of 3 cereal boxes by spinning the spinner 3 times. For each spin, record W (for winner) or N (for not winner). Then in the "At least one winner?" column write "yes" or "no," depending on whether there were any winners in the three-spin trial.

2. Repeat step 1 until the table is complete. Below are some sample results.

	Spin 1	Spin 2	Spin 3	At Least 1 Winner?
Trial 1	W	N	N	yes
Trial 2	N	N	N	no
Trial 3	N	W	W	yes

3. To compute the **experimental probability** of having at least one winner after purchasing 3 cereal boxes, add the number of successes ("yes" in the last column) and divide by the total number of trials. For our sample results above, this would be:

$$P(\text{at least one winner in 3 boxes}) \approx \frac{2}{3}$$

exercises

Answer the following questions about the simulation.

1. What did one spin of the spinner represent?

2. What did each three-spin trial represent?

3. What did the spinner stopping on "winner" represent?

4. What did the spinner stopping on "not a winner" represent?

5. *Explain* State whether the following tools could have been used instead of a spinner to conduct the simulation. Explain why or why not.

   a. Two red cards and one black card

   b. A coin

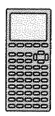
   c. Three red marbles

   d. One red, one white, one blue marble

   e. A number cube

6. *Discuss* We use simulation when the theoretical probability is difficult to compute and the experimental probability is impractical to find directly. In this case, why is it desirable to simulate the experiment rather than actually perform the experiment (purchasing the cereal boxes)?

7. Assuming a large number of cereal boxes were made for this contest, one can use probability theory (not simulation) to show that the theoretical probability of having at least one winner after purchasing three cereal boxes is $\frac{19}{27}$ (or about 0.704). What is the difference between the theoretical probability and the experimental probability you found? Why is it important to conduct many trials when computing an experimental probability?



Visit www.SaxonPublishers.com/ActivitiesC3 *for a graphing calculator activity.*

Activity 2

Design and Conduct a Simulation

Materials needed:

- coins
- number cubes
- spinners
- decks of cards
- color tiles or cubes available for students to select

Read each of the following descriptions of events and explain how you would simulate each one. Then choose one of the events and conduct a simulation. Select a simulation tool for the experiment. Draw the chart you would use to organize your results.

1. A basketball player makes an average of 4 in 6 free throws. If he shoots 3 free throws in a game, what is the probability he makes all of them?

2. John has 2 blue socks and 4 black socks in a drawer. On a dark morning, he reaches in and pulls out 2 socks. What is the probability that he has a matching pair?

3. One-half of the chairs at a party have stickers secretly placed under the seats entitling the person sitting in the chair to a door prize. What is the probability that Tina and her two friends will all sit in chairs with stickers under the seats?

LESSON 71

• Percent Change of Dimensions

Power Up

Building Power

facts Power Up O

mental math

a. **Algebra:** Evaluate: $\frac{1}{2}b(10)$ when $b = 2$

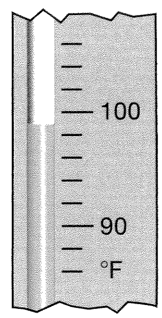
b. **Fractional Parts:** $\frac{5}{6}$ of 48

c. **Probability:** In a bag of 20 marbles, 4 are blue. What is the probability that one taken at random will not be blue?

d. **Proportions:** The ratio of DVD players to VCRs was 7 to 2. There were 35 DVD players. How many VCRs were there?

e. **Percent:** 5% of $40

f. **Measurement:** Find the temperature indicated on this thermometer.



g. **Select a Method:** To find the average high temperature for 31 days in August, which would you use?

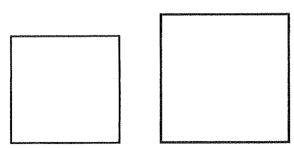
A mental math
B pencil and paper
C calculator

h. **Calculation:** Square 10, + 10, + 10, ÷ 2, + 4, $\sqrt{\ }$, + 1, $\sqrt{\ }$

problem solving Danielle has 5 different shirts and 4 different skirts. She wants to wear a different combination each day for as many days as she can before she repeats a combination. How many days can she wear a different combination?

New Concept

Increasing Knowledge



Projectors, copy machines, and graphics software can dilate and reduce the size of images by scale factors that are often expressed as percents.

Here we show two squares. The larger square is 20% larger than the smaller square.

Statements about percents of increase or decrease should be read carefully. A small word or single letter can affect the meaning.

Dilation: Percent of original = 100% + percent of increase

Reduction: Percent of original = 100% − percent of decrease

Referring to the square above, increasing the size of the smaller square 20% means that the size of the larger square is 120% of the smaller square. Notice that 120% is related to the scale factor. We find that the scale factor from the smaller square to larger square is 1.2 by converting 120% to a decimal.

Example 1

One square is 50% larger than another square.

a. The perimeter of the larger square is what percent of the perimeter of the smaller square?

b. The area of the larger square is what percent of the area of the smaller square?

c. Provide examples of two squares that fit the description.

Solution

Although we are not give the dimensions of the squares, we are given enough information to answer the questions.

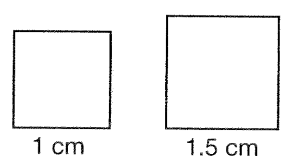
a. Since the dimensions of the larger square are 50% greater than the dimensions of the smaller square, each side is 150% of the length of the smaller square. That is, the scale factor from the smaller to larger square is 1.5. Perimeter is a length, so the perimeter of the larger square is 1.5 times, or **150%,** of the perimeter of the smaller square (50% greater).

b. Since area is the product of two dimensions, we square the scale factor to determine the relationship between the areas.

$$(1.5)^2 = 2.25$$

The area of the larger square is 2.25 times the area of the smaller square. In the language of percent, the area of the larger square is **225%** of the area of the smaller square (which means the area is 125% greater).

c. We may select any two side lengths for the squares so that the longer side length is 50% greater than the shorter side length. Here are some sample side lengths: 1 and 1.5, 2 and 3, 10 and 15, 100 and 150. It is often convenient to choose 1 for the dimension to which the other is compared. The larger square is compared to the smaller, so we choose 1 for the side length of the smaller square. Thus the side length of the larger square is 1.5. We choose to use cm as the unit of measure.



We compare the perimeter of the two squares.

Smaller Square	**Larger Square**
Perimeter = 4 cm	Perimeter = 6 cm

We see that the perimeter of the larger square is 150% *of* (50% *greater than*) the perimeter of the smaller square.

Now we compare the areas.

Smaller Square	**Larger Square**
Area $= 1\ \text{cm}^2$	Area $= 2.25\ \text{cm}^2$

The area of the larger square is 225% *of* (125% *greater than*) the area of the smaller square.

Example 2

The towel shrunk 10% when it was washed. The area of the towel was reduced by what percent? Provide an example of the "before" and "after" dimensions of the towel.

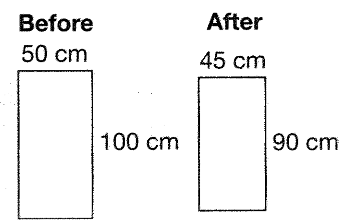
Solution

Assuming that the towel shrunk uniformly, the length was reduced 10% and the width was reduced 10%. Since the length and width are both 90% of the original size, the scale factor from the original size to the reduced size is 0.9. To find the effect of the contraction on the area we square the scale factor.

$$(0.9)^2 = 0.81$$

This number means that the area of the reduced towel is 81% of the area of the original towel. Thus the area was reduced by **19%.**

To provide a before and after example we select multiples of 10 for the dimensions so that a reduction of 10% is easy to calculate.



