

The slope-intercept equation of the line graphed on the previous page is $y = \frac{1}{2}x - 1$.

$$y = \frac{1}{2}x - 1$$

The slope is $\frac{1}{2}$. The y-intercept is -1 .

Many books show the slope-intercept form this way:

Slope-intercept equation

$$y = mx + b$$

The number for m is the slope.

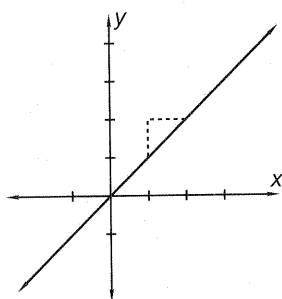
The number for b is the y-intercept.

Consider the following equations and their graphs.

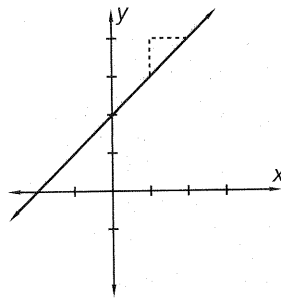
Thinking Skill

Analyze

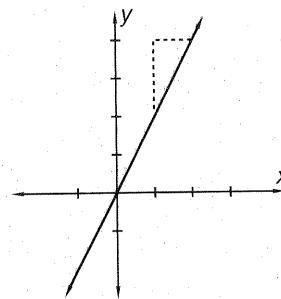
What is the value of b in the following equation: $y = 3x$.



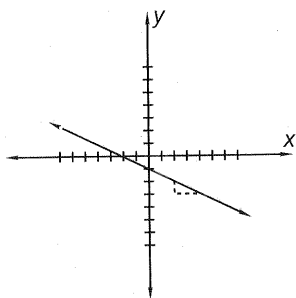
- Slope is 1
 - y-intercept is zero
- $y = x$



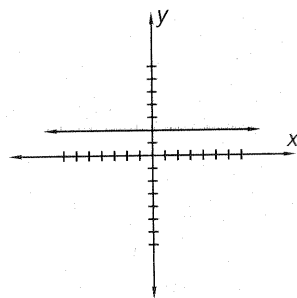
- Slope is 1
 - y-intercept is +2
- $y = x + 2$



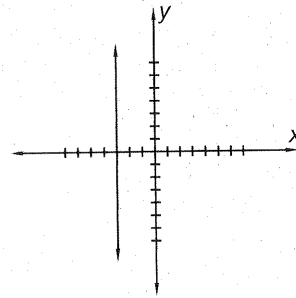
- Slope is 2
 - y-intercept is zero
- $y = 2x$



- Slope is $-\frac{1}{2}$
 - y-intercept is -1
- $y = -\frac{1}{2}x - 1$



- Slope is zero
 - Intersects y-axis at +2
- $y = 2$



- Slope is undefined
 - Every point has x-coordinate -3
- $x = -3$

Notice that a horizontal line has zero slope and can be expressed with slope-intercept form, $y = 0x + 2$, which simplifies to $y = 2$. A vertical line cannot be expressed in slope-intercept form.

Generalize Can the slope of a vertical line be determined? Why or why not?

Example 1

Refer to this equation to answer the questions that follow.

$$y = \frac{2}{3}x - 4$$

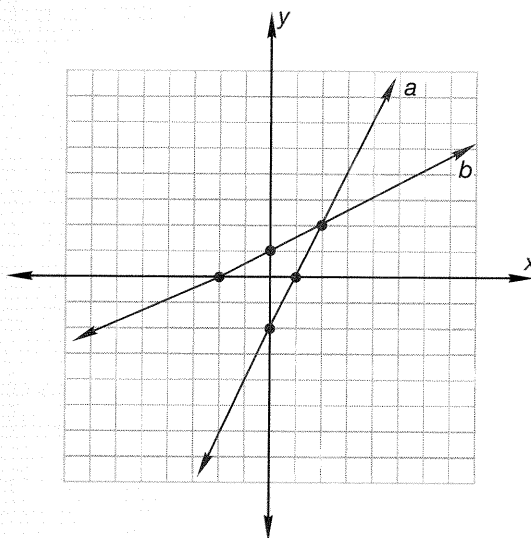
- Where does the graph of the equation cross the y -axis?
- Does the line rise to the right or fall to the right?

Solution

- The graph of the equation crosses (intercepts) the y -axis at -4 , which is 4 units down from the origin.
- The line **rises to the right** because the slope is $\frac{2}{3}$ which is positive.

Example 2

Write the equations of lines a and b in slope-intercept form. At what point do lines a and b intersect?



Solution

Line a has a slope of 2 and intercepts the y -axis at -2 .

$$y = 2x - 2$$

Line b has a slope of $\frac{1}{2}$ and intercepts the y -axis at $+1$.

$$y = \frac{1}{2}x + 1$$

Lines a and b intersect at **(2, 2)**.

Justify Prove that these lines intersect at (2, 2) by substituting for x in both equations and solving for y .

Example 3

Graph each equation using the given slope and y -intercept.

a. $y = 2x - 3$

b. $y = -\frac{1}{2}x + 2$

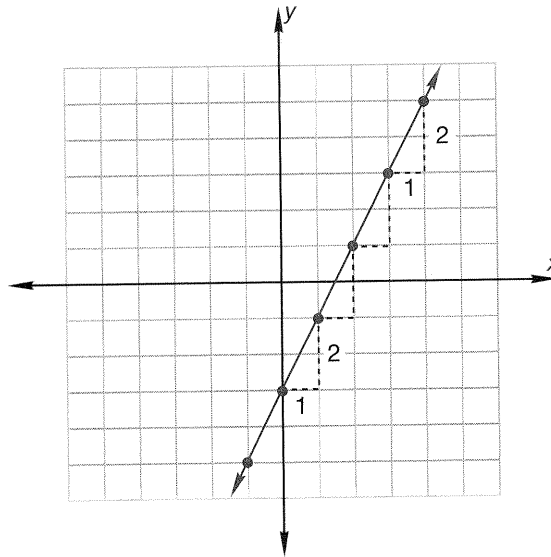
Solution

a. We study the equation to understand what the numbers mean.

$$y = 2x - 3$$

This number means the slope is positive 2 (over 1, up 2). ↑ ↑ This means the line intersects the y-axis at -3.

We start by graphing the point of the y-intercept. From there we graph additional points by going over 1 and up 2. The direction of “over” is to the right. Then we draw a line through the points.

