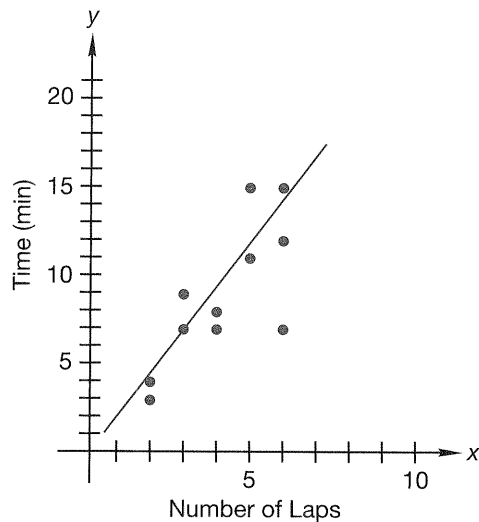Although the points do not lie on a straight line, we see that the points have a somewhat linear relationship. That is, a line can be drawn on the graph that passes near all of the points.

This means that there is a **correlation** between the number of laps completed and the total time completing laps. That is, there is a linear relationship between the quantities. As we follow the line above we see that as the number of laps increase, the total time increases.

Many lines can be drawn passing near these points. We choose a line that minimizes the distance between data points and the line; we call it a **best-fit line** or line of best fit. For now, we choose a best-fit line by visual inspection.

There is one data point that lies apart from the rest. The point (6, 7) does not seem to follow the trend of the other data points. It represents 6 laps in 7 minutes, a very fast run compared to the rest of the class. Points like these that lie away from the others are called **outliers** and are not taken into consideration when plotting a best-fit line. In this situation, the slope of the best-fit line indicates the average running pace of the students. That is, it indicates the average rate in minutes per lap.

Example 1

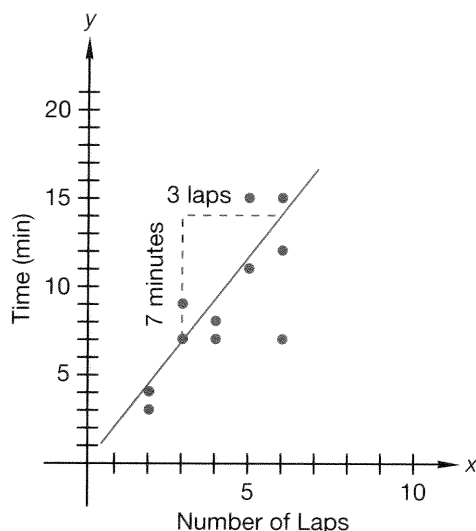**Draw a best-fit line that fits the data above. Find the equation of the line.**

Solution

Thinking Skill

Generalize

Should the best-fit line intersect the origin?

Plot the points on graph paper and use a ruler to find a best-fit line.

The slope of this line is about:

$$m = \frac{7 \text{ min}}{3 \text{ laps}} = 2\frac{1}{3} \text{ min/lap}$$

The y-intercept is zero, so with x representing the number of laps and y representing the time, the equation of the line is:

$$y = 2\frac{1}{3}x$$

This means the ratio of time to laps is about $2\frac{1}{3}$ min/lap (2:20/lap).

Predict Use the graph of the equation to predict how long it would take the students to run 7 laps.

It is probably not a surprise to us that the number of laps one completes and the elapsed time are correlated. However, some relationships are not as obvious as distance run and time. Researchers can create scatterplots to pair two sets of data in order to see if they are related or to investigate how they are related.
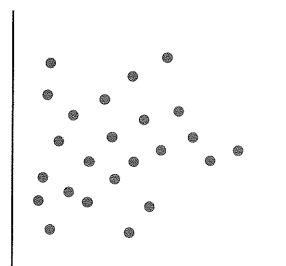
Quantities can be positively correlated (meaning the slope of the best-fit line is positive), negatively correlated (slope is negative), or not correlated (no linear relationship).
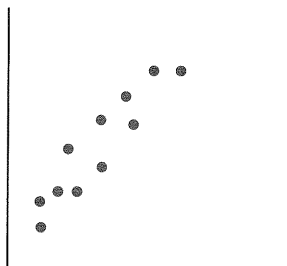
Positive correlation
as one variable increases, so does the other
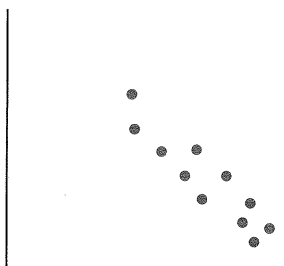
Negative correlation
as one variable increases, the other decreases

Zero correlation
no linear relationship

Example 2

Determine whether the wages of college graduates and the wages of high-school-only graduates in the listed jobs are correlated. If so, find the equation of a best-fit line.

Median Weekly Earnings, 2000

|	High School Diploma ($)	Bachelor Degree or Higher ($)
Cashier	303	384
Cook	327	396
Computer Programmer	864	1039
Data-Entry Keyers	475	514
Designers	633	794
Electricians	714	976
Police	726	886
Legal Assistants	563	725
Real Estate Sales	614	918

Solution

There is a **positive correlation** between the wages of a high school graduate and a college graduate in the given jobs.

The slope of a best fit line is $m = \dfrac{\$500}{\$400} = 1.25$

The equation of the line is $y = 1.25x$

This means that in these jobs, the wages of a college graduate are about 1.25 times the wages of a high-school-only graduate. In other words, college graduates in these jobs earn about 25% more than high-school-only graduates.
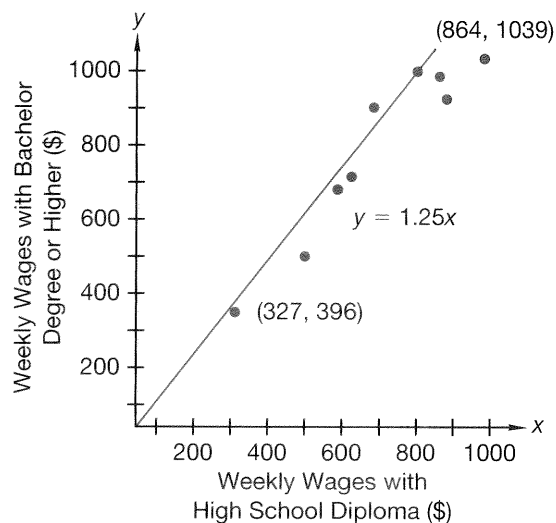
Scatterplots and best-fit lines can be easily graphed using a graphing calculator or a spreadsheet computer program. Calculators are programmed to use "least-squares fitting" to determine the line that best represents the data. Rather than multiple lines that are possible with an inspection method, *exactly one best-fit line* is computed for a given data set.

To use a graphing calculator (TI-83+) to compute the best-fit line, follow the steps below:

1. **Enter the data**

   STAT, choose "edit", then type the data points into the lists L_1 and L_2.

2. **Make a scatterplot**

   2nd StatPlot, choose "Plot1:on", scatterplot (the graph with points only), "X list:L_1", "Y list:L_2", "Mark:0".

   GRAPH If the window does not display all the points, select WINDOW, then type 0 for XMin and YMin and 1100 for XMax and YMax. Type 100 for XScl and YScl. GRAPH

3. **Compute best-fit line**

   STAT, choose "CALC," "LinReg $(ax + b)$," enter L_1, L_2, Y by pressing 2nd L_1 . 2nd L_2 . VARS, "Y Vars," "1. Function," Y_1.

   ENTER Write down the equation and numbers you see.

4. **Graph the scatterplot and best-fit line together.**

   GRAPH

Example 3

Use a graphing calculator to find the best-fit line (using least squares fitting) for the data in example 2.

Solution

Lin Reg
$y = ax + b$
$a = 1.291547346$
$b = -12.06506654$

The equation of the best-fit line is $y = ax + b$, with $a = 1.3$ and $b = -12.1$ (rounding to one decimal place).

The equation of the line is $y = 1.3x - 12.1$

Make a Scatterplot and Graph a Best-fit Line

Materials needed:

- multiplication worksheet
- stopwatch

Are the time you spend working and the number of math problems you complete positively correlated?

To study this correlation, conduct the following experiment:

5. Complete a multiplication worksheet while a timer keeps track of how long you work. Work in order—you may skip problems but not go back to complete previously skipped problems. The timer will say "time" at 30-second intervals and "stop" after 3 minutes. When the timer says "time," circle the most recently completed problem. When the timer says "stop," put down your pencil.

6. Correct your work and record data. After checking your answers, fill in the chart to indicate how many problems (in total) you completed correctly after $\frac{1}{2}$ min, 1 min, $1\frac{1}{2}$ min, etc.

7. **Model** Make a scatterplot and if the quantities are correlated, determine the equation of a best-fit line. The slope is indicative of how many problems per minute you completed.