Example 3

The vertices of a square are (4, 4), (4, −4), (−4, −4), and (−4, 4).

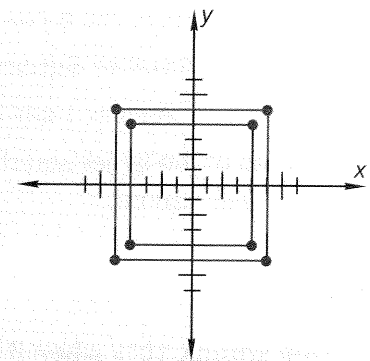
a. Sketch the square and its image after a 125% dilation.

b. The perimeter of the dilation is what percent greater than the perimeter of the original square?

c. The area of the dilation is what percent greater than the area of the original square?

Solution

a. The scale factor of the dilation is 1.25 (or $1\frac{1}{4}$). Since 4×1.25 is 5, the vertices of the dilation are (5, 5), (5, −5), (−5, −5), and (−5, 5). We sketch the square and its dilation.



b. The perimeters of the two squares are 20 units and 16 units, so the perimeter of the image is 125% of the perimeter of the original square.

$$\frac{20}{16} = 1.25$$

Therefore, the perimeter of the image is **25%** greater than the perimeter of the original square.

c. The areas of the two squares are 25 square units and 16 square units, so the area of the image is $156\frac{1}{4}\%$ of the area of the original square.

$$\frac{25}{16} = 1.5625$$

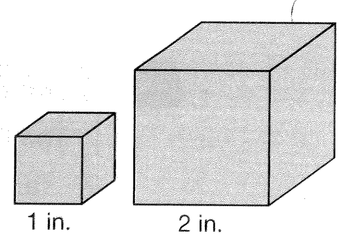
Therefore, the image is **$56\frac{1}{4}\%$ (or 56.25%)** greater than the area of the original square.

Example 4

If the dimensions of a cube are increased 100%, by what percent is the volume increased?

Solution

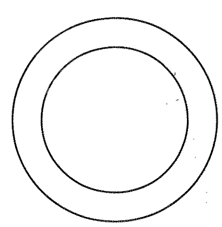
We will make the calculations using a specific example. We start with a cube with edges 1 inch long. If the dimensions are increased 100%, then the larger cube has edges 2 inches long.



The volume of the smaller cube is 1 in.^{3}. The volume of the larger cube is 8 in.^{3}. The larger cube is 8 times, or 800%, as large as the smaller cube. Therefore, the volume is *increased* **700%.**

Practice Set

a. There are two concentric circles on the playground. The diameter of the larger circle is 40% greater than the diameter of the smaller circle. The area of the larger circle is what percent greater than the area of the smaller circle?



b. Becky reduced the size of the image on the computer screen by 40%. By what percent was the area of the image reduced?

c. If the original square in example 3 were dilated 150%, then by what percent would the perimeter increase, and by what percent would the area increase?

d. Suppose the dimensions of the larger cube in example 4 (the 2-inch cube) were increased by 100%. What would be the edge length of the expanded cube? The volume of the expanded cube would be what percent of the 2-inch cube?

Written Practice *Strengthening Concepts*

1. (45) Pencils outnumbered pens in the drawer 17 to 3. If there were 80 pens and pencils in all, how many were pens?

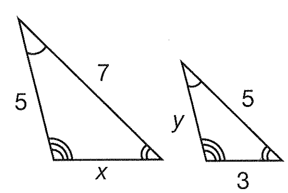
2. (48) Thirty-eight percent of the students watched the ball game. If there are 150 students in all, how many did not watch the ball game?

* **3.** (67) **Analyze** The price of the art piece increased by 90% after the artist became famous. If the new price is $2470, what was the original price?

* **4.** (69) The table shows the perimeter of a polygon given a side length. Does the table show direct variation? If so, state the constant of variation. What type of polygon could this table represent?

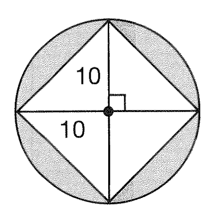
P	*s*
3	1
6	2
15	5
150	50

* **5.** (35) **Analyze** Find the lengths of x and y in the similar triangles to the right.



* **6.** (56, 69) Graph $y = 2x - 2$. Does the graph indicate direct variation? Explain your answer.

7. (37) Find the area of the shaded region in the figure to the right. Use $\pi = 3.14$. (Units are in cm.)



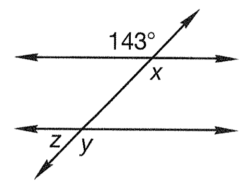
8. (48) When sales tax is 6%, the amount paid for an item is 106% of the price. Write 106% **a** as a decimal and **b** as a fraction. **c** Show how you would use one of these three forms to compute 106% of $21.

9. (32) Izzy rolls one number cube in a game. He must roll a 3 or more in order to win on his next turn. What is the probability he wins on his next turn?

10. (71) If the dimensions of a square are increased 10%, by what percent is the area increased?

11. (62) Graph the solution on a number line. $7 - x \geq 5$

12. (54) A transversal cuts two parallel lines. Find z.



13. (53) Six students estimated the length of the window, in meters: 1.1, 1.3, 1.5, 1.7, 1.9, and 1.5.

a. Display the data in a line plot.

b. Find the mean, median, mode, and range of the estimates.

c. Which measure indicates the spread of the estimates?

14. (64) Alex swims 100 meters in 50 seconds. Use a unit multiplier to convert 100 meters per 50 seconds to meters per minute.

* **15.** (Inv. 5) **Model** Sketch $\triangle ABC$ with vertices at $A(-2, -2)$, $B(2, -2)$, and $C(0, 2)$. Then sketch its image $\triangle A'B'C'$ with a scale factor of 3. What are the coordinates of the vertices of $\triangle A'B'C'$?

* **16.** (71) In problem **15,** the dimensions of $\triangle ABC$ are what percent of the dimensions of $\triangle A'B'C'$? (Hint: AB is what fraction of $A'B'$?)

Simplify.

17. (27) $\frac{15x^2y^{-1}}{5xy}$

18. (13, 22) $\frac{5}{6} - \frac{2}{3} \div \frac{4}{5}$

19. (36) $-3 + (-4)(-5)$

20. (24, 25) $\frac{1.2 + 0.24}{0.3}$

Solve.

21. (50) $\frac{1}{4} - \frac{2}{3}x = \frac{11}{12}$

22. (44) $\frac{3}{4} = \frac{15}{x}$

23. (50) $0.02x - 0.3 = 1.1$

24. (50) $6x - 12 = 84$

25. (Inv. 6) The frequency table below indicates how many of the 60 Movie Express members rented 1, 2, 3, or 4 movies during a one-week period.

Number of Movies Rented	Tally	Frequency
1	卌 卌 卌 卌 III	23
2	卌 卌 卌 III	18
3	卌 卌 I	11
4	IIII	4

Which is the most reasonable conclusion based on the information in the chart?

A Most members rent 1 movie per week.

B About $\frac{1}{3}$ of the members rent 3 movies per week.

C Few members rent 2 or 3 movies per week.

D Less than half of the members rent more than 2 movies per week.

Early Finishers
Real-World Application

A small accounting firm has seven employees. Their annual salaries are as follows:

\$38,500 \$41,500 \$45,000 \$52,000 \$43,500 \$34,000 \$45,000

The employees are voting on two different plans for salary raises for the coming year. Both plans are one-year proposals and apply to all employees.