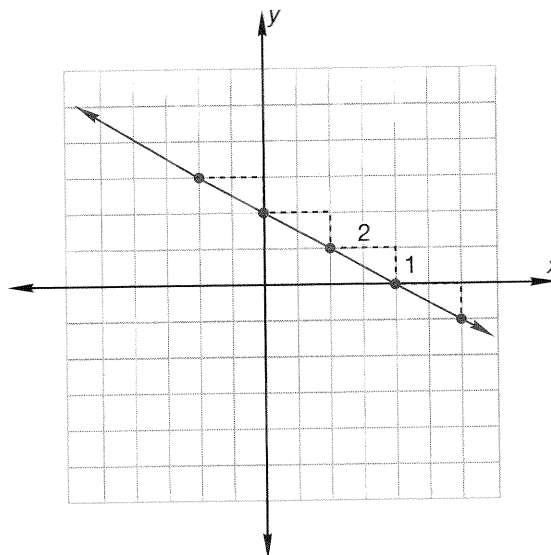


b. If the slope is a fraction, the denominator represents the “over” number and the numerator represents the “up or down” number.

$$y = -\frac{1}{2}x + 2$$

The slope is $-\frac{1}{2}$ (over 2, down 1). ↑ ↑ y-intercept is at +2.

We start at +2 on the y-axis and count over 2, then down 1.



Practice Set

a. **Analyze** Which of the following equations is written in slope intercept form?

A $x = 2y + 3$

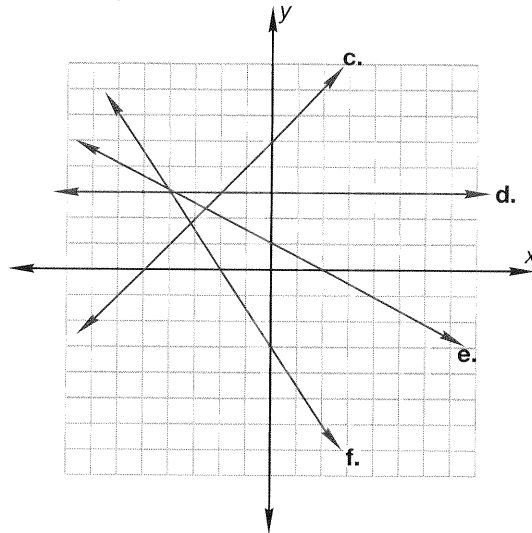
B $y + 2x = 3$

C $y = 2x + 3$

b. What is the slope and y-intercept of the graph of this equation?

$$y = -\frac{1}{3}x + 2$$

Represent Write equations for lines **c-f** in slope-intercept form.



Graph the following equations using the given slope and y-intercept.

g. $y = x - 2$

h. $y = -2x + 4$

i. $y = \frac{1}{2}x - 2$

Written Practice

Strengthening Concepts

* 1. **Analyze** ⁽⁴⁵⁾ The ratio of arable land (land that can be used for growing crops) to non-arable land in a certain county is 3 to 7. If the county has an area of 21,000 sq km, what area of the land is arable?

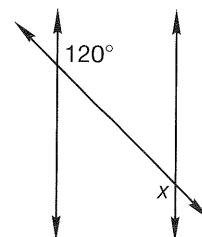
2. ^(3, 4) A lamp costs \$10 and a package of 4 light bulbs costs \$3. How much would it cost to buy 8 lamps and one light bulb for each lamp?

* 3. **Represent** ⁽⁵³⁾ Mary measures the heights of the young plants in her garden to the nearest cm and collects this data: 4 cm, 5 cm, 6 cm, 6 cm, 7 cm, 8 cm, 6 cm, 7 cm, 5 cm

a. Display the data with a line plot.

b. Find the range and the mean, median, and mode of the data.

* 4. **Conclude** ⁽⁵⁴⁾ Two of the lines are parallel. Find x .



5. Use the slope-intercept method to graph the equation $y = 2x - 4$. Is $(3, 2)$ a solution? (56)

6. Find the **a** area and **b** circumference of the circle with radius 1. Use 3.14159 for π . (39, 40)

* 7. The Mid-Atlantic Ridge spreads along the ocean floor an average of about 10 cm every 4 years. (52)

a. Express this rate as a unit rate.

b. **Connect** This rate is equal to what distance of spread every one million years? (Express in km.)

* 8. Convert 1600 m to km. Use a unit multiplier. (52)

Solve.

9. $\frac{0.6}{x} = \frac{0.12}{5}$ (44)

* 10. $2x - x = 1.5$ (50)

* 11. $0.6x + 1.2 = 3$ (50)

* 12. $7m - 9m = -12$ (50)

* 13. a. Write $\frac{2}{5}$ as a decimal and as a percent. (12)

b. Select any one of these three forms to represent the fact that 10 students out of 25 had visited the national park.

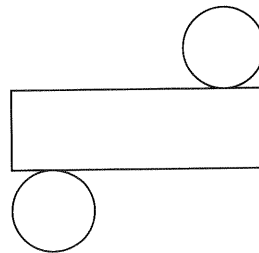
* 14. Factor: (21)

a. $2x^2 + 14x$

b. $15x - 20$

* 15. A square with vertices at $(2, 2)$, $(-2, 2)$, $(-2, -2)$, and $(2, -2)$ is dilated by a scale factor of 2. The area of the original square is what fraction of its dilated image? (Inv. 5)

* 16. The figure is a net for a geometric solid. Sketch and name the solid. (55)



Generalize Simplify:

17. $-5(x + 2) - 2x + 9$ (31, 36)

* 18. $(-4)^2 - (-4)^3$ (33, 36)

* 19. $\frac{(-15)(-12)}{(-15) - (-12)}$ (33, 36)

20. $\frac{wr^2d}{r^3d}$ (27)

21. $2 \cdot 1\frac{1}{2} - \left(1\frac{1}{2}\right)^2$ (13, 23)

22. $\frac{6}{8} = \frac{9}{x}$ (44)

23. $\frac{4.8 \times 10^7}{1.6 \times 10^4}$ (46)

24. $\sqrt{10^2 - 8^2}$ (15, 21)

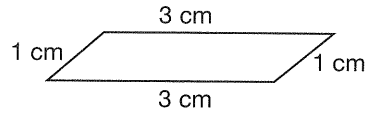
25. Simplify and express using only positive exponents. (27, 51)

$a^0b^1ab^{-1}c^{-1}$

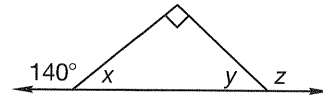
26. Arrange in order from least to greatest:
(12, 30)

$$0.3, \frac{1}{3}, 0.33, 3\%$$

27. **Represent** Sketch a quadrilateral similar to the one shown enlarged by a scale factor of 1.5. Label the side lengths.
(35)