8. Freddy ran for eight minutes wearing a heart rate monitor. He checked his pulse at every minute of his run. Freddy's heart rate at each minute is charted below:

Minutes Run	Heart Rate
1	95
2	110
3	133
4	142
5	150
6	158
7	170
8	175

Make a scatterplot of the data.

Analyze   Which axis should we use for minutes run?

If there is a correlation, then find a best-fit line by visual inspection, and state whether the correlation is positive or negative. What does the slope of the line represent? What does the intercept represent?

Explain   Do you think Freddy's heart rate would continue to increase at the same rate if he ran for 30 minutes? Explain your answer.

9. Shayna conducted a survey to find out how much money her customers are willing to pay for a new item. She charted the number of people willing to buy the item for each proposed price.

Proposed Price ($)	Number of Purchasers
1.00	10
1.50	9
2.00	6
2.50	5

Represent   Make a scatterplot of the data. Are the quantities positively or negatively correlated? Sketch a best-fit line.

• Central Angles and Arcs

**facts**

mental math

Power Up Q

a. **Statistics:** Find the mode: 13, 15, 15, 15, 12, 11

b. **Estimation:** $4 - \sqrt{10}$

c. **Fractional Parts:** $\frac{3}{5}$ of 20

d. **Measurement:** Find the temperature indicated on this thermometer.

e. **Percent:** 10% more than $70

f. **Proportion:** $\frac{9}{12} = \frac{x}{8}$

g. **Scientific Notation:** Write 41,600,000 in scientific notation.

h. **Calculation:** Three fourths of an hour is how many minutes more than two thirds of an hour?
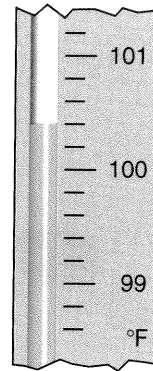
problem solving

Find the next three numbers in this sequence: 1, 1, 2, 4, 7, 13, 24, 44
(Hint: Look at the previous terms.)

New Concept | *Increasing Knowledge*

An angle whose vertex is the center of a circle is called a **central angle**. Below we see four central angles of a circle, each measuring 90°. Notice each angle intercepts $\frac{1}{4}$ of the circle, and the sum of the four angle measures is 360°.
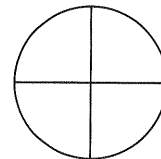
For any circle, the sum of its central angle measures is 360°.

Conclude What is the greatest number of obtuse central angles into which a circle can be divided? Explain your answer.

Example 1

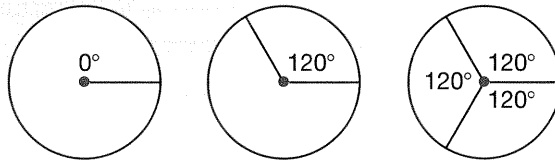**Draw a circle with central angles that divide the circle evenly into thirds.**

Solution

The three angle measures are equal and total 360°.

$$\text{angle measure} = 360° \div 3$$
$$\text{angle measure} = 120°$$

Use a compass to draw a circle. Then use a protractor as a guide to sketch a 120° angle with its vertex at the center of the circle. Rotate the paper or protractor to draw the next angle.
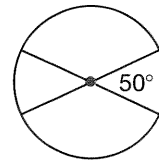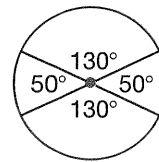

Example 2

The court of a playground game is painted on the blacktop. The court is formed by a circle with two intersecting diameters, forming four central angles. One of the central angles measures 50°. Find the measures of the other angles.

Solution

The court looks like this:

Thinking Skill

Explain

Describe another way to find the measures of the other angles in the circle without using a protractor.

In the figure, one acute and one obtuse angle together form a straight angle (180°). Thus, the other angle measures are **130°, 50°, and 130°.**
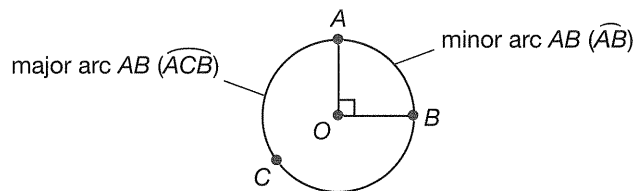

An **arc** is a portion of a circle intercepted by a central angle. One way to describe an arc is by the portion of the circle it represents. An arc that forms one half of a circle is a semicircle. An arc less than a semicircle is a minor arc. An arc greater than a semicircle is a major arc.

In the figure on the previous page, the quarter circle from A to B is arc AB (abbreviated $\overset{\frown}{AB}$) and is a minor arc. The arc from A to B through C is named major arc AB ($\overset{\frown}{AB}$) or arc ACB ($\overset{\frown}{ACB}$). One way to describe the size of an arc is by the measure of its central angle. Arc AB ($\overset{\frown}{AB}$) measures 90°.

Conclude Major arc AB ($\overset{\frown}{AB}$) measures how many degrees?

We may also refer to the length of an arc. The length of an arc is proportional to the fraction of the circumference of the circle the arc represents. For example, if the circumference of the circle above is 100 cm, then minor $\overset{\frown}{AB}$ is 25 cm long and major $\overset{\frown}{AB}$ is 75 cm long.

Example 3

What is the measure of $\overset{\frown}{AD}$ in degrees? What fraction of the circumference is the arc intercepted by $\angle ABD$? If the radius of the circle is 6 cm, what is the length of the arc in terms of π?
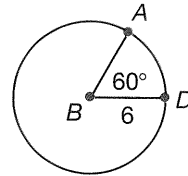
Solution

The measure of the arc equals the measure of its central angle, so $\overset{\frown}{AD}$ measures 60°. The arc is $\frac{60°}{360°} = \frac{1}{6}$ of the circle. The arc length is $\frac{1}{6}$ of the circumference.

$$C = \pi d$$
$$C = 12\pi \text{ cm}$$
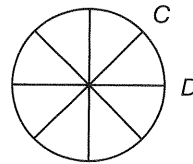$$\text{Arc} = \frac{1}{6}(12\pi \text{ cm})$$
$$= 2\pi \text{ cm}$$

Explain What is the measure of a central angle that intercepts $\frac{1}{9}$ of a circle? How do you know?

Practice Set

For **a–d** refer to the figure.

a. A circle is divided into eighths. What is the measure of each acute central angle?

b. What is the measure of arc CD?

c. What is the measure of major arc CD?

d. **Justify** If the diameter of the circle is 20 cm, what is the length of arc CD? Use 3.14 for π. Describe the steps you follow to solve the problem.

Written Practice | *Strengthening Concepts*

1. Between the starting line and finish line there was an 8% decrease in the number of runners. If there were 75 runners at the starting line, how (67) many made it to the finish line?

2. The weight of the load varies directly with the number of crates that
(70) make up the load. If a load of 15 crates weighs 2 tons, how many crates
would weigh 10 tons?

3. Draw quadrilateral *ABCD* with points *A* (2, 2), *B* (2, −2), *C* (−2, −2), and
(Inv. 5) *D* (−2, 2). Then draw its image after a dilation of scale factor 1.5.

* 4. **a.** Find the perimeter of the two squares you drew in problem **3.**
(8,
Inv. 5) **b.** Does the scale factor apply to the perimeters? Justify your answer.

c. Find the area of the two squares.

d. How does the scale factor apply to the areas?

* 5. **Evaluate** A circle with a diameter of 12 inches is divided into
(40, 81) 6 congruent sectors. Write answers to **b** and **c** in terms of π.

a. Find the measure of one of the central angles.

b. Find the length of the arc of one sector.

c. Find the area of one sector.

*** 6.** Expand: $3d(d + 4)$
(21, 36)

Generalize Simplify.

* 7. $\sqrt{50}$ *** 8.** $5\sqrt{20}$
(74)   (74)

9. $\dfrac{5x^4m^{-1}}{3mx}$ **10.** $\dfrac{(-5)(-10)}{(-5) - (-10)}$
(25, 51)   (36)

* 11. **Model** Solve and graph the solution on a number line: $x - 2x - 1 < 3$
(77)

* 12. Transform this equation to solve for *y*: $y + 5 = 2x$
(79)

13. Can a right triangle have sides of length 1, $\sqrt{3}$, and 2? Explain.
(Inv. 2)

14. Suppose a plumber charges $45 per hour. Use unit multipliers to
(64, 72) convert this rate to cents per minute.

15. The following (*x, y*) points represent the number of baked goods Tina
(69) orders (*x*) and the corresponding price in dollars (*y*).

$$(2, 5), (4, 10), (6, 15), (0, 0)$$

Does this set of points show direct variation? If so, what is the constant
of variation?