Plan 1: Each employee receives a raise of \$1,800.

Plan 2: Each employee receives a raise of 4.2%.

a. Make a table of current salaries and proposed salaries under both plans.

b. Write a brief explanation of how the two plans differ and their potential financial impact on the highest- and lowest-paid employees.

LESSON 72

• Multiple Unit Multipliers

Power Up

Building Power

facts Power Up O

mental math

a. **Statistics:** Find the range: 27, 26, 26, 24, 28

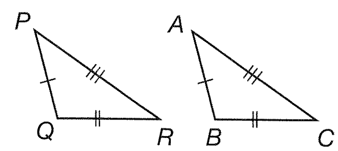
b. **Estimation:** $3\sqrt{63}$

c. **Number Sense:** Compare $2 \times 0.5 \bigcirc (0.5)^2$

d. **Powers/Roots:** $\sqrt{5 \cdot 5} \cdot \sqrt{3 \cdot 3}$

e. **Proportion:** $\frac{x}{5} = \frac{20}{25}$

f. **Geometry:** Copy and complete this congruence statement: $\triangle ABC \cong$ ________



g. **Scientific Notation:** Write 45,000,000 in scientific notation.

h. **Calculation:** $3 \times 3, \times 3, \times 3, \sqrt{\ }, \sqrt{\ }$

problem solving

List the three pairs of one-digit numbers whose product has a 9 in the ones place. Then find the missing digits in the problem below.

$$\begin{array}{r} \_\,\_ \\ \times \quad \_ \\ \hline 539 \end{array}$$

New Concept

Increasing Knowledge

Math Language

Recall that a **unit multiplier** is a form of 1, because the numerator and denominator are equivalent measures.

We have used single unit multipliers to convert measures and rates. For some conversions we use multiple unit multipliers. Using multiple unit multipliers is similar to multiplying repeatedly by 1.

$$5 \cdot 1 \cdot 1 \cdot 1 = 5$$

We can use two unit multipliers to convert units of area and three unit multipliers to convert units of volume.

Example 1

Thinking Skill

Connect

Does this example involve area or volume? Explain.

The Ortegas want to replace the carpeting in some rooms of their home. They know the square footage of the rooms totals 855 square feet (ft^2). Since carpeting is sold by the square yard (yd^2), they want to find the approximate number of square yards of carpeting they need. Use unit multipliers to perform the conversion.

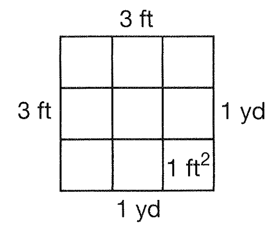
Solution

To convert square feet (ft^2) to square yards (yd^2), we need to cancel feet twice. We will use two unit multipliers to perform the conversion.

$$855\ \cancel{ft^2} \times \underbrace{\frac{1\text{ yd}}{3\ \cancel{ft}} \times \frac{1\text{ yd}}{3\ \cancel{ft}}}_{\text{unit multipliers}} = \frac{855\text{ yd}^2}{9} = 95\text{ yd}^2$$

The Ortegas will need about **95 square yards** of carpet.

To understand visually why it is necessary to cancel feet twice in the example above, consider this diagram of a square yard.



We see that one square yard is equivalent to 9 square feet. Therefore, $855\text{ ft}^2 = \frac{855}{9}\text{ yd}^2 = 95\text{ yd}^2$

Analyze What unit multipliers would we use to convert square feet to square miles?

Example 2

Concrete is delivered to a job site in cement trucks and is ordered in cubic yards. Ray is pouring a concrete driveway that is 18 feet wide, 36 feet long, and 4 inches ($\frac{1}{3}$ foot) thick. Ray calculates the volume of concrete he needs is 216 cubic feet. Use unit multipliers to find the number of cubic yards of concrete Ray needs to order.

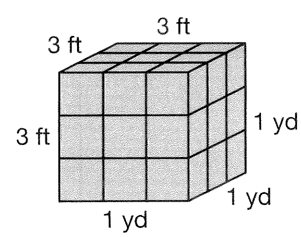
Solution

To convert cubic feet (ft^3), we need to cancel feet three times. We use three unit multipliers.

$$216\ \cancel{ft^3} \times \underbrace{\frac{1\text{ yd}}{3\ \cancel{ft}} \times \frac{1\text{ yd}}{3\ \cancel{ft}} \times \frac{1\text{ yd}}{3\ \cancel{ft}}}_{\text{unit multipliers}} = \frac{216\text{ yd}^3}{27} = 8\text{ yd}^3$$

Ray needs to order **8 cubic yards** of concrete.

Using the three unit multipliers in example 2 essentially divides the number of cubic feet by 27 (since $3 \cdot 3 \cdot 3 = 27$). This is reasonable because one cubic yard is equivalent to 27 cubic feet.



Analyze What unit multipliers would we use to convert cubic inches to cubic feet?

Example 3

Keisha set a school record running 440 yards in one minute (60 seconds). Find Keisha's average speed for the run in miles per hour (1 mi = 1760 yd).

Solution

We are given Keisha's rate in yards per minute. We are asked to find her rate in miles per hour.

We need one unit multiplier to convert yards to miles and another unit multiplier to convert minutes to hours. We select from the following.

$$\frac{1760 \text{ yd}}{1 \text{ mi}} \qquad \frac{1 \text{ mi}}{1760 \text{ yd}} \qquad \frac{1 \text{ hr}}{60 \text{ min}} \qquad \frac{60 \text{ min}}{1 \text{ hr}}$$

We choose the unit multipliers that cancel the units we want to remove and that properly position the units we want to keep.

$$\frac{440 \cancel{\text{yd}}}{1 \cancel{\text{min}}} \times \underbrace{\frac{1 \text{ mi}}{1760 \cancel{\text{yd}}} \times \frac{60 \cancel{\text{min}}}{1 \text{ hr}}}_{\text{unit multipliers}} = \frac{440 \times 60 \text{ mi}}{1760 \text{ hr}}$$

We perform the arithmetic and find that Keisha's average speed for her run was **15 mph.**

Practice Set

a. *Analyze* Examine problems **b–h.** For which problems will you need to use two unit multipliers? Three unit multipliers? How do you know?

Use multiple unit multipliers to perform each conversion.

b. 9 sq. ft to sq. in.

c. 9 sq. ft to sq. yd

d. 1 m^3 to cm^3

e. 1,000,000 mm^3 to cm^3

f. 12 dollars per hour to cents per minute

g. 10 yards per second to feet per minute

h. 1 gallon per day to quarts per hour

Written Practice *Strengthening Concepts*

1. (45) The ratio of fish to ducks at the water's edge was 5 to 2. If there were 28 in all, how many were fish?

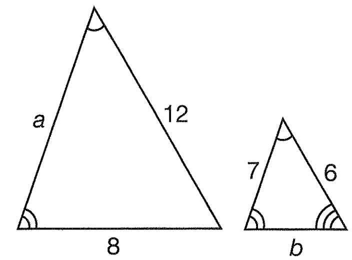
2. (58) Forty-six percent of the pigeons wore bands around one leg. If there were 81 pigeons without bands, how many pigeons were there in all?

*** 3.** (67) *Analyze* After the revision, the book had 5% more pages. If the book now has 336 pages, how many pages did the book have before the revision?

*** 4.** *(69)* **Justify** Does the table to the right show direct variation? If so, state the constant of variation. If not, state why not.

x	y
6	22
15	165
18	176

5. *(35)* Find the lengths of a and b in the similar triangles to the right. What is the ratio of the perimeter of the smaller triangle to the perimeter of the larger triangle?