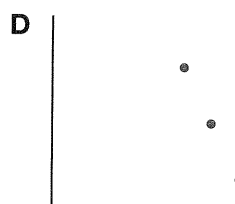
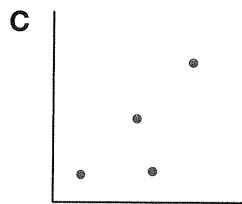
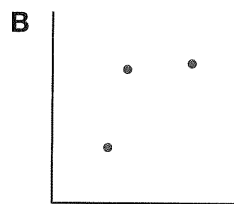
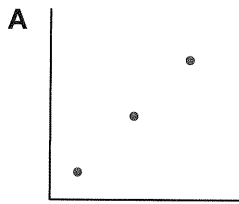


28. Find the measures of angles x , y , and z .
(18, 20)



29. Find the slope of the line passing through the points $(3, 4)$ and $(-4, -3)$.
(44)

30. Which graph represents a proportion? Explain how you know.
(41)



• Operations with Small Numbers in Scientific Notation

Power Up

Building Power

facts

Power Up L

mental math

- Number Sense:** 25×36
- Statistics:** Find the mean of 30 and 40.
- Fractional Parts:** $66\frac{2}{3}\%$ of \$300
- Probability:** What is the probability of rolling a number greater than 1 with one roll of a number cube?
- Proportions:** $\frac{x}{24} = \frac{3}{8}$
- Geometry:** Two angles of a parallelogram measure 80° and 80° . What are the measures of the other two angles?
- Measurement:** Find the length of this nail.



- Calculation:** $10 \times 8, + 1, \sqrt{\quad}, \sqrt{\quad}, \times 10, + 6, \sqrt{\quad}, - 7$, square it.

problem solving

A plane leaves Minneapolis, Minnesota, at 1 p.m. and flies non-stop to Honolulu in 8 hours. If Hawaii is four time zones earlier than Minnesota, what time does the plane arrive? If the return flight also takes 8 hours, what time does the plane arrive back in Minneapolis if it leaves Honolulu at 8 p.m.?

New Concept

Increasing Knowledge

In Lesson 51 we practiced writing small numbers between 0 and 1 in scientific notation. In this lesson we will multiply and divide those numbers. Recall that to multiply powers of 10 we add the exponents and to divide we subtract.

$$10^5 \cdot 10^3 = 10^8 \quad \frac{10^5}{10^3} = 10^2$$

The Laws of Exponents implied on the previous page apply to negative exponents as well. We exercise care adding and subtracting the exponents. Here we show some examples.

$$10^{-5} \cdot 10^{-3} = 10^{-8} \quad 10^5 \cdot 10^{-3} = 10^2$$

$$\frac{10^{-5}}{10^{-3}} = 10^{-2} \quad \frac{10^5}{10^{-3}} = 10^8$$

Example 1

If a sheet of notebook paper is 0.01 cm thick, how tall is a stack of 2500 sheets of notebook paper? Express each number in scientific notation and perform the calculation in scientific notation.

Solution

We multiply 1×10^{-2} and 2.5×10^3 .

$$(1 \times 10^{-2})(2.5 \times 10^3) = 2.5 \times 10^1$$

The stack of paper is 2.5×10^1 cm which is **25 cm**.

Example 2

Perform each indicated calculation and express the result in scientific notation.

- a. $(2.4 \times 10^{-6})(2 \times 10^{-2})$ b. $\frac{2.4 \times 10^{-8}}{2 \times 10^{-2}}$
- c. $(4 \times 10^{-6})(5 \times 10^{-4})$ d. $\frac{4 \times 10^{-8}}{5 \times 10^{-2}}$

Solution

- a. $(2.4 \times 10^{-6})(2 \times 10^{-2}) = 4.8 \times 10^{-8}$
- b. $\frac{2.4 \times 10^{-8}}{2 \times 10^{-2}} = 1.2 \times 10^{-6}$
- c. $(4 \times 10^{-6})(5 \times 10^{-4}) = 20 \times 10^{-10}$ (improper form)
Adjustment: $20 \times 10^{-10} = 2 \times 10^1 \times 10^{-10} = 2 \times 10^{-9}$
- d. $\frac{4 \times 10^{-8}}{5 \times 10^{-2}}$ (improper form)
Adjustment: $0.8 \times 10^{-6} = 8 \times 10^{-1} \times 10^{-6} = 8 \times 10^{-7}$

Thinking Skill

Explain

Describe how the power of 10 for each product or quotient in a–d is found.

Practice Set

Find each product or quotient.

- a. $(4 \times 10^{10})(2 \times 10^{-6})$ b. $(1.2 \times 10^{-6})(3 \times 10^3)$
- c. $(1.5 \times 10^{-5})(3 \times 10^{-2})$ d. $(7.5 \times 10^{-3})(2 \times 10^{-4})$
- e. $\frac{7.5 \times 10^5}{3 \times 10^{-2}}$ f. $\frac{4.8 \times 10^{-3}}{3 \times 10^2}$
- g. $\frac{8.1 \times 10^{-4}}{3 \times 10^{-7}}$ h. $\frac{1.2 \times 10^{-8}}{3 \times 10^{-4}}$

- i. A dollar bill weighs about 0.001 kg. What is the weight of 1,000,000 dollar bills? Express each number in scientific notation and perform the calculation in scientific notation.

Written Practice

Strengthening Concepts

1. In a certain state, the ratio of weddings held in spring or summer to weddings held in autumn or winter is about 3 to 2. If there were 46,000 weddings in a year, how many were in spring or summer?
(45)
- * 2. Sixty percent of the days in the desert were hot and dry. Out of the 365 days, how many were hot and dry?
(48)
- * 3. **Analyze** Martin completed 60% of his math assignment during class. If his math assignment consists of 30 problems, how many problems does he have left to do?
(48)

- * 4. Allison collected these donations for a charity:
(53)

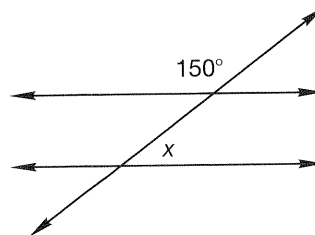
\$50, \$75, \$100, \$50,

\$60, \$75, \$80, \$50, \$75

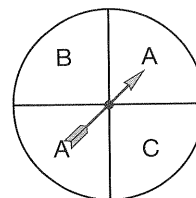
- Display the data with a line plot.
- Find the range and the mean, median, and mode.
- Write a description of the data using the mode.

5. $\frac{6}{7} = \frac{x}{10.5}$
(44)

- * 6. Two of the lines are parallel. Find x .
(54)



7. The spinner is spun twice.
(32)
- Write the sample space of 16 equally-likely outcomes.
 - What is the probability the spinner will stop on A at least once in two spins?