16. Write $\frac{4}{15}$ as **a** a decimal and **b** a percent. **c** Is $\frac{4}{15}$ closer to $\frac{1}{3}$ or $\frac{1}{4}$?
(63)

17. Find the area of the trapezoid.
(75)
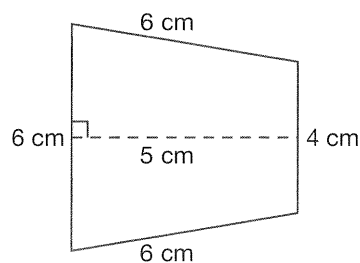
* 18. Preston's pup tent has the dimensions shown. Find the volume of the
(76) tent.

3 ft →

7 ft

4 ft
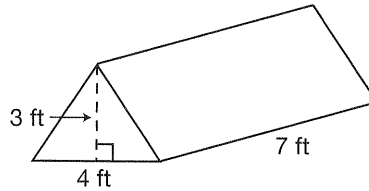
19. Find *a, b,* and *c* in the similar triangles below. (Hint: First find side *a*
(35) using the Pythagorean Theorem.)

b

16

c

a

17

15
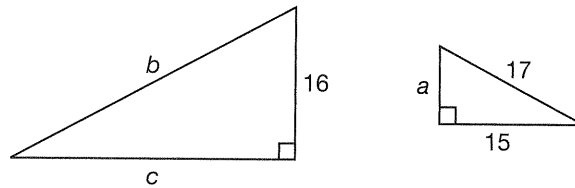
Solve.

20. $\frac{1}{3}m - \frac{2}{3} = 0$
(23, 38)

21. $3x - 5 = x$
(79)

22. **a.** Factor: $10x^2 - 15x$ **b.** Expand: $10(2x + 1.5)$
(21, 36)

23. Mrs. Ortiz is 5 ft 4 in. tall. Her four-year-old son is 3 ft $8\frac{1}{2}$ in. tall.
(80) Mrs. Ortiz is how much taller than her son?

24. Which rule below best describes the sequence 2, 5, 10, 17, ...?
(73)
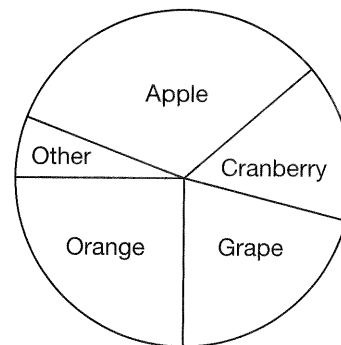
A $a_n = 2n$ **B** $a_n = n^2 + 1$ **C** $a_n = n + (n + 2)$

What is the 10th term of the sequence?

25. The circle graph shows the preferred
(Inv. 6) juice drinks among the students in
Meerca's class.

a. List the juices in order of
preference, starting with the juice
that is preferred by the most
students.

b. About what percent of the class
prefers orange juice?

Apple

Other

Cranberry

Orange   Grape

• Graphing Equations Using Intercepts

| *Building Power*

facts | Power Up Q

mental math

a. **Algebra:** Simplify: $6x - 2y - x + 2y$

b. **Estimation:** $10\sqrt{120}$

c. **Number Sense:** $5^2, 15^2, 25^2, 35^2, \ldots$ What are the last two digits of these squares?

d. **Rate:** 3000 revolutions per minute is how many revolutions per second?

e. **Powers/Roots:** $\frac{x^7}{x^2}$

f. **Geometry:** If two angles are supplementary, the sum of their measures is 180°. $\angle A$ and $\angle B$ are supplementary. $m\angle A = 100°$. Find $m\angle B$.

g. **Select a Method:** To calculate the approximate distance he can drive on a tank of gas, Gilbert will probably use which method?

  A mental math   **B** pencil and paper   **C** calculator

h. **Calculation:** $8 + 8, \sqrt{}, \times\, 12, +\, 1, \sqrt{}, \times\, 9, +\, 1, \sqrt{}, +\, 1, \sqrt{}$

problem solving | Pente gazed at the clock and wrote this sequence:

$$5, 10, 3, 8, 1, 6, 11, 4, 9, \ldots$$

What are the next three numbers in the sequence? What is the rule of the sequence?

| *Increasing Knowledge*

Previously we have graphed equations by making a table of (x, y) pairs that satisfy the equation and graphing the corresponding points. We have also graphed linear equations using the slope and y-intercept. In this lesson we will graph equations that are expressed in a form called the **standard form** of a linear equation.

Standard Form

$$Ax + By = C$$

We could transform equations in this form to slope-intercept form, but there is an easier way to graph these equations. A quick way to graph linear equations in standard form is to choose zero for each variable and then solve for the other variable. We illustrate this with the following equation:
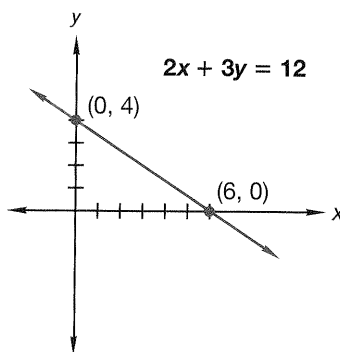
$2x + 3y = 12$

x	y
0	
	0

If we choose 0 for x, then $3y = 12$, so $y = 4$. If we choose 0 for y, then $2x = 12$, so $x = 6$.

$$2(0) + 3y = 12$$
$$3y = 12$$
$$y = 4$$
$$(0, 4)$$

x	y
0	4
6	0

$$2x + 3(0) = 12$$
$$2x = 12$$
$$x = 6$$
$$(6, 0)$$

We find that two solutions to the equation are (0, 4) and (6, 0). Notice that **by choosing 0 for x and 0 for y we have found the x- and y-intercepts of the equation's graph.** The equation is linear, so we graph all the solutions to the equation by drawing a line through these two points.

Example 1

Find the x- and y-intercepts of $3x + 4y = 24$. Then graph the equation.

Solution

The y-intercept is where $x = 0$.

$$3(0) + 4y = 24$$
$$4y = 24$$
$$y = 6$$

The y-intercept is **(0, 6).**

x	y
0	6
8	0

The x-intercept is where $y = 0$.

$$3x + 4(0) = 24$$
$$3x = 24$$
$$x = 8$$

The x-intercept is **(8, 0).**

Once we know the intercepts of a line, we can graph the line by connecting the two points.
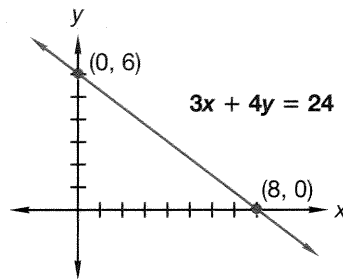
Example 2

Martin is thinking of two numbers. He gives these hints. The sum of the numbers is 3. If you double the first number and add the second number, the sum is 8. Graph this system of equations to find the two numbers.
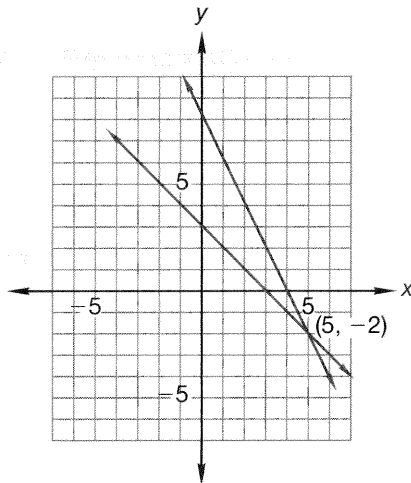
$$\begin{cases} x + y = 3 \\ 2x + y = 8 \end{cases}$$

Solution

We can graph each equation by finding the x- and y-intercepts. We can mentally calculate the value of each variable when the other is zero.

x + y = 3			2x + y = 8	
x	**y**		**x**	**y**
0	3		0	8
3	0		4	0

Now we graph both equations by drawing lines through the x- and y-intercepts for each equation.

Thinking Skill

Verify

Look back at the original problem. Do 5 and -2 satisfy the conditions described in the hints?

Every point on a line represents a pair of numbers that satisfy the equation. The point where the lines intersect represents the pair of numbers that satisfies both equations. We see that the pair of numbers that satisfies both equations is (5, −2). This means that one of Martin's numbers is **5** and the other is **−2.**

Practice Set

Formulate Find the x- and y-intercepts of the following equations, then graph them.

a. $2x + y = 8$

b. $3x − 2y = 12$

c. $−x + 3y = 6$

d. $x + 2y = −4$

e. Tanisha is thinking of two numbers. The sum of the numbers is 5. If the lesser number is subtracted from the greater number the result is 7. Graph this system of equations to find the two numbers.

$$\begin{cases} x + y = 5 \\ x − y = 7 \end{cases}$$

Written Practice   *Strengthening Concepts*

1. A book was lengthened by 8 pages for the second edition. If the book
(67) was originally 200 pages long, what was the percent increase?