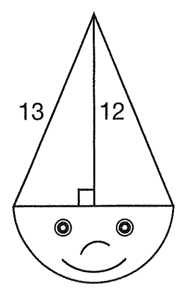


*** 6.** *(64, 72)* Gabrielle trains on her bicycle, riding at a rate of 21 miles per hour during some parts of her training. Use two multipliers to find her rate in yards per minute (1 mi = 1760 yd).

*** 7.** *(64, 72)* Amanda window shops, walking 90 feet per minute. Use unit multipliers to convert 90 feet per minute to yards per second.

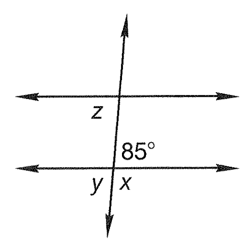
*** 8.** *(56, 69)* Graph $y = \frac{1}{2}x$. Does the graph indicate direct variation? Explain.

9. *(Inv. 2, 37)* Stephanie baked a cake according to this plan. In order to know how much batter to use, Stephanie needs to know the area of the top of the cake. Find the area of the figure. (Use 3.14 for π. Dimensions are in inches.)



10. *(63)* Write $33\frac{1}{3}\%$ as a **a** decimal and **b** reduced fraction. **c** Tyrone has read $\frac{2}{3}$ of the book. What percent remains?

11. *(54)* Two of the lines are parallel. Find x, y, and z.



12. *(32)* A coin will be flipped three times.

a. What is the sample space for the experiment?

b. What is the probability of getting heads two or more times in three flips?

c. **Predict** If a coin were flipped four times, would the probability of getting heads two or more times increase or decrease?

13. (58) What is the tax on a \$85.97 purchase if the rate is 8%?

14. (28) The distance from the earth to the moon is approximately 2.5×10^5 miles. Write that distance in standard form.

15. (21) Factor:

a. $7x^2 + 35x - 14$

b. $-3x - 15$

Simplify.

16. (27, 51) $\dfrac{8a^2b^{-1}c}{6ab^2c}$

17. (13, 33) $-\dfrac{1}{2} - \left(-\dfrac{3}{4}\right)$

18. (31) $x + y - x + 3 - 2$

19. (24, 25) $0.12 + (0.6)(0.02) + 2$

Solve.

20. (50) $2 + \dfrac{1}{3}t = \dfrac{2}{3}$

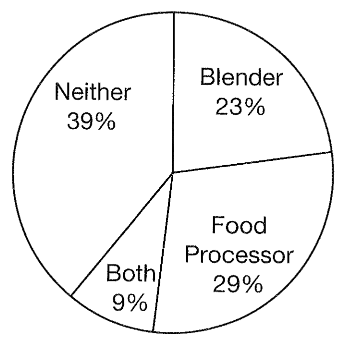
21. (50) $0.9x - 1.3 = 0.5$

22. (44) $\dfrac{x}{6} = \dfrac{6}{9}$

23. (50) $2(x + 7) - 4 = 24$

24. (62) Graph the solution on a number line: $x + 5 \geq 3$

25. (11, Inv. 6) A small appliance company surveyed 680 consumers to determine which of the appliances the consumers had used within the past month. What number of consumers had used *neither* a blender *nor* a food processor in the past month?



LESSON 73

Formulas for Sequences

Power Up

Building Power

facts Power Up O

mental math

a. **Algebra:** Evaluate: $\frac{1}{2}(8)h$ when $h = 12$

b. **Estimation:** $4\sqrt{99}$

c. **Fractional Parts:** $\frac{2}{7}$ of 42

d. **Percent:** 90% of $30

e. **Rate:** Lance drives west at 65 miles per hour. Courtney starts at the same place and drives west at 50 miles per hour. How far apart are they after 3 hours?

f. **Measurement:** How many quarts is $1\frac{1}{2}$ gallons?

g. **Geometry:** What is the specific name for an equilateral quadrilateral with right angles?

h. **Calculation:** 10 − 9, + 8, − 7, + 6, − 5, + 4, − 3, + 2, − 1

problem solving

A dentist's drill can spin at 300,000 revolutions per minute. How many revolutions per second is that? A certain carver's drill spins at 450,000 revolutions per minute. How many revolutions per second is that? In ten seconds, how many more times does the carver's drill spin than the dentist's drill?

New Concept

Increasing Knowledge

Recall that a sequence is an ordered list of numbers that follows a certain rule. In this lesson we will practice using formulas to express the rule.

Also recall that a term in a sequence has a value and a position. We distinguish between the number of the term (its position, such as 1st, 2nd, 3rd) and the value of a term (such as 3, 6, 9).

Number of Term (*n*)	1	2	3	4	. . .
Value of Term (*a*)	3	6	9	12	. . .

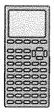
We can write a formula that shows how to find the value of a term if we know its number.

The formula $a_n = 3n$ will generate any term of the sequence in the table.

In place of n we substitute the number of the term we wish to find. Thus, to find the 7th term we substitute 7 for n in the formula.

$$a_7 = 3(7) = 21$$

Example 1



Visit www.SaxonPublishers.com/ActivitiesC3 *for a graphing calculator activity.*

Find the 8th and 9th terms of the sequence generated by the formula $a_n = 3n$.

Solution

The eighth and ninth terms correspond to $n = 8$ and $n = 9$.

$a_8 = 3(8)$ $\qquad$ $a_9 = 3(9)$

$a_8 = \mathbf{24}$ $\qquad$ $a_9 = \mathbf{27}$

When a formula is not given, be creative when looking for the rule or pattern of a sequence. The rule may include addition, subtraction, multiplication, division, powers, arithmetic between several previous terms, or other patterns that are not immediately obvious.

Example 2

Find the next two terms in each sequence.

a. **{2, 4, 8, 16, 32, . . .}**

b. **{1, 1, 2, 3, 5, 8, . . .}**

c. **{1, −2, 2, −1, 3, 0, 4, . . .}**

Solution

a. Each term is twice the previous term.

×2 ×2 ×2 ×2 ×2 ×2

2, 4, 8, 16, 32, **64, 128**

b. Each term is the sum of the two previous terms.

1+1 1+2 2+3 3+5 5+8 8+13

1, 1, 2, 3, 5, 8, **13, 21**

Thinking Skill

Analyze

Find another way of describing the pattern of example **2c.**

c. The pattern alternates between subtracting three and adding four.

−3 +4 −3 +4 −3 +4 −3 +4

1, −2, 2, −1, 3, 0, 4, **1, 5**

Example 3

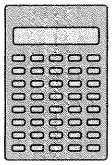
A rule for the sequence in Example 2a is: $a_n = 2^n$

Find the tenth term in the sequence.

Solution

The first term in the sequence corresponds with $n = 1$, so the tenth corresponds with $n = 10$.

$$a_{10} = 2^{10}$$



To use your calculator to compute this power, press

2 y^x 1 0 =

We find that the tenth term is **1024.**

Example 4

Find the first three terms of the sequence given by the formula

$$a_n = 1 + 2n$$

Solution

To find the first three terms we replace n with 1, 2, and 3 and simplify.