8. Use the slope-intercept method to graph $y = x - 1$. Then graph the point $(-2, -3)$. Is $x = -2, y = -3$ a solution?
(56)

Solve.

9. $0.3m - 0.3 = 0.3$
(50)

10. $4x + 7x = 99$
(50)

11. $\frac{2}{3}x + \frac{1}{2} = \frac{2}{3}$
(50)

12. $7 = -2p - 5p$
(50)

Refer to the following information to solve problems 13 and 14.

A circular tetherball court 6 meters in diameter is painted on the playground.

A stripe divides the court into two equal parts.

13. What is the circumference of the tetherball court to the nearest meter?
(39)

14. What is the area of each semicircle to the nearest square meter?
(40)

15. Collect like terms: $x(x + 2) + 2(x + 2)$
(31, 36)

* 16. Simplify and compare: $\left(\frac{1}{2}\right)^2 \bigcirc \left(-\frac{1}{2}\right)^2$
(22)

* 17. **Represent** Maggie drove 195 miles in 3 hours.
(49)

a. At that rate, how many miles will she drive in 5 hours?

b. Express 195 miles in 3 hours as a unit rate.

* 18. A mile is 5280 ft. Use a unit multiplier to convert 5280 ft to yards.
(52)

Evaluate For 19 and 20 refer to this description.

Diego is building an architectural rendering of a house. For a portion of the roof Diego cuts and folds a net (pattern) for a triangular prism. The base of the prism is an isosceles triangle. The triangle has a base of 24 in. and a height of 5 in. The distance between the triangular bases is 20 in.

* 19. a. Make an isometric sketch of the folded prism.
(Inv. 4, 55)

b. Make a sketch of the net of the prism.

* 20. a. Find the volume of the prism.
(42, 43)

b. Find the surface area of the prism.

21. **Generalize** Simplify: $\frac{b^3r^4}{mb^3r^2}$
(27)

22. Simplify, then compare: $\sqrt{\frac{4}{9}} \bigcirc \frac{\sqrt{4}}{\sqrt{9}}$
(15)

23. Write $\frac{11}{12}$ a as a percent and b as a decimal.
(30)

* 24. **Classify** The fare charged for a taxi ride was \$1.50 plus 40¢ for each $\frac{1}{5}$ of a mile. What term best describes the relationship between the distance traveled and the fare charged?
(47)

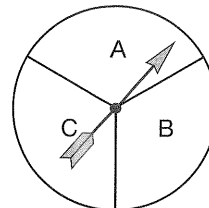
A continuous function B step function C direct proportion

25. The spinner is spun twice.
(32)

a. What is the sample space of the experiment?

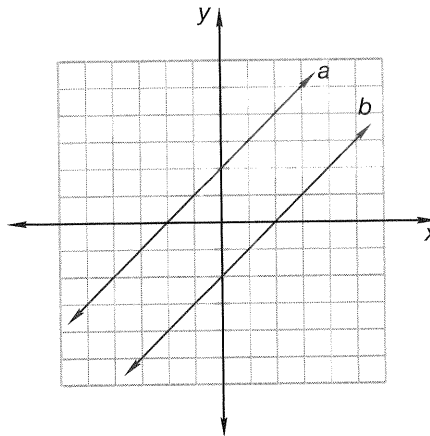
b. What is the probability the spinner stops on A at least once?

c. What is the probability the spinner does not stop on A in two spins?



26. Evaluate the expression $-b + \sqrt{b^2 - 4ac}$ when $a = 3$, $b = 4$, and $c = 1$.
 (14, 15)

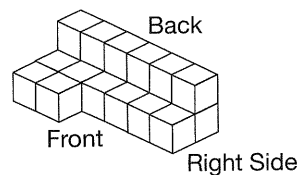
27. Using the slope-intercept form, write the equation of each line at the right.
 (56)



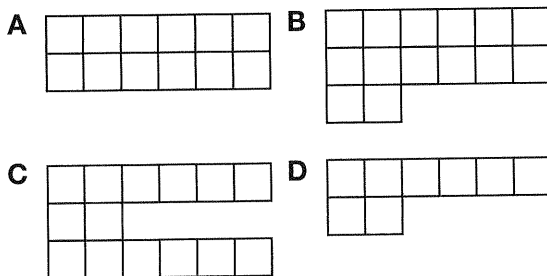
28. If a line has a slope of $\frac{1}{2}$ and if it intersects the y -axis at -1 , then what is the equation of the line in slope-intercept form?
 (56)

29. Find all solutions: $9x^2 = 36$
 (14, 15)

30. Use this figure to answer the following questions.
 (Inv. 4)



a. Which of the following is the front view of this figure?



b. Which of these is the right-side view?



• Solving Percent Problems with Equations

Power Up

Building Power

facts

Power Up L

mental math

- Number Sense:** Can we produce a 2-digit whole number by multiplying two 2-digit whole numbers?
- Scientific Notation:** Write 0.0019 in scientific notation.
- Geometry:** Three angles of a trapezoid measure 50° , 90° and 100° . Find the measure of the fourth angle.
- Fractional Parts:** If coats are on sale for 40% **off** the regular price, then the sale price is what percent **of** the regular price?
- Algebra:** $12x + 1 = 145$
- Measurement:** The bicycle odometer read 24.2 miles when she left. It read 29.5 miles when she returned. How far did she ride?
- Rate:** Kenneth read 35 pages per hour. Shana read 40 pages per hour. After 2 hours, how many more pages has Shana read than Kenneth?
- Calculation:** $\sqrt{9}, \times 8, \times 2, + 1, \sqrt{\quad}, \times 8, - 1, \div 5, \times 3, - 1, \div 4$

problem solving

A store sells three sizes of the same candle—a “regular” size with radius r and height h . Candle B is twice as tall as Candle A but has the same radius. Candle C is just the reverse, with twice the radius of Candle A but the same height. If both large Candles B and C are made of the same material, which of them is heavier? Why?

New Concept

Increasing Knowledge

We may solve percent problems using proportions (as we did in Lesson 48) or by solving a percent equation.

A percent of a whole is a part.

$$\% \times W = P$$

The factors and product in the equation above are the percent, the whole, and the part. If two of the three numbers are known, we can write and solve an equation to find the unknown number. If we are given the percent, we convert the percent to a fraction or decimal before performing the calculation.

For any percent problem we may choose to express the percent as a decimal or as a fraction. We might make the choice based on which form seems easier to calculate. For some problems the calculations are tedious, so we might choose to express the percent as a decimal and use a calculator. For other problems, expressing the percent as a fraction is the best choice.