2. If measured at the same moment, the length of a person's shadow varies
(71) directly with the height of the person. If a 3 foot child casts a 5 foot shadow, what is the length of the shadow of a 6 foot adult?

* 3. **Evaluate** On a coordinate plane graph two points representing the
(47) height (x) and shadow lengths (y) of the two people described in problem **2.** Then draw a line through the two points. What are the coordinates of the point on the line that corresponds to a shadow length of $2\frac{1}{2}$ ft? What do those numbers mean for this function?

4. The beans were packed in a can that had a height of 10 cm and a radius
(76) 3 cm. Sketch the cylinder and find its volume in terms of π.

For problems **5** and **6** refer to the similar triangles. Units are cm.

5. Draw the two similar triangles separately. Then find x, y, and z.
(Inv. 2, 35)
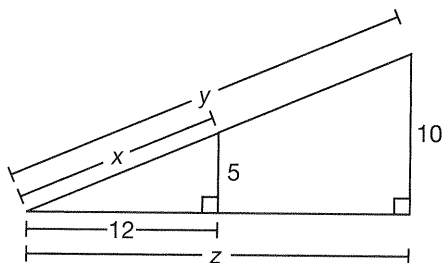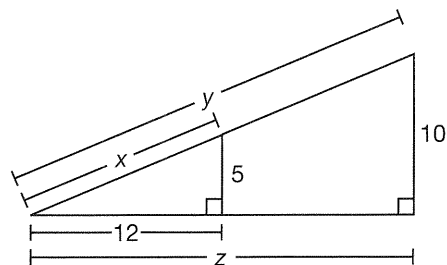
6. **a.** What is the scale factor from the smaller to larger triangle in
(20, 35)   problem **5**?

b. Find the area of each triangle.

c. The area of the larger triangle is how many times the area of the smaller triangle?

* 7. *Analyze* A circle with a diameter of 15 inches is divided into fifths.
(81)   **a** What is the degree measure of each central angle? **b** What is the length of one arc? Use 3.14 for π.

* 8. *Model* Graph the equation $2x + 3y = 6$ using the method taught in this
(82)   lesson.

9. During middle school Ting read seventy percent of the 110 books on
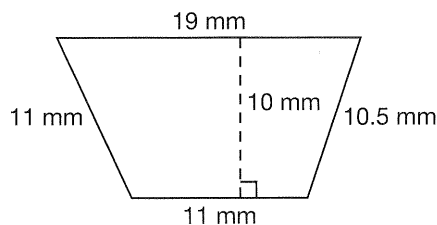(48)   the recommended list. How many of the recommended books did she read?

Generalize Simplify.

* 10. $3\sqrt{27}$ *** 11.** $\sqrt{2}\sqrt{22}$
(78)   (78)

12. $\dfrac{6xm}{12x^2}$ *** 13.** $\left(\dfrac{2}{3}\right)^2\left(\dfrac{1}{3}\right)^2\left(\dfrac{4}{9}\right)^{-2}$
(27, 51)   (23, 63)

* 14. Find the area of the trapezoid. Is the area greater than or less
(75)   than 2 cm^2?

* 15. Can a right triangle have sides of length 2, 2, and $2\sqrt{2}$? Explain.
(Inv. 2,
78)

* 16. Sam's car averages 30 miles per gallon on the highway. Traveling
(64, 72)   at 60 miles per hour, Sam's car uses how many gallons per hour?

17. Write $1\frac{1}{6}$ as a **a** decimal and **b** percent. **c** Round the decimal form to the
(63)   nearest thousandth.

* 18. Transform this formula to solve for s. $A = \pi rs$
(79)

* 19. *(32)* **Evaluate** The Abbot, Benitez, and Cheung mailboxes are side by side. A postal worker with a bundle of mail for each of the three families places a bundle in each box without checking the recipient's name.

a. List the possible arrangements of the bundles.

b. What is the probability the postal worker placed the bundles in the correct boxes?
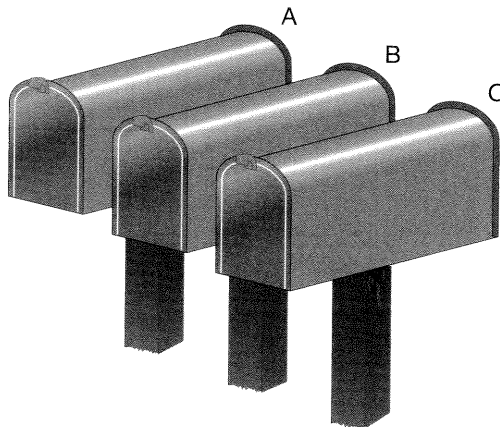
20. *(40)* Lilly plans a circular garden that she will border with pavers as in the shaded region of this figure. What is the area that the pavers will cover? (Round to the nearest whole unit.)
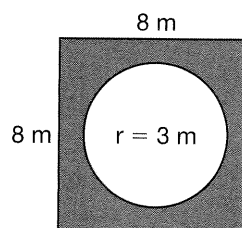
21. *(21)* Use the distributive property to factor -3 from $-3x^2 - 9x - 42$.

Solve.

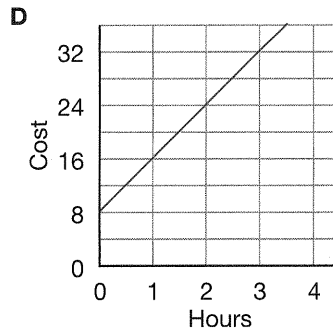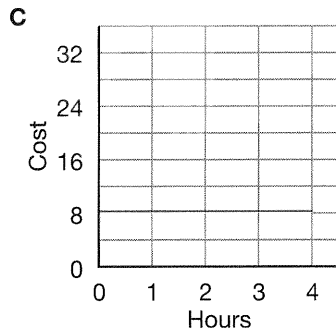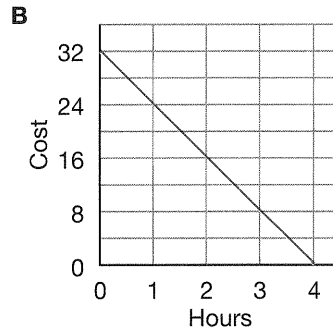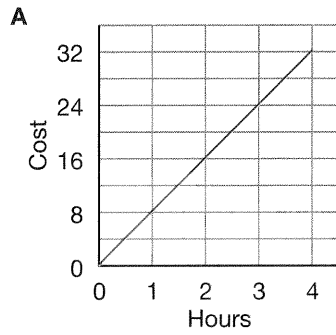**22.** *(23, 38)* $\frac{1}{2}w - \frac{2}{3} = \frac{2}{3}$

23. *(38, 79)* $0.3m = 0.4m - 0.5$

24. *(80)* Arrange each trinomial in descending order and then find their sum.

$$5 + x^2 + 2x \qquad x + x^2 - 1$$

25. Sandy's Boat Rental rents paddleboats for $8 per hour. Which graph
(69) represents the relationship of number of hours rented to the cost? Is
this a proportional relationship?

A

B

C

D

• Probability of Dependent Events
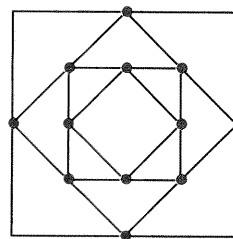
facts | Power Up Q

mental math |
a. **Algebra:** Simplify: $5x + 3x + 1$

b. **Statistics:** What is the mean of 24 and 42?

c. **Probability:** A spinner has five equal sectors labeled 1–5. Find the probability that the spinner stops on an even number.

d. **Percent:** 25% more than $80.

e. **Rate:** Micah swam laps at a rate of 2 laps per minute. How long does it take Micah to swim 5 laps?

f. **Measurement:** The odometer read 2150.5 miles at the beginning of the trip and 3876.5 miles at the end. How long was the trip?

g. **Select a Method:** To find the 7.75% sales tax on a $46.85 purchase, a retailer would probably use
A   mental math.
B   pencil and paper.
C   a calculator.

h. **Calculation:** $6 \times 11, -2, \sqrt{\ }, +1, \sqrt{\ }, \times 12, \sqrt{\ }, -7$

problem solving | The vertices of each square are the midpoints of the sides of the next larger square. If the area of the inner square is 1 cm², what are the perimeter and area of each of the four squares? Make a table listing the perimeters and areas in order. Then use the table to predict the perimeter and area of a square drawn inside the inner square.