$n = 1 \qquad a_1 = 1 + 2(1) = 3$

$n = 2 \qquad a_2 = 1 + 2(2) = 5$

$n = 3 \qquad a_3 = 1 + 2(3) = 7$

The first three terms are **3, 5, and 7.**

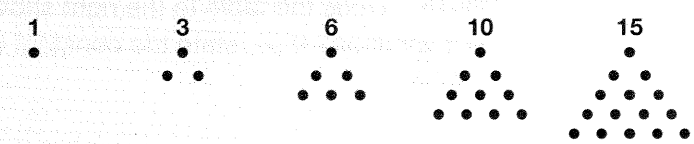
Connect One term of the sequence described in example 4 is 37. How could you use an algebraic method to find the number of this term?

Example 5

The numbers in this sequence are sometimes called triangular numbers.

$$\{1, 3, 6, 10, 15, \ldots\}$$

These numbers of objects can be arranged in triangular patterns.



Triangular numbers can be found using this formula.

$$a_n = \frac{n(n + 1)}{2}$$

Find the 10th triangular number.

Solution

Using the formula we replace n with 10 and simplify.

$$a_{10} = \frac{(10)(10 + 1)}{2}$$

$$= \frac{110}{2}$$

$$= \mathbf{55}$$

Practice Set

a. What is the 7th term of this sequence?

1, 4, 9, 16, . . .

b. The terms of the following sequence are generated with the formula $a_n = 5n - 2$. Find the tenth term.

$$3, 8, 13, 18, \ldots$$

c. Which formula below generates the terms of the following sequence?

$$0, 3, 8, 15, \ldots$$

A $a_n = n - 1$ **B** $a_n = 2n - 1$ **C** $a_n = n^2 - 1$

d. What are the first four terms in the sequence generated by the formula $a_n = n^2 + 2n + 1$?

e. **Formulate** Can you write another formula that generates the same first four terms in problem **d?** If yes, write the formula.

f. Find the 20th triangular number. (See Example 5.)

Written Practice *Strengthening Concepts*

1. (71) Renaldo increased the image size on the computer screen by 20%. By what percent did the area of the image increase?

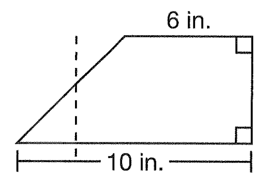
2. (48) Eighty-eight percent of the students volunteered in their community. If 110 students volunteered, how many students were there in all?

*** 3.** (67) **Analyze** The basketball regularly sold for $18. If it is on sale for 20% off, what is the new price of the basketball?

*** 4.** (69) **Evaluate** Does the table to the right show direct variation? If so, state the constant of variation.

W	*D*
4	20
7	35
9	45

5. (7) Daniel cut the trapezoid and arranged the pieces to form a rectangle. What is the length of the rectangle Daniel formed if it is equal to the average of the base lengths of the trapezoid?

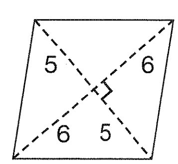


6. (52) Brianne walked 6 yards across the room in 3 seconds. Use a unit multiplier to write 6 yards in 3 seconds as feet per second.

*** 7.** (72) Aidan walked 440 feet in one minute. Find his average rate in miles per hour.

8. (56) Graph $y = -x + 5$.

9. (37) Find the area of the figure to the right. (Dimensions are in m.)



10. Write $\frac{1}{12}$ as a **a** decimal and **b** percent. **c** If $\frac{1}{12}$ of the customers pay cash, which of these statements is correct:
(63)

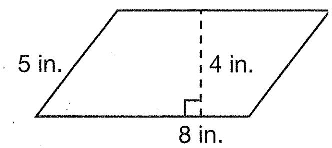
A More than one in ten pay cash

B Less than one in ten pay cash

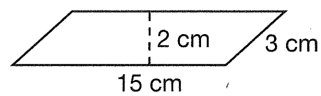
11. Factor:
(21)

a. $4x^2 + 12x - 4$ **b.** $-2x - 16$

12. Find the perimeter and area of the parallelogram.
(60)



13. Find the perimeter and area of the parallelogram.
(60)



14. The hypotenuse of a right triangle is $\sqrt{39}$ inches. The length of the hypotenuse is between:
(16)

A 3 and 4 in. **B** 4 and 5 in. **C** 5 and 6 in. **D** 6 and 7 in.

15. A company charges (c) a flat fee of \$4 for shipping plus \$0.85 per pound (p). Which equation expresses this relationship?
(41)

A $c = 4p + 0.85$ **B** $c = 0.85p + 4$

C $c = (4 + 0.85)p$ **D** $4c = 0.85p$

Simplify.

16. $\frac{24m^4b^{-1}}{18m^{-3}b^3}$
(27, 51)

17. $(-3)^1 - (-2)^2 - 4^0$
(17, 36)

Solve.

*** 18.** **Analyze** $\frac{2}{3} - \frac{1}{6}x = \frac{1}{2}$
(50)

*** 19.** $\frac{4}{5}x + \frac{1}{10} = \frac{1}{2}$
(50)

20. $\frac{4}{3} = \frac{6}{x}$
(44)

21. $\frac{3}{4}x = 12$
(38)

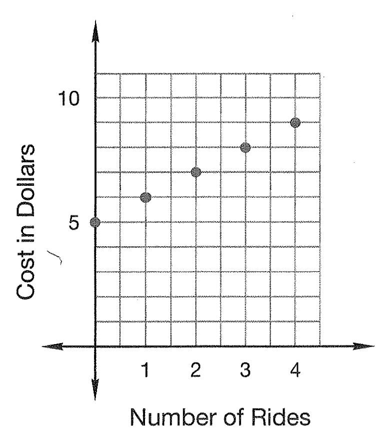
22. $0.09x + 0.9 = 2.7$
(50)

23. $3(2x - 1) = 45$
(50)

*** 24.** **Generalize** A rule for the following sequence is $a_n = 2n - 3$. Find the 20th term of the sequence.
(73)

$$-1, 1, 3, 5, \ldots$$

***25.** Describe a real-life situation illustrated by the graph. Explain why the graph looks the way it does. Is the relationship between the number of rides and the cost proportional? If the graph is not proportional, how could you change the situation to make it proportional?
(41)



LESSON 74

• Simplifying Square Roots

Power Up

Building Power

facts Power Up O

mental math

a. **Algebra:** If $2a = 10$, then a^2 equals what?

b. **Number Sense:** Arrange A, B, C, and D from least to greatest:

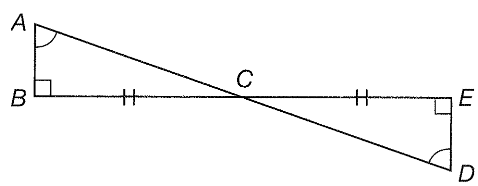
A $\frac{3}{4}$ **B** $\frac{1}{4}$ **C** $\frac{3}{4} + \frac{1}{4}$ **D** $\frac{3}{4} \times \frac{1}{4}$

c. **Powers/Roots:** $x \cdot x^5 \cdot x^3$

d. **Probability:** In a box of blocks, $\frac{3}{4}$ are rectangular. What is the probability that one taken at random will not be rectangular?

e. **Rate:** At 150 mph, how long would it take to fly 750 miles?

f. **Geometry:** Copy and complete this congruence statement: $\triangle ABC \cong \triangle D\_\,\_$



g. **Select a Method:** Howard wants to leave the waiter a 20% tip. Howard will probably

A estimate an approximate tip.
B calculate an exact tip.

h. **Calculation:** Square 5, $\times$ 2, $-$ 1, $\sqrt{}$, $\times$ 9, $+$ 1, $\sqrt{}$, $\times$ 2, $\sqrt{}$

problem solving

A cube has six faces. Depending on your perspective, you can see different numbers of faces. Draw the following views of a cube, when possible.