Example 1

Thirty-two ounces is 25% of a gallon. How many ounces is a gallon?

Solution

We are given the percent and the part. We are asked for the whole. We write the percent as a decimal or as a fraction and solve for the unknown.

As a decimal:

32 oz is 25% of a gallon

$$32 = 0.25g$$

$$\frac{32}{0.25} = g$$

$$g = 128$$

As a fraction:

32 oz is 25% of a gallon

$$32 = \frac{1}{4}g$$

$$\frac{4}{1} \cdot 32 = \frac{4}{1} \cdot \frac{1}{4}g$$

$$g = 128$$

A gallon is **128 ounces**.

Example 2

Mr. Villescas bought a used car for \$8,500. The sales-tax rate was 7%. How much did Mr. Villescas pay in tax?

Solution

The sales tax will be added to the price, but the amount of tax is based on the price. In this case the tax is 7% of the price. We translate the sentence to an equation using = for "is" and \times for "of." We substitute the known numbers and solve for the unknown. We may write the percent as a decimal or a fraction before performing the calculation.

As a decimal:

Tax is 7% of the price

$$t = 0.07 \times \$8500$$

$$t = \$595$$

As a fraction:

Tax is 7% of the price

$$t = \frac{7}{100} \times \overset{\$85}{\$8500}$$

$$t = \$595$$

The sales tax on the car was **\$595**. Notice that the amount of tax on large-dollar purchases might seem high. However, it is correct. Seven percent of the price is less than 10% but more than 5%. Since 10% of \$8500 is \$850, and 5% is half of that (\$425), our answer of \$595 is reasonable.

Example 3

For the following percent problems, decide whether it is better to express the percent as a decimal or as a fraction.

- The sales-tax rate is 8.25%. What is the sales tax on a \$18.97 purchase?
- Shirts are on sale for $33\frac{1}{3}\%$ off the regular price. How much is saved on a \$24 shirt?

Solution

Thinking skill

Connect

What is the decimal form of $33\frac{1}{3}\%$? Why would we need to round the decimal to perform the calculation?

- Whether we change the percent to a fraction or decimal, we must multiply a 4-digit number by a 3-digit number. Converting the percent to a fraction further complicates the arithmetic. To calculate an exact answer we would **convert the percent to a decimal** and use a calculator if one was readily available.
- To write the percent as a decimal we would need to round. Expressing the percent as the fraction $\frac{1}{3}$ is more accurate, and we can perform the calculation mentally. We would **express the percent as a fraction**.

Example 4

Blanca correctly answered 23 of the 25 questions. What percent of the questions did she answer correctly?

Solution

We are given the whole and the part. We are asked for the percent. After solving the equation, we convert the fraction or decimal solution to a percent.

What percent of 25 is 23?

$$P \cdot 25 = 23$$

$$\frac{P \cdot 25}{25} = \frac{23}{25}$$

$$P = \frac{23}{25} \text{ or } 0.92$$

We convert 0.92 to **92%**.

Practice Set

Solve by writing and solving equations. In **a–d** choose whether to express the percent as a fraction or as a decimal.

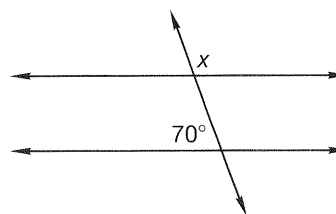
- Justify** Six percent of \$4500 is how much money? Explain why your answer is reasonable.
- Twenty percent of what number is 40?
- How much is a 15% tip on a \$13.25 meal? Round your answer to the nearest dime.
- How much money is $16\frac{2}{3}\%$ of \$1200?
- What percent of 50 is 32?
- Dixon made a \$2,000 down payment on an \$8,000 car. The down payment was what percent of the price?
- Kimo paid \$24 for the shirt which was 75% of the regular price. What was the regular price?

- * 1. **Analyze** In water, the ratio of hydrogen atoms to oxygen atoms is 2 to 1. If there are 3×10^{23} atoms in a sample of water, how many are hydrogen atoms?
(45)
- * 2. Sixty percent of the voters favored the initiative, and 12,000 did not favor it. How many voters favored the initiative?
(48, 58)
- 3. A bank charges customers \$10 per month for a checking account, plus 10 cents for each check that is written. If Jessie uses 13 checks in a month, what will this bank charge for the month?
(3, 4)
- * 4. **Evaluate** The sales revenue of a retail store for one week was (in thousands of dollars):
(53)
2.9, 1.6, 1.4, 1.3, 1.5, 2.5, 2.8
 - a. Display the data in a line plot.
 - b. Find the mean, median, mode, and range of the data.
 - c. Which measure would you choose to report the sales most favorably?
- 5. Brad is trying to remember the combination to a lock. He knows the three numbers are 15, 27, and 18, but he cannot remember the order.
(32)
 - a. What are the possible orders of the numbers?
 - b. What is the probability that Brad finds the correct order on his first try?

Solve for x .

6. Solve for x . $\frac{x}{5} = \frac{14}{20}$
(44)

- * 7. These two lines are parallel. Find x .
(54)



- 8. Use the slope-intercept method to graph $y = \frac{1}{2}x - 2$.
(56)
- 9. The circumference of the earth at the equator is about 25,000 miles.
(39) Estimate the diameter of the earth to the nearest thousand miles. Explain how you found your answer.

Analyze Solve.

* 10. $3m - 6 + 2m = 4$
(50)

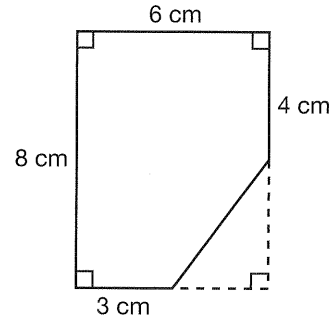
* 11. $x - 3x = 18$
(50)

* 12. $\frac{2}{3}x + \frac{1}{2} = \frac{5}{6}$
(50)

13. $\frac{y}{8} = 0.375$
(38)

14. Isabel wonders whether a beach ball with a 50 inch circumference will pass through a basketball hoop with an 18 inch diameter. What do you predict? Justify your answer.
(39)

15. One corner is cut from a small rectangle of note paper. Find the **a** area and **b** perimeter of the resulting piece of paper.
(37)



16. Sarah just got her hair cut. If Sarah's hair grows $1\frac{1}{4}$ inches every month, how much longer will it be in $4\frac{1}{2}$ months if she does not cut it?
(23)

17. Use a unit multiplier to convert 2.5 ft to inches.
(52)

Simplify.