We know that when events are independent, the probability that both will occur is the product of their probabilities.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$
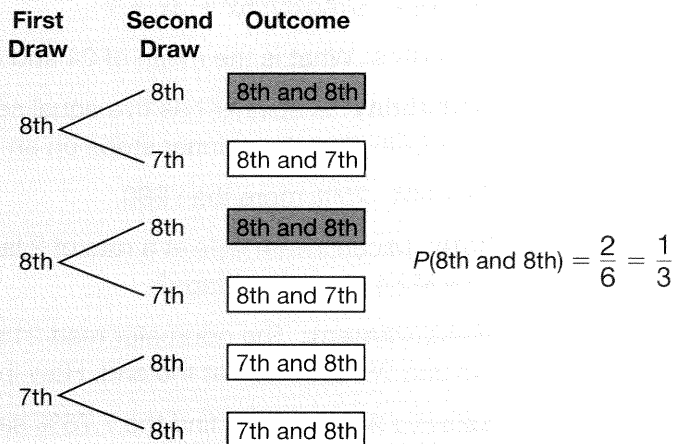
In this lesson, we will compute probabilities of events that are **not independent.** Events that are not independent are sometimes called **dependent** events because the probability of one event *depends* on the other event.

Example 1

Two eighth graders and one seventh grader want to go on the trip, but only two students will be selected. If their names are drawn at random, what is the probability that the two eighth graders will get to go?

Solution

We make a tree diagram to find the probability.

First Draw	Second Draw	Outcome
8th	8th	8th and 8th
	7th	8th and 7th
8th	8th	8th and 8th
	7th	8th and 7th
7th	8th	7th and 8th
	8th	7th and 8th

$P(\text{8th and 8th}) = \dfrac{2}{6} = \dfrac{1}{3}$

Rather than counting outcomes in example 1, we can consider probabilities at each stage of the experiment. In the first stage (the selection of the first student), $\frac{2}{3}$ of the students are eighth graders.

After an eighth grader is selected, there is a 50% chance (or probability of $\frac{1}{2}$) that the second eighth grader will be selected. The probability that two eighth graders are selected is $\frac{2}{3} \cdot \frac{1}{2}$, which is $\frac{1}{3}$.

Stage 1:

2 out of 3 are eighth graders

Stage 2:

1 out of 2 are eighth graders
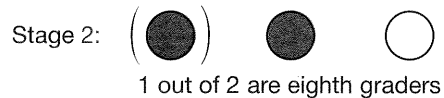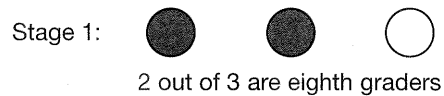
This illustrates a multiplication rule for events *A* and *B* that are **not independent.**

$$
\begin{array}{ll}
P(A) & \text{under initial conditions} \\
\times\ P(B) & \text{under new conditions} \\
\hline
P(A \text{ and } B) &
\end{array}
$$

The problem above is an example of **conditional probability.**

The multiplication rule would look like this:

$P(\text{8th and 8th}) = P(\text{eighth first draw}) \cdot P(\text{eighth second draw})$

$$= \frac{2}{3} \cdot \frac{1}{2}$$

$$= \frac{1}{3}$$

The selection of the first eighth grader and the second are not independent events. When one name is selected, the probability of selecting the next name changes.

In an experiment, two marbles are drawn from a bag with two white marbles and three blue marbles. Do you think you will be more likely to select two blue marbles if the marble is replaced after the first draw, or if the marble is not replaced after the first draw? Explain your reasoning.

This differs from independent events in which one event does not change the probability of future events. For example, consider an experiment in which a marble is selected from a bag, its color is recorded, and it is replaced. Then another marble is drawn, recorded, and replaced. The events, "blue" (for the first marble) and "blue" (for the second marble), are independent because every time a marble is selected the probability of selecting a blue marble remains $\frac{3}{5}$.

However, if a blue marble is drawn and not replaced, the conditions have changed and the probability of selecting another blue marble changes to $\frac{1}{2}$.
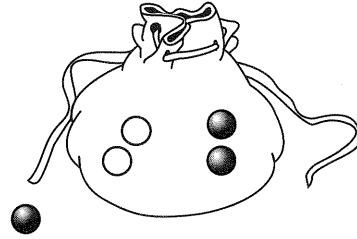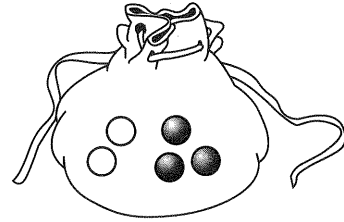
Example 2

A bag contains two white marbles and three blue marbles. If two marbles are selected, what is the probability of selecting two blue marbles if the marbles are selected a. with replacement? b. without replacement?

Solution

a. The events are independent:

$$P(\text{blue and blue}) = P(\text{blue}) \cdot P(\text{blue})$$

$$= \frac{3}{5} \cdot \frac{3}{5}$$

$$= \frac{9}{25}$$

The probability is $\frac{9}{25}$, which is 0.36.

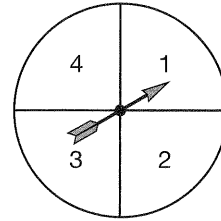**b.** The events are not independent.

$$P(\text{blue and blue}) = P(\text{blue}) \cdot P(\text{blue after one blue was selected})$$

$$= \frac{3}{5} \cdot \frac{\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}}$$

$$= \frac{3}{10}$$

The probability is $\frac{3}{10}$, which is 0.30.

Practice Set

**a.** If a spinner is spun twice, are the events independent or not independent? Justify your answer.

b. Four different cards lettered *A, B, C,* and *D* are face down on a table with the order scrambled. As a card is turned over it is left letter side up. Find these probabilities.

- Turning over the *A* card first.
- Turning over *A,* then *B.*
- Turning over *A,* then *B,* then *C.*
- Turning over *A,* then *B,* then *C,* then *D.*

c. The four letter cards in **b** are face down on the table. Michelle turns over one card and finds that it is *A.* If she turns over the remaining cards one at a time, what is the probability that she turns them over in alphabetical order?

d. Recall that the probabilities of an event and its complement total 1. In a bag are six marbles: 3 red, 2 white, and 1 blue. Three marbles are drawn one at a time without replacement. What is the probability of drawing red, then white, then blue? What is the probability of not drawing red then white, then blue?

e. *Analyze* Gerry's drawer has two black socks and four blue socks. If he selects two socks without looking, what is the probability they are both black?

Written Practice *Strengthening Concepts*

1. The number of calories of a food item varies directly with the size of the portion. If a 2-inch slice of a certain delicacy contains 170 calories, how many calories are in a 3-inch slice?
(71)

* 2. *Explain* Greg rode his bike 30 miles to the summit averaging 10 miles per hour. Then, he rode back down averaging 30 miles per hour. How far did he ride? How long did it take him? What was his average speed? Explain why Greg's average speed is not 20 miles per hour.
(49)

3. The sweater was normally $50. It was on sale for $12 off. What was the percent of discount?
(67)

* 4. *Analyze* **a.** Find the measure of the central angle *x.*
(81)

b. Find the length of the arc *AB.*
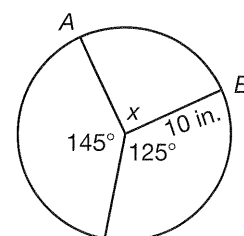
* 5.
(8, 40)
A square washer 4 cm on a side has a 2 cm diameter hole. When the washer is bolted in place, what is the area of the washer against the object to which it is bolted? Round your answer to the nearest square centimeter.

4 cm

radius = 1

←—2 cm—→
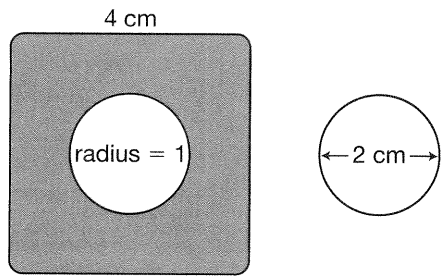
* 6.
(42, 72)
a. How many cubic feet of concrete is needed to build a walkway 36 feet long, 3 feet wide, and 3 inches thick?

b. The number of cubic feet needed equals how many cubic yards?

7.
(65)
Tim bikes at a park that has two triangular paths, as shown here. Some of the path lengths are marked. Assuming the triangles are similar, find x and y. Then find the ratio of the perimeter of the shorter path to the longer path.

x

350 m y 450 m

400 m   200 m
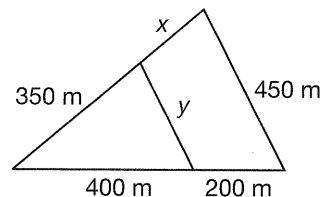
* 8.
(77)
Model Solve and graph the solution set on a number line: $5x - 7x \leq -4 - (-4)$

* 9.
(Inv. 2, 78)
Justify Can a right triangle have sides of 1, 2, and $\sqrt{5}$? Justify your answer.
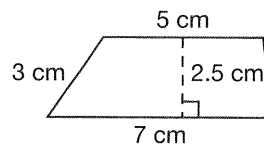
* 10.
(49)
Sue swims 400 m in 6 minutes. Which equation could be used to find how far Sue swims in 9 minutes?