1. Only one face visible
2. Exactly two faces visible
3. Exactly three faces visible
4. Exactly four faces visible

New Concept

Increasing Knowledge

Math Language

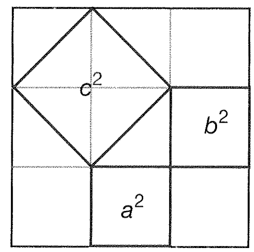
Recall that the **Pythagorean Theorem** states that the sum of the squares of the lengths of the two legs of a right triangle is equal to the square of the length of its hypotenuse.

Thinking Skill

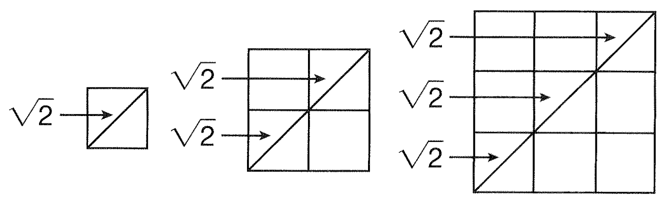
Explain

Why can we use the Pythagorean Theorem to find the length of the diagonal of a square if we know the length of the sides of the square?

We have used the Pythagorean Theorem to find that the diagonal of a 1 by 1 square is $\sqrt{2}$.



Since the area of c^2 is 2 (see the four half squares), the length of the diagonal is $\sqrt{2}$. The diagonal of a 2 by 2 square is $\sqrt{8}$, but notice that it is also 2 times $\sqrt{2}$. We simplify $\sqrt{8}$ to $2\sqrt{2}$.



Diagonal is $\sqrt{2}$. Diagonal is $\sqrt{8} = 2\sqrt{2}$. Diagonal is $\sqrt{18} = 3\sqrt{2}$.

The diagonal of a 3 by 3 square is $\sqrt{18}$, but we see it is also 3 times $\sqrt{2}$. We simplify $\sqrt{18}$ to $3\sqrt{2}$. In this lesson we will learn how to simplify some square roots.

We simplify square roots by using the **product property** of square roots, which states that square roots can be multiplied and factored:

$\sqrt{2}\sqrt{6} = \sqrt{12}$	$\sqrt{12} = \sqrt{4}\sqrt{3}$
multiplied	factored

We symbolize the product property of square roots this way:

The Product Property of Square Roots
$\sqrt{ab} = \sqrt{a}\sqrt{b}$

Above we see that $\sqrt{12}$ has a perfect square factor: $\sqrt{12} = \sqrt{4}\sqrt{3}$. Since $\sqrt{4}$ equals 2, we simplify $\sqrt{12}$ to $2\sqrt{3}$.

Step:	Justification:
$\sqrt{12} = \sqrt{4}\sqrt{3}$	Factored $\sqrt{12}$
$\sqrt{12} = 2\sqrt{3}$	Simplified $\sqrt{4}$

Similarly, we find perfect square factors in $\sqrt{8}$ and $\sqrt{18}$.

Step:	Justification:	Step:
$\sqrt{8} = \sqrt{4}\sqrt{2}$	Factored	$\sqrt{18} = \sqrt{9}\sqrt{2}$
$\sqrt{8} = 2\sqrt{2}$	Simplified	$\sqrt{18} = 3\sqrt{2}$

To help us find perfect square factors of a square root, we can write the number under the radical sign as a product of prime factors. Then we look for pairs of identical factors. Each pair is a perfect square. Below we simplify $\sqrt{600}$. First we write the prime factorization of 600.

Step:	**Justification:**
$\sqrt{600} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5}$	Factored 600
$\sqrt{600} = \sqrt{2 \cdot 2}\sqrt{5 \cdot 5}\sqrt{2 \cdot 3}$	Grouped factors under separate radical signs
$\sqrt{600} = 2 \cdot 5\sqrt{2 \cdot 3}$	Simplified perfect squares
$\sqrt{600} = 10\sqrt{6}$	Multiplied factors

Square roots can be simplified by removing perfect square factors from the radicand. If the perfect square factor is apparent, we can factor in one step.

$$\sqrt{600} = \sqrt{100} \cdot \sqrt{6} = 10\sqrt{6}$$

If the perfect square factors are not apparent, write the prime factorization and look for pairs of common factors.

Example 1

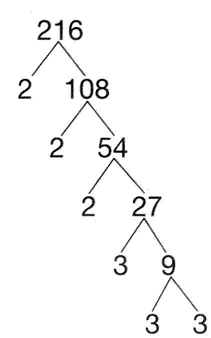
Simplify:

a. $\sqrt{216}$

b. $\sqrt{210}$

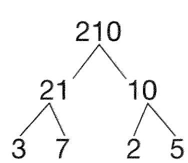
Solution

a. We can draw a factor tree to find prime factors. Then we pair identical factors and simplify.



$$\sqrt{216} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3} = \sqrt{2 \cdot 2}\sqrt{3 \cdot 3}\sqrt{2 \cdot 3} = 2 \cdot 3\sqrt{6} = \mathbf{6\sqrt{6}}$$

b. We find the prime factors. There are no pairs of identical factors.

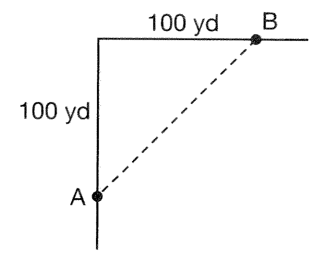


Since 210 has no identical factors, $\sqrt{\mathbf{210}}$ cannot be further simplified.

Analyze The radical expression $\sqrt{210}$ is between what two consecutive integers on the number line?

Example 2

Instead of walking 200 yards from point A to point B, Arnold takes a shortcut across a field. How long is the shortcut?



Solution

The shortcut is the hypotenuse of a right triangle with legs 100 yd long. We apply the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$100^2 + 100^2 = c^2$$
$$10{,}000 + 10{,}000 = c^2$$
$$20{,}000 = c^2$$
$$\sqrt{20{,}000} = c$$

Now we simplify the square root.

$$\sqrt{20{,}000} = \sqrt{100} \cdot \sqrt{100}\sqrt{2} = 100\sqrt{2}$$

The length of the shortcut is $\mathbf{100\sqrt{2}}$ **yd.** Since $\sqrt{2} \approx 1.41$ the shortcut is about 141 yd.

In example 2 we estimated the length of the shortcut using $\sqrt{2} \approx 1.41$. It is helpful to remember some approximate values of frequently encountered square roots to one or two decimal places.

Approximate Values of Square Roots

Square Root	Approximate Value
$\sqrt{2}$	1.41
$\sqrt{3}$	1.73
$\sqrt{5}$	2.24
$\sqrt{10}$	3.16

Practice Set

Simplify if possible.

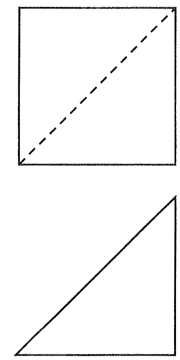
a. $\sqrt{20}$ **b.** $\sqrt{24}$ **c.** $\sqrt{27}$

d. $\sqrt{30}$ **e.** $\sqrt{125}$ **f.** $\sqrt{48}$

g. $\sqrt{50}$ **h.** $\sqrt{90}$ **i.** $\sqrt{1000}$

j. Using the numbers in the table in this lesson, calculate to the nearest tenth the values of the square roots in problems **g, h,** and **i.**

k. Jenny folded a 10-inch square piece of paper in half diagonally, making a triangle. What is the length of the longest side of the triangle?



l. *Justify* Were there any square roots in problems **a–i** that you could not simplify? Explain why or why not.