18. $\left(\frac{2}{5}\right)^2$
(22)

19. $\frac{ssr^5}{s^2r^4}$
(27)

20. $\frac{2}{15} + \frac{2}{5} \cdot \frac{1}{6}$
(13, 22)

21. $x^2 + 2x + x + 2$
(31)

22. a. Write 55% as a decimal and reduced fraction.
(12, 48)

b. Choose one form and use it to find 55% of \$1200.

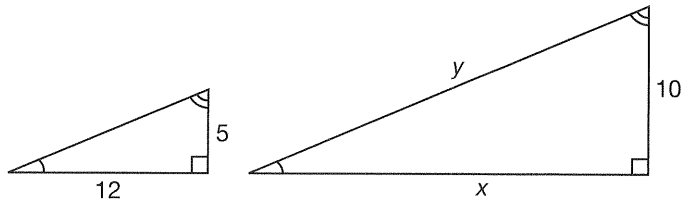
23. Express $\frac{1}{7}$ as a decimal rounded to three decimal places.
(30)

24. Find $\frac{n+m}{m}$ when $n = 100$ and $m = -10$.
(31, 36)

25. a. Explain why the triangles are similar.
(35)

b. Find x .

c. Find y .



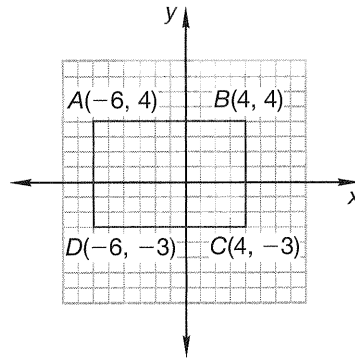
26. An ant weighs as little as 0.00001 kilograms. Write the minimum weight of an ant using an exponent.
(51)

27. Last month, Andrea missed two days of school because she had a cold. What percent of school days did Andrea miss that month. (Hint: Assume that there are 20 school days in a month.)
(48)

28. The Giant Water Lily of the Amazon River is the world's largest lily. Its pads are almost perfectly round. One lily pad usually measures about 6 feet in diameter. What is the approximate surface area of one lily pad?
(40)

29. On the equator, the earth rotates about its axis about 25,000 miles in 24 hours. Express this rate in miles per hour. (Round to the nearest hundred.)

30. What set of coordinates represents the translation of rectangle $ABCD$ 3 units to the right and 2 units down? Sketch the rectangle on graph paper to illustrate the transformation.



Early Finishers

*Real-World
Application*

Below are the weight in grams (g) of a neutron, a proton and an electron. First write each particle's weight in standard form. Then list the particles in order from lightest to heaviest.

neutron 1.6750×10^{-24}

proton 1.6726×10^{-24}

electron 9.1083×10^{-28}

• Experimental Probability

Power Up

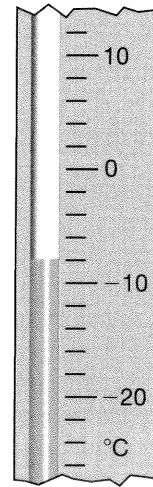
Building Power

facts

mental math

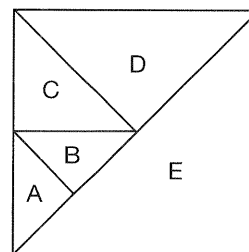
Power Up L

- a. **Number Sense:** Can we produce a 4-digit number by multiplying two 2-digit numbers?
- b. **Statistics:** The mean of equally spaced numbers equals the median of the numbers. Find the mean of 30, 40, and 50.
- c. **Fractional Parts:** A tip of 15% to 20% is customary. Sam tipped $\frac{1}{6}$ of the bill on a bill of \$24. What was the tip?
- d. **Ratio:** There are 16 pens in a drawer. Six of the pens are blue and the other 10 are black. What is the ratio of black pens to blue pens?
- e. **Probability:** What is the probability of rolling an even number with one roll of a number cube?
- f. **Geometry:** Two sides of a triangle are 5 inches and 13 inches. Can the third side be 12 inches long? 7 inches? 19 inches?
- g. **Measurement:** What temperature is indicated on this thermometer?
- h. **Calculation:** Start with a dollar, add 50%, give away half, subtract 25¢, add a dime, give away half. How much is left?



problem solving

A square is divided as shown. Regions B and C combine to equal what fraction of the area of the square?



When discussing probability, we distinguish between the **theoretical probability**, which is found by analyzing a situation, and the **experimental probability**, which is determined statistically. Experimental probability is the ratio of the number of times an event occurs to the number of trials.

$$\text{experimental probability} = \frac{\text{number of times an event occurs}}{\text{number of trials}}$$

If we want to experimentally determine the probability that a flipped coin will land heads up, we would flip the coin a number of times, counting the number of trials and the number of heads.

Trial	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Outcome	H	H	T	H	T	H	T	T	T	H
Probability $H \div t$	1.00	1.00	0.67	0.75	0.60	0.67	0.57	0.50	0.44	0.50

As the number of trials grows, the experimental probability tends to approach the theoretical probability of an experiment. Therefore, we want a large number of trials when conducting probability experiments.

Experimental probability is used widely in sports. If a basketball player makes 80 free throws in 100 attempts, then the probability of the player making a free throw is $\frac{80}{100}$ or $\frac{4}{5}$. The player might be described as an 80% free throw shooter. In baseball and softball, a player's batting average is the probability, expressed as a decimal, of the player getting a hit.

Example 1

A softball player has 21 hits in 60 at-bats. Express the probability the player will get a hit in her next at-bat as a decimal number with three decimal places.

Solution

The ratio of hits to at-bats is 21 to 60. To express the ratio as a decimal we divide 21 by 60.

$$21 \div 60 = 0.35$$

It is customary to write the probability (batting average) with three decimal places and without a zero in the ones place: **.350**

Thinking Skill

Analyze

Marcia has a batting average of .330. In one game she has 2 hits in 5 at bats. Will this raise or lower her batting average?

Experimental probabilities are used not only in sports but in many fields such as business, weather forecasting, insurance, and banking. Theoretical probability is commonly applied to designing games of chance.

Example 2

A salesperson distributed free samples at the mall. Fifty shoppers accepted the samples and thirty did not. What is the probability that a shopper at the mall will accept the salesperson's samples?

Solution

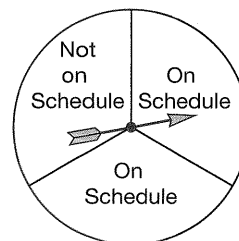
Fifty out of eighty shoppers accepted the sample, so the probability is $\frac{5}{8}$.

Predict If the salesperson from example 2 expects to see 400 customers at the mall during the next shift, how many samples should the salesperson have ready? Explain your reasoning.