A $\dfrac{400}{x} = \dfrac{6}{9}$ **B** $\dfrac{400}{9} = \dfrac{x}{6}$ **C** $\dfrac{400}{6} = \dfrac{9}{x}$ **D** $400x = 9 \cdot 6$

11.
(72)
Uriel buys a one-acre lot (4,840 yd^2). Use unit multipliers to convert this area to square feet.

* 12.
(Inv. 5)
Model Sketch $\triangle ABC$ with vertices $A(0, 0)$, $B(0, 5)$, and $C(2, 3)$. Then sketch the image of $\triangle ABC$ after a translation of $(-2, -3)$.

13.
(Inv. 3, 75)
Identify the type of quadrilateral and find the area.

5 cm

3 cm   2.5 cm

7 cm

Simplify.

* 14.
(78)
$6\sqrt{12}$

* 15.
(78)
$5\sqrt{5}\sqrt{5}$

16.
(27, 51)
$\dfrac{9x^2y^4 2m^{-1}}{2x^2y^3 9m^{-1}}$

**17.**
(36)
$\dfrac{(-15)(10)}{-15 - 10}$

* 18.
(76)
Find the capacity (volume) of a cylindrical cup with an inside diameter of 6 cm and a height of 9 cm. Use 3.14 for π.

* 19. In the metric system, small quantities of liquid are measured in milliliters
(6) (mL) or cubic centimeters (cm^3 or cc). A liter is 1000 mL or 1000 cc. The
capacity of the cup in problem **18** is about what fraction of a liter?

A $\frac{1}{2}$ **B** $\frac{1}{4}$ **C** $\frac{1}{5}$ **D** $\frac{2}{5}$

* 20. *Explain* Graph $y = -x - 1$ using slope-intercept. Then on another
(56, 82) coordinate plane graph $x + y = -1$ using the standard form
method. What do you notice about the two graphs? Explain your
observation.

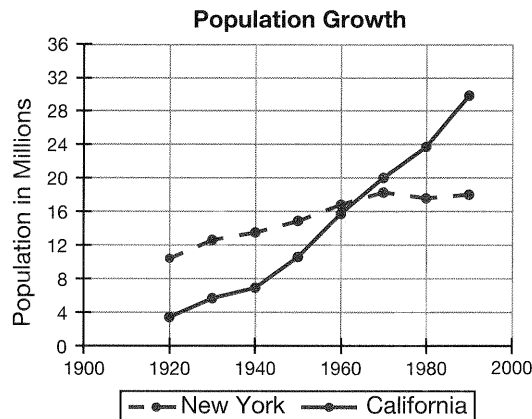*** 21.** Arc *AB* measures 100°. What is the measure
(81) of major arc *AB?*
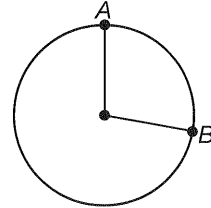
Solve.

22. $\dfrac{0.2}{0.5} = \dfrac{8}{x}$
(25, 44)

23. $\dfrac{2}{3}x = \left(\dfrac{3}{2}\right)^{-1}$
(38, 68)

* 24. Find the sum of the trinomials $2x + 3y - 4$ and $x - 3y + 7$.
(80)

25. The graph shows the growth in the populations of California and New
(Inv. 6) York from 1920 to 1990.

Population Growth

a. About how much did the population of California increase between
1980 and 1990?

b. In which decade were the populations of California and New York
about the same?

A 1920–1930

B 1940–1950

C 1960–1970

D 1970–1980

• Selecting an Appropriate Rational Number

facts

mental math

Power Up Q

a. **Statistics:** Find the range of this data: 13, 15, 15, 15, 12

b. **Estimation:** $2 \times \$1.47 + \$12.97$

c. **Number Sense:** The numbers 2^2, 12^2, 22^2, 32^2, etc., all have the same digit in the ones place. What is that digit?

d. **Powers/Roots:** $\frac{x^4}{x^3}$

e. **Proportion:** $\frac{7}{8} = \frac{56}{x}$

f. **Geometry:** Angle P and $\angle R$ are supplementary. The measure of $\angle P$ is 50°. Find $m\angle R$.

g. **Scientific Notation:** Write 4.2×10^{-1} in standard notation.

h. **Calculation:** Two dozen $\div$ 3, $\times$ 4, $+$ 1, $\div$ 3, $\times$ 5, $+$ 1, $\div$ 7

problem solving

A rectangle has vertices at (5, 7), (5, −1), (1, 7), and (1, −1). How many units and in which directions should the rectangle be translated (moved) so that the center of the rectangle is at the origin?

Consider how you would calculate the answers to these two questions.

• The tax rate is 7%. How much money is 7% of $12.60?

• The discount is $33\frac{1}{3}\%$. How much money is $33\frac{1}{3}\%$ of $12.60?

To find a percent of a number, we usually convert the percent to a decimal or fraction before we multiply. Sometimes we choose to use the decimal form and sometimes we choose to use the fraction form. The better choice depends upon the situation. To answer the two questions above, many people would convert 7% to a decimal because it is relatively easy to multiply by 0.07. Since $33\frac{1}{3}\%$ is a repeating decimal, many people would convert it to the fraction $\frac{1}{3}$ to perform the calculation.

Example 1

a. **How much is 40% of $\frac{1}{2}$ of $12.60?**

b. **How much is $66\frac{2}{3}\%$ of $\frac{5}{6}$ of $12.60?**

a. Both 40% and $\frac{1}{2}$ easily convert to decimal form. We rewrite the numbers as decimals and perform the calculations.

Step:	Justification:
40% of $\frac{1}{2}$ of $12.60	Given
$0.4 \times 0.5 \times \$12.60$	Converted 40% and $\frac{1}{2}$ to decimals
$0.2 \times \$12.60$	Multiplied 0.4 and 0.5
$2.52	Multiplied

b. Both $66\frac{2}{3}\%$ and $\frac{5}{6}$ convert to repeating decimals. We choose to convert the percent to a fraction before performing the calculations.

Step:	Justification:
$66\frac{2}{3}\%$ of $\frac{5}{6}$ of $12.60	Given
$\overset{1}{\cancel{\frac{2}{3}}} \times \frac{5}{\underset{3}{\cancel{6}}} \times \$12.60$	Convert to fraction
$\frac{5}{\underset{1}{\cancel{9}}} \times \$\overset{1.40}{\cancel{12.60}}$	Multiplied $\frac{2}{3}$ and $\frac{5}{6}$
$7.00	Multiplied

Example 2

In a public opinion survey, 180 of the 280 women surveyed agree or strongly agree with the mayor, while 150 of the 275 men surveyed agree or strongly agree with the mayor. Which choice listed below is the most appropriate way to describe the results of the survey?

A $\frac{9}{14}$ **of the women and** $\frac{6}{11}$ **of the men agree with the mayor.**

B 0.643 of the women and 0.545 of the men agree with the mayor.

C 64% of the women and 55% of the men agree with the mayor.

Solution

Thinking Skill

Discuss

The question says that 180 of the 280 women and 150 of the 275 men surveyed "agree or strongly agree" with the mayor. Each answer choice only says "agree." Why might we combine these two categories?

When stating comparisons it is appropriate to use forms of numbers that are easy to compare and are readily understood.

- Choice A uses two fractions with different denominators, which makes it difficult to compare.

- Choice B states the comparison with decimals, which is a less common way to express a comparison.

- **Choice C** expresses the comparison to the nearest percent. The numbers are easy to compare and understand.

Practice Set

a. Find 20% of $\frac{1}{4}$ of $12.00.

b. Find $33\frac{1}{3}$% of $\frac{3}{8}$ of $12.00.

c. *Explain* To find 80% of $21.50, would you convert 80% to a fraction or a decimal? Why?

d. *Explain* To find 25% of $24, would you convert 25% to a fraction or decimal? Why?

e. Which expression is the most appropriate rational number to announce a discount of $14 from a regular price of $40?

  A save $\dfrac{7}{20}$ **B** save 0.35   **C** save 35%

---

Written Practice   *Strengthening Concepts*

1.
(45) At a certain store, there are about 11 CD's sold for every 4 cassettes. If the store sells 270 CD's and cassettes in all, about how many are CD's?

2.
(48) Irina made 85% of her free-throw attempts. If she missed only six free throws, how many did she attempt?

3.
(67) Robert saved 15% on his backpack. If he paid $17, what was the original price?

* 4.
(81) *Analyze* **a.** How many degrees is arc *RS*?

  b. Find the length of the arc *RS*.
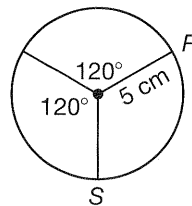
* 5.
(71) If the diameter of a circle is reduced 20%, then its area is reduced by what percent?