Written Practice

Strengthening Concepts

1. (45) At basketball practice, every five players shared two basketballs. If there were 45 players, how many basketballs were there?

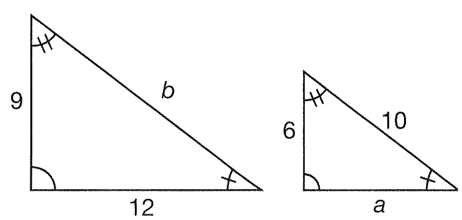
* **2.** (67) During the year student enrollment increased from 600 to 630. Student enrollment increased by what percent?

* **3.** (67) An item was regularly $90. It was marked 20% off. What was the new price?

* **4.** (69) Does the table show direct variation? If so, give the constant of variation.

x	0	1	2	3
y	0	3.1	6.2	9.3

For **5** and **6** refer to the two triangles.



5. (35) Find *a* and *b* in the similar triangles to the right.

6. (35) The dimensions of the smaller triangle are what fraction of the dimensions of the larger triangle? The area of the smaller triangle is what fraction of the area of the larger triangle?

Simplify.

7. (15) $\sqrt{2500}$

* **8.** (74) $\sqrt{18}$

* **9.** (74) $\sqrt{75}$

10. (27, 51) $\dfrac{36x^3y^{-1}}{24x^2y^2}$

* **11.** (72) With a few calculations, Kenan estimated that the tank has a capacity of about 86,400 in.^{3}. What is its capacity in cubic feet? Use three unit multipliers.

12. (56) Graph $y = -3x + 1$. Is $(2, -5)$ a solution?

13. (73) The rule of the following sequence is $a_n = n(n + 1)$. Find the 12th term of the sequence: 2, 6, 12, 20, ...

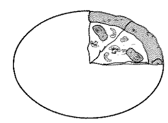
14. (63) Write $\frac{4}{15}$ **a** as a decimal and **b** as a percent. **c** Select one of these three forms to state that 16 of the 60 athletes had played in the finals in a prior season.

15. (21) Factor:

a. $6x^2 - 30x - 18$

b. $2x^3 + 2x$

16. (40) A 12-inch diameter pizza is sliced into sixths. Which is the best estimate of the area of the plate covered by two slices?



A 108 in.^{2}

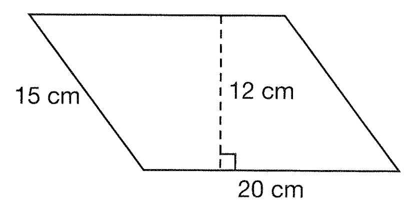
B 36 in.^{2}

C 12 in.^{2}

D 4 in.^{2}

17. (71) If the diameter of a circle is increased 40%, then its perimeter is increased 40% and its area is increased by what percent?

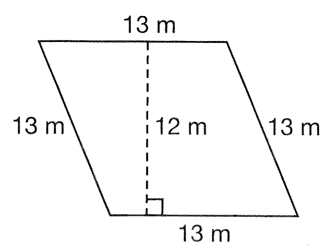
* **18.** (60) Find the perimeter and area of the parallelogram.



* **19.** (Inv. 3, 60)

a. Find the area of the parallelogram.

b. What kind of parallelogram is this?



Solve.

* **20.** (50) $\frac{3}{8} - \frac{2}{3}x = \frac{11}{12}$

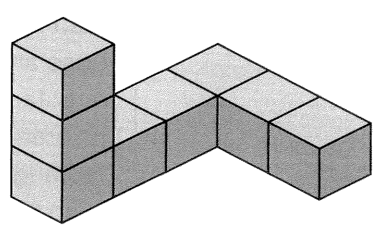
* **21.** (44) $\frac{5}{9} = \frac{x}{12}$

22. (38) $\frac{4}{3}m = -8$

23. (50) $0.07 - 0.003x = 0.1$

24. (50) $4x - x - 7 = 5$

25. (Inv. 4) Draw the front, top, and right-side views of this figure.



LESSON 75

• Area of a Trapezoid

Power Up

Building Power

facts

Power Up O

mental math

a. **Statistics:** Find the mean of these numbers: 13, 15, 15, 15, 12

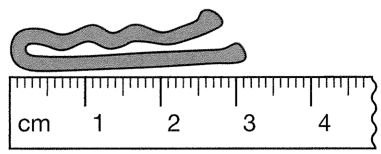
b. **Estimation:** $5 - \sqrt{3}$

c. **Fractional Parts:** $\frac{4}{5}$ of 60

d. **Percent:** 80% of $20

e. **Proportion:** $\frac{13}{x} = \frac{39}{12}$

f. **Measurement:** Find the length of this object.



g. **Scientific Notation:** Write 0.00805 in scientific notation.

h. **Calculation:** 5280 ÷ 10, − 8, ÷ 10, − 2, ÷ 10, − 5

problem solving

Find the missing reduced fraction:

$$\frac{2}{3} \times \frac{7}{4} \times \frac{9}{10} \times \frac{5}{3} \times \frac{?}{?} = 1$$

New Concept

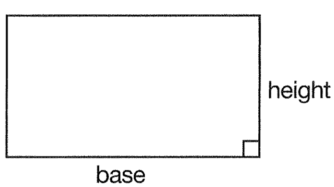
Increasing Knowledge

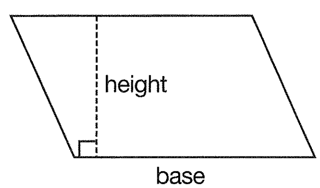
Thinking Skill

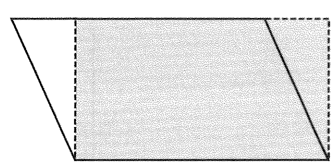
Discuss

How are parallelograms and trapezoids similar? How are they different?

Recall that we multiply the base and height of a parallelogram to find its area. The base and height are perpendicular.

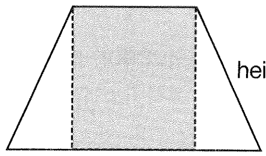


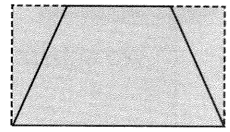




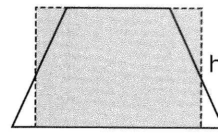
Also recall that the product of multiplying the base and height of a parallelogram is the area of a rectangle. If the parallelogram is not a rectangle, its area is nevertheless equal to the area of a rectangle with the same base and height. This is so because the portion of the parallelogram outside the rectangle matches the “hole” inside the rectangle.

To find the area of a trapezoid we can find the area of a rectangle with the equivalent area. A trapezoid has two bases, which are the parallel sides. The perpendicular distance between the bases is the height. If we multiply either one of the bases by the height the result is a rectangle that is either too small or too large.





Instead of multiplying one of the bases by the height, we multiply the **average length of the bases** by the height. The average length of the bases is the length halfway between the length of the shorter base and the length of the longer base.

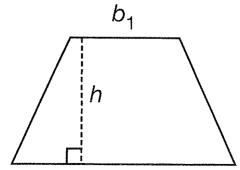


Notice that the parts of the trapezoid outside the rectangle match the "holes" inside the rectangle.

We can express this method for finding the area of a trapezoid with a formula.

Area of trapezoid = average length of the bases · height

$$A = \frac{1}{2}(b_1 + b_2) \cdot h$$

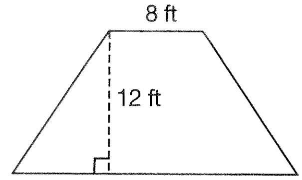


The expression $\frac{1}{2}(b_1 + b_2)$ indicates a way to find the average length of the bases.