Sometimes experiments are impractical to conduct, so to find an experimental probability, we **simulate** the event using models such as spinners, number cubes, coins, or marbles.

Example 3

The commuter plane flight arrives on schedule $\frac{2}{3}$ of the time on any given weekday. Heather wonders what the probability is that it will arrive on schedule on a Tuesday, Wednesday, and Thursday in the same week.



- How could the experimental probability be found? Why is this impractical?
- How could the spinner at right be used to simulate the experiment?

Solution

- Heather could check the flight arrival time on Tuesdays, Wednesdays, and Thursdays for several weeks. The experimental probability is the number of times the flight arrives on schedule all 3 days in a row divided by the total number of selected 3-day periods. **This experiment would take a very long time to conduct.**
- Each spin can represent one flight. The probability of an on-time arrival is $\frac{2}{3}$. **Heather can spin the spinner three times in a row and repeat this for many trials to find the experimental probability.**

Explain How might a number cube or marbles be used to simulate this experiment?

Represent Select one of the models described in the lesson and use it to simulate Heather's experiment. Conduct 12 trials, which is 36 spins, rolls, or draws. What experimental probability do you find?

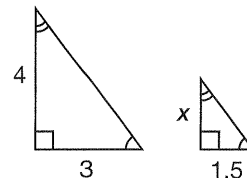
Practice Set

- a. **Explain** When a member of the opposing team fouls a basketball player who is shooting, the player shoots two free throws. Near the end of a close game a player has made 6 out of 20 free throws, and he has made 60% of his 2-point shots. When the player has the ball in 2-point range, should the opposing team foul him or risk the shot? Explain your answer in terms of probability.
- b. Quinn runs a sandwich shop. Since she added a turkey melt to the menu, 36 out of 120 customers have ordered the new sandwich. What is the probability that the next customer will order a turkey melt?
- c. **Predict** Quinn ordinarily has 200 customers on a busy afternoon, about how many turkey melts should she expect to sell?
- d. To prepare premium rates (the amounts customers pay) for an insurance plan, an insurance company conducts an extensive risk study to determine the probability that a member of a certain group may require a pay-off. Based on these probabilities, the expected amount of payoff is charged to the customer as part of the premium. Explain why car insurance companies charge a higher premium for teenage drivers with a clean driving record than for adult drivers with a clean driving record.
- e. Meghan is a 50% free throw shooter. Select a model to simulate a game in which Meghan shoots 10 free throws.

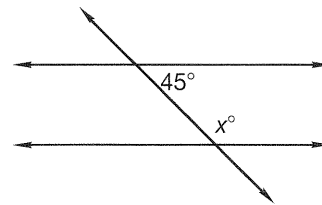
Written Practice

Strengthening Concepts

- * 1. **Analyze** ⁽⁴⁵⁾ In carbon dioxide, the ratio of carbon atoms to oxygen atoms is 1 to 2. If there are 12×10^{23} oxygen atoms in a sample of carbon dioxide, how many atoms are there in all?
- * 2. ⁽⁵³⁾ In the triple-jump competition, Lee jumped 28 ft, 32 ft, 29 ft, 30 ft, and 33 ft.
- a. Find the mean, median, mode, and range of Lee's marks.
- b. Which measure gives the difference between her shortest and longest jumps?
- * 3. **Formulate** ^(48, 58) Twelve of the 30 students in the class brought their lunches from home. What percent of the students brought their lunches from home?
- * 4. ⁽⁵⁹⁾ If a student from the class in problem 3 were selected at random, what is the probability that the student would be one who brought his or her lunch from home?
5. ⁽³⁵⁾ These triangles are similar. Find the scale factor from the larger triangle to the smaller triangle and solve for x .



- * 6. Two of the lines are parallel. Solve for x .
(54)



7. Use slope-intercept to graph $y = -2x + 4$. Is the point $(0, -2)$ a solution?
(56)

Analyze Solve.

* 8. $3(x - 4) = 15$
(14, 21)

* 9. $2x - x - 2 = 12$
(14, 31)

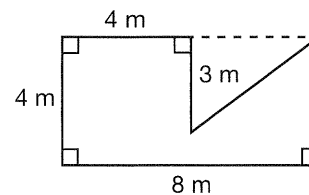
10. $\frac{x}{3} = 0.7$
(25, 38)

11. $\frac{1}{2}x = \frac{1}{3}$
(22, 38)

12. Find **a** the area and **b** the circumference of a circle that has a radius of 13 units. Express your answer in terms of π .
(39, 40)

- * 13. **Evaluate** Compare the figure to a 4 m by 8 m rectangle.
(37)

- a. Is the area of the figure greater than or less than the area of the rectangle?
b. Is the perimeter of the figure greater than or less than the perimeter of the rectangle?



14. Express as a unit rate: 2400 miles in 48 hours.
(7)

- * 15. Use a unit multiplier to convert 3.5 minutes to seconds.
(52)

16. The door is 6 ft 9 in. high. Express the height of the door as a mixed number of feet and as a decimal number of feet.
(52)

Simplify.

17. $3^{-2} \cdot 2^{-2}$
(51)

18. $\frac{h^3 p^2}{ph}$
(27)

19. $\frac{5}{18} - \frac{5}{18} \cdot \frac{1}{5}$
(13, 22)

20. $0.3 - 0.2(0.1)$
(24, 25)

21. a. Write 0.95 as a percent and reduced fraction.
(12)

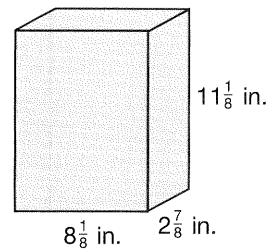
- b. Order 0.95, $\frac{39}{40}$, and $\frac{9}{10}$ from least to greatest.

- * 22. **Evaluate** Van selected two unmarked paint cans from a shelf with three unmarked cans. Inside two cans is white paint and inside one can is yellow paint. Van will open the cans as he looks for the yellow paint.
(32)

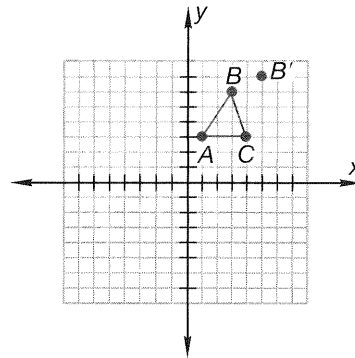
- a. In a list, record the sample space for the experiment. (Hint: Use the abbreviations W_1 , W_2 , Y .)

- b. What is the probability that one of the two cans Van selected contains yellow paint?