6.
(65) Refer to the triangles to the right to answer parts **a–c**.

  a. How do we know the triangles are similar?

  b. Find *x* and *y*.

  c. What is the scale factor from $\triangle DEF$ to $\triangle ABC$?
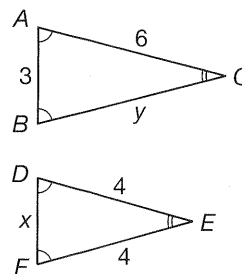
* 7.
(77) This number line shows the solution set to which of these inequalities?

  A $x > 3$ **B** $-x < 3$ **C** $x + 1 \geq 2$ **D** $1 - x \leq -2$

* 8.
(80) *Classify* Classify the following polynomial by the number of terms and by degree. Then arrange the terms in descending order.

$$5 + x^2 - 2x$$

* 9. **Analyze** Find the surface area of a 1-yard cube in square yards and in
(43, 52) square feet.

Generalize Simplify.

10. $\sqrt{6}\sqrt{8}$
(77)

11. $3x + 2x^2 + x^2 - x$
(31)

12. $\dfrac{4x^2y^{-1}2x^3}{6x^4y^2}$
(27, 51)

* 13. 50% of $\dfrac{2}{3}$ of $1.20
(84)

A triangle has vertices at (0, 0), (3, 3), and (−4, 4). Use this information in
problems **14** and **15.**

* 14. **a.** Classify the triangle by angles.
(Inv. 2, 20)
b. What is the area of the triangle?

* 15. **a.** What is the length of the longest side of the triangle?
(Inv. 2, 74)
b. What is the perimeter of the triangle?

16. Write $1\frac{1}{3}$ as **a** a decimal and **b** a percent.
(63)

17. When U.S. coins were silver the weight of the coin was proportional to
(29) its value.

a. What is the ratio of a quarter to a dime?

b. What percent of a quarter is a dime?

18. A piece has been cut from a sheet
(75) of construction paper. The remaining
portion covers how many square
inches?

12 in.

12 in.

17 in.

13 in.
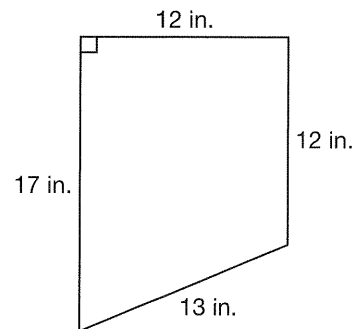
19. Graph $y = x$ and graph $x + y = 0$.
(56, 82)

20. The rule of a sequence is $a_n = 2n^2 - 1$. What is the 10th term of the
(73) sequence?

Solve.

21. $\dfrac{x}{5} = \dfrac{0.42}{0.3}$
(25, 44)

22. $3x + 2 = x + 14$
(79)

23. $\dfrac{3}{4}x = \left(\dfrac{4}{3}\right)^{-2}$
(38, 63)

24. $-\dfrac{x}{5} = 10$
(23, 38)

25. The paint store uses the amounts shown on the table to make a gallon
(69) of a certain shade of paint. Use the data in the table to make a graph. Is
the relationship between the amount of yellow paint and the amount of
red paint per gallon proportional? If it is a proportional relationship, write
an equation that describes the relationship and state the constant of
proportionality. (see below.)

	Parts Red Paint (x)	Parts Yellow Paint (y)
1 gal	3	5
2 gal	6	10
3 gal	9	15
4 gal	12	20

Early Finishers
Real-World Application

The Marquez family has four children born in different years.

a. List all possible permutations of boy and girl children that the Marquez
family could have.

b. Find the probability that the Marquez family has three boys and one girl in
any order.

# • Surface Area of Cylinders and Prisms

facts   Power Up Q

mental   **a. Algebra:** Simplify: $5x - 6x + 3$
math
              b. Geometry: If a right triangle is isosceles, then each acute angle measures how many degrees?

              c. Fractional Parts: $\frac{4}{7}$ of 49

              d. Rate: Kurt and Dash clear debris from fields. If Kurt cleans 1.5 acres per week and Dash cleans 1 acre per week, how much can they clean together in 4 weeks?

              e. Measurement: Find the length of this object.
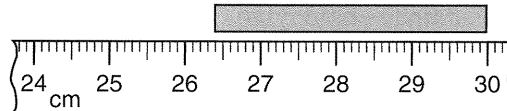
              f. Percent: 50% more than $90.

              g. Select a method: While grocery shopping, Reagan keeps track of the total price of the items in the basket. Reagan probably uses
              A mental calculation.
              B mental estimation.
              C pencil and paper.
              D a calculator.

              h. Calculation: $100 \div 5, + 5, \div 5, + 5, \div 5, + 5$

problem   There are 3 numbers whose sum is 77. The second number is twice the first,
solving   and the third number is twice the second. What is the second number?
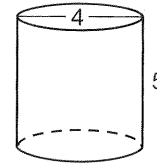
To make a soup can, the manufacturer assembles two circles and one rectangle of metal in the shape of a cylinder.
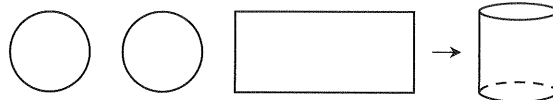
The area of the material used on one soup can represents the **surface area** of the cylinder. We compute the surface area of an object by adding the areas of all its faces or surfaces.
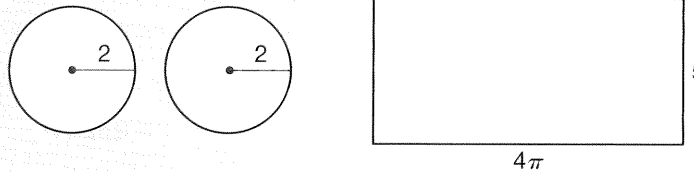
Example 1

Compute the surface area of this cylinder in terms of π.

Solution

We find the area of each face or surface of the cylinder. The curved surface of the cylinder is equivalent to a rectangle with a length equal to the circumference of the circular bases, which is $2\pi r$, or 4π units. The rectangle's height is equal to the height of the cylinder (5 units)

Thinking Skill

Analyze

What parts of the cylinder does 4π represent in the expression? What does 20π represent?

Surface Area $= A_1 + A_2 + A_3$

Surface Area $= 4\pi + 4\pi + 20\pi$

Surface Area $= \textbf{28}\pi$ **square units**

Example 2

A tent in the shape of a triangular prism is shown. What is the surface area of the tent in square feet?
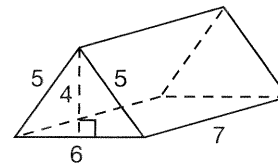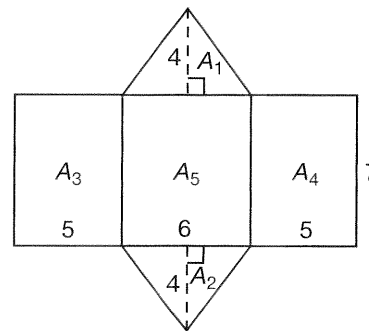

Solution

The triangular prism has five faces.

$$A = A_1 + A_2 + A_3 + A_4 + A_5$$
$$A = 12 + 12 + 35 + 35 + 42$$
$$A = 136$$

The tent is made with **136 ft^2** of material.

The **lateral surface area** of a prism is the area of the faces between its bases. The lateral surface area of a cylinder is the area of the surface between its bases. For example, the lateral surface area of a soup can is represented by the label but not by the circular top and bottom of the can. A quick way to find the lateral surface area of a cylinder or prism is to multiply the *perimeter* of the base by the *height* of the cylinder or prism.

Example 3

Hector wants to paint the exterior walls of a
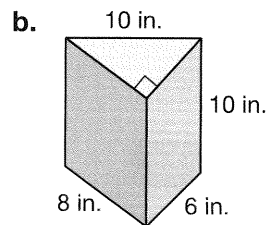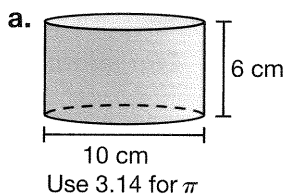shed. Find the lateral surface area of the shed.

7 ft

8 ft
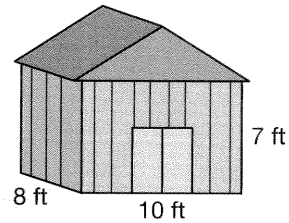
10 ft

Solution

Instead of finding and adding the areas of the four walls, we multiply the
perimeter of the base by the height.

$$\text{lateral surface area} = \text{perimeter of base} \cdot \text{height}$$
$$= (8 + 10 + 8 + 10) \text{ ft} \cdot 7 \text{ ft}$$
$$= 36 \text{ ft} \cdot 7 \text{ ft}$$
$$= \textbf{252 ft}^2$$

Practice Set

Find the total surface area of these figures.

a.