Explain Use what you know about averages to explain why the expression $\frac{1}{2}(b_1 + b_2)$ represents the average length of the bases.

Example 1

Erik needs to replace the shingles on the south side of his roof. The section of roof is a trapezoid with the dimensions shown. How many square feet of shingles does he need to cover this section of the roof?



Solution

The parallel sides are the bases. We find the average length of the bases.

$$\text{average length of the bases} = \frac{8 \text{ ft} + 24 \text{ ft}}{2} = 16 \text{ ft}$$

Then we multiply the average of the bases by the height.

$$\begin{aligned}\text{Area of trapezoid} &= \text{average length of the bases} \cdot \text{height}\\ &= 16 \text{ ft} \cdot 12 \text{ ft}\\ &= 192 \text{ ft}^2\end{aligned}$$

The section of roof is covered with **192 ft^2** of shingles.

Example 2

The bases of a trapezoid are 12 cm and 18 cm. The height of the trapezoid is 8 cm. Find the area of the trapezoid using the formula $A = \frac{1}{2}(b_1 + b_2) \cdot h$.

Solution

We substitute the values into the equation and then we simplify.

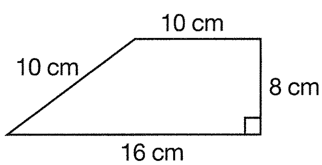
Step:	Justification:
$A = \frac{1}{2}(b_1 + b_2) \cdot h$	Area formula for trapezoid
$A = \frac{1}{2}(12 \text{ cm} + 18 \text{ cm})8 \text{ cm}$	Substituted
$A = \frac{1}{2}(30 \text{ cm})8 \text{ cm}$	Simplified within parentheses
$A = 15 \text{ cm} \cdot 8 \text{ cm}$	Multiplied $\frac{1}{2} \cdot 30$ cm
$A = \mathbf{120\ cm^2}$	Multiplied 15 cm · 8 cm

Model Make a drawing of this trapezoid with the bases and height labeled.

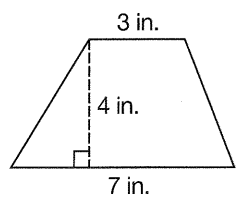
Practice Set

Find the area of each trapezoid.

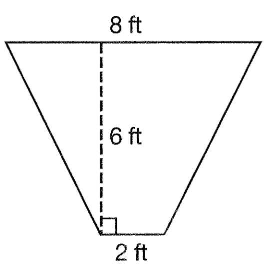
a.



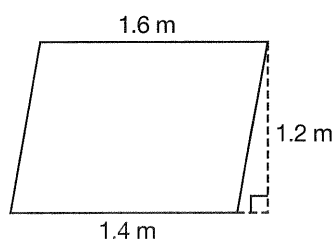
b.



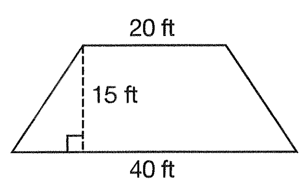
c.



d.



e. The shingles on the south side of Tamika's roof need to be replaced. The shingles she wants come in bundles that cover $33\frac{1}{3}$ square feet. Tamika calculates the area of the section of roof and then determines the number of bundles she needs. She wants to buy two extra bundles to allow for cutting and waste. What is the area of the section of roof and how many bundles should Tamika buy?



Written Practice   *Strengthening Concepts*

1. (45) Three out of four doctors surveyed recommend the product. If 200 of the doctors surveyed do not recommend it, how many doctors were surveyed?

2. (67) Sally bought a collectible at a flea market for \$120. She sold it on-line for \$156. The selling price was what percent more than her purchase price?

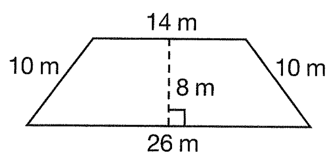
3. (48) Sixty-five percent of the tuba players had red cheeks. If there were fourteen tuba players without red cheeks, how many were there in all?

*** 4.** (69) **Analyze** If y varies directly with x, then a and b in the table equal what numbers?

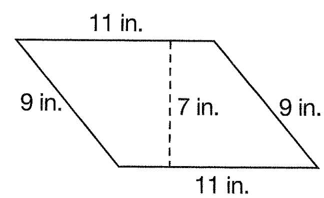
x	2	8	10	a
y	6	24	b	45

Classify For **5** and **6,** name the type of quadrilateral and find its area.

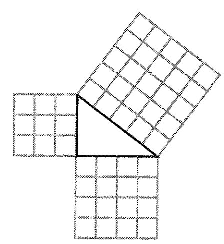
*** 5.** (74)



*** 6.** (60)



7. (Inv. 2) This figure best illustrates



A the area of a triangle.

B the perimeter of a triangle.

C the Pythagorean Theorem.

D the volume of a prism.

Solve.

8. (50) $0.04 - 0.02x = 0.5$

9. (50) $\frac{7}{3}x + 1 = \frac{17}{3}$

10. (38) $\frac{3}{4}r = 33$

11. (50) $7x + 1 - x = 19$

12. (44) $\frac{4}{x} = \frac{28}{56}$

*** 13.** (25, 44) $\frac{4}{x} = \frac{2.8}{5.6}$

Generalize Simplify.

*** 14.** (74) $\sqrt{45}$

*** 15.** (74) $\sqrt{50}$

16. (43) **Formulate** Write $\frac{8}{11}$ as a **a** percent and **b** decimal. **c** Then round the decimal to the nearest thousandth.

17. (26, 71) If a square is dilated by a scale factor of 2, then the area of the dilated square is what percent greater than the area of the original square?

18. During the season a basketball player has made 42 out of 60 free throws. What is the probability the player will make a free throw?
(59)

19. Use unit multipliers to convert 15 miles per hour to feet per second.
(72)

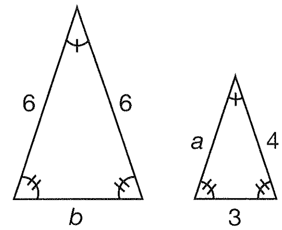
20. Graph $y = \frac{2}{3}x - 3$. Is (9, 3) a solution?
(41)

21. **a.** Find a and b.
(35)

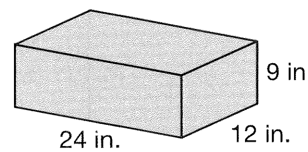
b. How do we know the triangles are similar?

c. Classify the triangles by sides.

d. What is the scale factor from the smaller to the larger triangle?



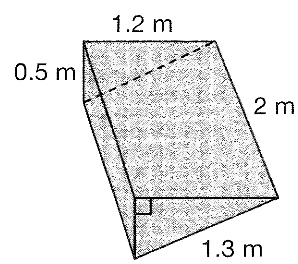
22. A cardboard carton has the given dimensions. Convert the dimensions to feet and then find the capacity of the carton in cubic feet and the surface area of the carton in square feet.
(42, 43)



23. A formula for the following sequence is $a_n = 3(n - 1)$. Find the 20th term of the sequence.
(73)

0, 3, 6, . . .

24. Sketch a net of this triangular prism.
(36)



25. Kathy has a prepaid phone card worth \$6.00. Every minute, m, she uses the phone, \$0.02 is deducted from the phone card balance, b. The graph indicates this relationship. Is this relationship an example of direct variation? If so, what is the constant of proportionality? If not, why?
(69)