6 cm

10 cm
Use 3.14 for π

b.

10 in.

10 in.

8 in.   6 in.

c. What is the lateral surface area of the cylinder in problem **a?**

d. What is the lateral surface area of the triangular prism in **b?**

e. _Analyze_ Find the lateral surface area of a
30-ft-high water tower with a 40 ft diameter.
Use 3.14 for π.

30 ft

40 ft
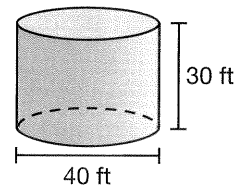
Written Practice   _Strengthening Concepts_

1. The ratio of cyclists to pedestrians on a certain trail was 2 to 7. If there
(45)   were 27 people on the trail, how many more pedestrians were there than
        cyclists?

* 2. _Connect_ A jogger ran three miles down the trail then turned around and
(72)   ran back, completing the roundtrip in 45 minutes.

    a. The jogger averaged how many minutes per mile?

    b. The jogger's average rate was how many miles per hour?

3. The traffic to the website increased $16\frac{2}{3}$% during the month. If there
(67, 84) were 3,000,000 visitors per day at the beginning of the month, how many were there by the end of the month? What rational number did you use to help you find the answer? Why did you use that number?

* 4. **Analyze** A circle is divided into ten equal sectors.
(81)
a. Find the measure of each central angle.

**b.** If the diameter is 50 in., find the length of an arc of one of the sectors.

5. The thickness of a dime is about one millimeter, which is 0.001 meters.
(51) Write 0.001 in scientific notation.

6. Refer to the similar triangles to the right to
(65) answer parts **a–c.**

a. Find x and y in the similar triangles.

b. Which angle has the same measure as $\angle C$?

c. What is the scale factor from the smaller to larger triangle?
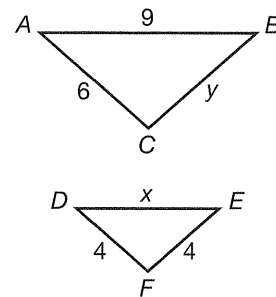
* 7. **Justify** Will a 9 inch-by-13 inch picture
(Inv. 2) frame fit completely inside a box of this size? Justify your answer.
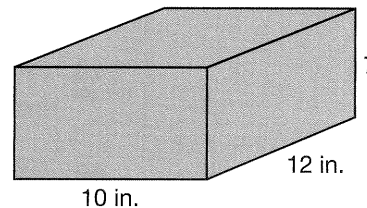
* 8. A rectangle's length is $(x + 4)$ and width is $(x + 3)$. By adding binomials
(80) give its perimeter.

9. Solve the inequality and graph the solution on a number line:
(77)
$$-2x + 1 > 1$$

* 10. The label of a can represents the lateral surface area of a cylinder. What
(85) is the lateral surface area of a can of beans with a diameter of 7 cm and a height of 11 cm? (Use $\frac{22}{7}$ for π.)

11. Expand $-7x(5 - x)$ and arrange the terms of the product in descending
(36, 80) order.

12. There are 2 red marbles and 3 green marbles in a bag. If two marbles
(83) are drawn at the same time from the bag, what is the probability both will be green?

Generalize Simplify.

* 13. $\sqrt{500}$
(74)

* 14. $2\sqrt{5}\sqrt{20}$
(78)

* 15. Ming is playing a board game with a red number cube and a green
(59) number cube. She needs to roll a sum of 10 or more with the two
number cubes to win on this turn.

    a. Of the 36 possible combinations, how many result in a sum of 10 or
more? List them.

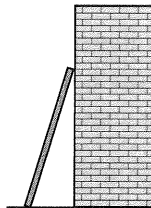   **b.** What is the probability she will win on this turn?

16. **Evaluate** A 10-foot ladder leans against a building. The foot of the
(Inv. 2) ladder is 3 feet from the building. Which choice below best indicates
how far up the building the ladder reaches?

    A Between 7 and 8 feet   **B** Between 8 and 9 feet

    C Between 9 and 10 feet   **D** Between 10 and 11 feet

17. Laura walked the 440 feet from history class to math class in
(64, 72) 2 minutes. Use unit multipliers to convert 440 feet in 2 minutes to
miles per hour.

* 18. Graph $y = \frac{1}{2}x + 2$. Then graph $x + y = 5$. What are the coordinates of
(56, 82) the point at which the graphed lines intersect?

19. **Explain** One wall of an attic room is shown. The window is 4 feet wide
(75) and 3 feet high. Fernando wants to paint the wall. Find the area of the
wall to be painted. Explain how you found your answer.
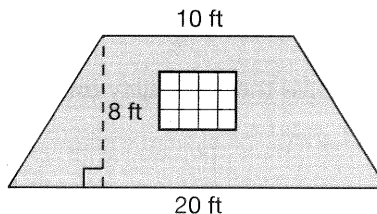
20. Use your ruler to find the scale of the drawing in problem **19.** If the
(35) length of the room is 25 feet, which would be the length of the room in a
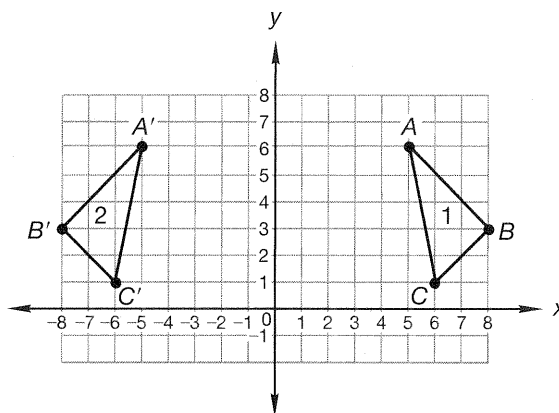drawing at this scale?

Solve.

21. $\dfrac{x}{3} = \dfrac{1.5}{0.5}$
(25, 44)

22. $\dfrac{2}{3}x + 2 = \left(\dfrac{1}{3}\right)^{-1}$
(38, 63)

23. $0.2x - 0.02x = 0.018$
(38, 50)

24. $2x^2 - 2 = 48$
(35, 50)

25. Describe the transformation(s) needed to align figure 1 with figure 2.
(Inv. 5)

Sherman just purchased a dance hall. One of the hall's badly worn dance surfaces is shaped like an isosceles trapezoid. The trapezoid's bases measure 40 feet and 60 feet, and its height measures 30 feet. If hardwood flooring costs $2.25 per square foot, what will it cost Sherman to install new floors on four such dance surfaces?

• Volume of Pyramids and Cones

facts | Power Up R

mental math

a. **Algebra:** If x is 10, then what is the product of $x + 1$ and $x - 1$?

b. **Number Sense:** The perfect squares 9^2, 19^2, 29^2, 39^2, etc. all have what digit in the one's place?

c. **Ratio:** The ratio of statues to live creatures was 6 to 7. If there were 39 in all, how many were statues?

d. **Number Sense:** The sum of two numbers is 8. Their product is 12. What are the two numbers?

e. **Probability:** The probability that his name will be drawn is $\frac{1}{1000}$. Find the probability that his name will not be drawn.

f. **Geometry:** Angle X and $\angle Y$ are supplementary. If $m\angle Y$ is 90°, find $m\angle X$.

g. **Scientific notation:** Light travels at about 300,000 km/sec. Express this number in scientific notation.

h. **Calculation:** 7×20, $+ 4$, $\sqrt{}$, $+ 4$, $\sqrt{}$, $+ 4$
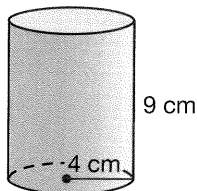
problem solving | A shoe manufacturer loads boxes of shoes onto pallets and ships them to a retailer. Each pallet can hold a stack of shoeboxes 3 ft high, 3 ft wide, and 4 ft long. If each shoebox is 12 inches long, 6 inches wide, and 6 inches high, how many shoeboxes fit on each pallet?
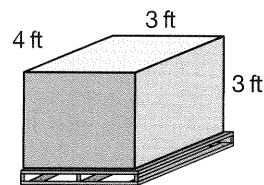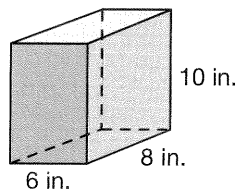
We know how to compute the volume of cylinders and prisms.

The volume of this cylinder is:
$V = B \cdot h$
$V = \pi(4 \text{ cm})^2 \cdot 9 \text{ cm}$
$\quad = 144\pi \text{ cm}^3$

The volume of this prism is:
$V = B \cdot h$
$V = (6 \text{ in.} \cdot 8 \text{ in.}) \cdot 10 \text{ in.}$
$\quad = 480 \text{ in.}^3$

In this lesson we will learn the formulas to calculate the volumes of cones and pyramids, beginning with the formula for the volume of a cone.