- 23.** ⁽⁷⁾ The Wilsons traveled the 2400 miles of Route 66 from Chicago, IL to Santa Monica, CA. If their total driving time was 50 hours, what was their average rate?
- * **24.** ⁽⁵⁸⁾ What is the amount of sales tax for \$315.90 if the tax rate is 7%? (Round to the nearest cent.)
- * **25.** ⁽⁵⁸⁾ If Dori answers 23 of 25 questions correctly, what percent of the questions did she answer correctly?
- 26.** ^(31, 36) Combine like terms: $x(x + 1) - 1(x + 1)$
- 27.** ^(42, 43) Estimate the volume and surface area of a cereal box that has these dimensions.



- 28.** ⁽⁵¹⁾ Some lichens grow at a rate of 0.000000000625 miles per hour. What is the same number expressed in scientific notation?
- 29.** ⁽⁵⁷⁾ If the ink in the period at the end of this sentence weighs 1.3×10^{-8} lb, and if there are 30,000 periods printed in this book, then how much does this ink weigh altogether? Express each number in scientific notation and perform the calculation in scientific notation.
- 30.** ^(Inv. 5) The coordinates of the vertices of $\triangle ABC$ are $A(1, 3)$, $B(3, 6)$ and $C(4, 3)$. A translation of $\triangle ABC$ is used to produce $\triangle A'B'C'$. Vertex B' is located at $(5, 7)$. Choose the coordinates of the vertices A' and C' .



- A** $A'(3, 5)$ and $C'(6, 5)$
- B** $A'(2, 4)$ and $C'(4, 7)$
- C** $A'(5, 3)$ and $C'(4, 7)$
- D** $A'(3, 4)$ and $C'(6, 4)$

• Area of a Parallelogram

Power Up

Building Power

facts

Power Up L

mental math

- a. **Number Sense:** Can we produce a 5-digit whole number by multiplying two 2-digit whole numbers?
- b. **Algebra:** If $x^3 = 8$, then x equals what number or numbers?
- c. **Geometry:** Two sides of a triangle measure 17 inches and 12 inches. Can the third side measure 4 inches? 19 inches? 29 inches?
- d. **Fractional Parts:** $\frac{5}{9}$ of the 72 athletes were on the home team. How many were on the home team?
- e. **Scientific Notation:** Write 0.000507 in scientific notation.
- f. **Rate:** Brad ran north up the trail at 8 miles per hour. Skip walked the trail at 4 miles per hour north. After 30 minutes, how far apart are they if they started at the same place and time?
- g. **Proportions:** $\frac{18}{6} = \frac{12}{x}$
- h. **Calculation:** $6 \times 7, - 2, \div 2, + 1, \div 3, + 2, \sqrt{\quad}, \times 12, \sqrt{\quad}, - \frac{1}{2}$

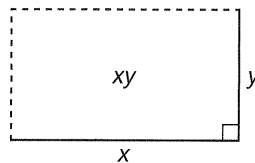
problem solving

Victor makes three different purchases at three different stores one afternoon. Each time, he spends $\frac{1}{3}$ of the money he is carrying. If he ends the day with \$16, how much money did he have at the start of the day?

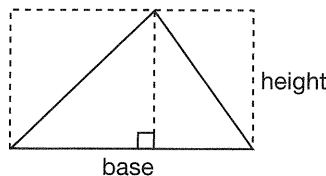
New Concept

Increasing Knowledge

Recall that to find the area of a rectangle we multiply the length and width, which are the perpendicular dimensions of a rectangle.



Also recall that to find the area of a triangle, we multiply the perpendicular base and height as the first step to calculating the area. Then we find half of that product.



Thinking Skill

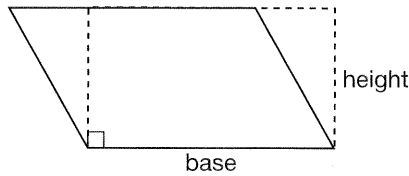
Explain

Why does the formula $A = \frac{1}{2}bh$ work for finding the area of a triangle?

Math Language

Recall that a **parallelogram** is a quadrilateral with two pairs of parallel sides. Rectangles, squares, and rhombuses are special parallelograms.

Likewise, to find the area of a parallelogram we multiply the perpendicular base and height. Again, the product is the area of a rectangle, but the area of the rectangle is equal to the area of the parallelogram. In the figure below, notice that the part of the parallelogram outside the rectangle matches the missing portion of the rectangle.



The formula for the area of a parallelogram is

$$A = bh$$

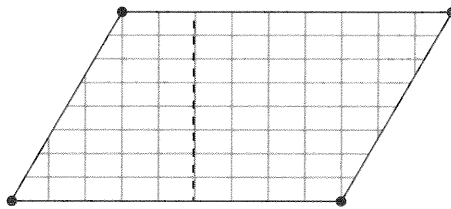
in which A represents area, b represents the length of the base, and h represents the height. Two parallel sides are the bases of a parallelogram. The perpendicular distance between the bases is the height.

Activity

Area of a Parallelogram

Materials needed: $\frac{1}{2}$ sheet of graph paper per student, scissors, ruler, pencil

On graph paper, draw a parallelogram that is not a rectangle. Trace over lines on the background grid to make one pair of parallel sides. Make sure that the vertices of the parallelogram are at intersections of the background grid and then cut out your parallelogram.

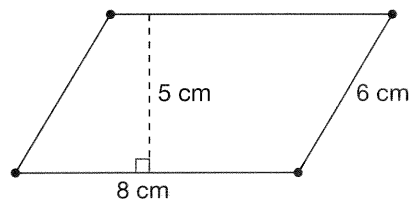


Next, on a grid line (such as the dashed line in the example), cut the parallelogram into two parts. Then, rearrange the two pieces to form a rectangle and find the area of the rectangle. Does the area of the rectangle equal the product of the base and height of the parallelogram?

Connect Repeat this activity by drawing a parallelogram and one of its diagonals. How does this area of each triangle you formed compare to the area of the parallelogram? How does the formula for the area of a triangle compare to the formula for the area of a parallelogram?

Example 1

Find the perimeter and area of this parallelogram.

**Solution**

We find the perimeter of the parallelogram by adding the lengths of the four sides.

$$8 \text{ cm} + 6 \text{ cm} + 8 \text{ cm} + 6 \text{ cm} = \mathbf{28 \text{ cm}}$$

We find the area of the parallelogram by multiplying the perpendicular dimensions of the base and the height.

Step:	Justification:
$A = bh$	Formula
$A = 8 \text{ cm} \cdot 5 \text{ cm}$	Substituted
$A = \mathbf{40 \text{ cm}^2}$	Simplified

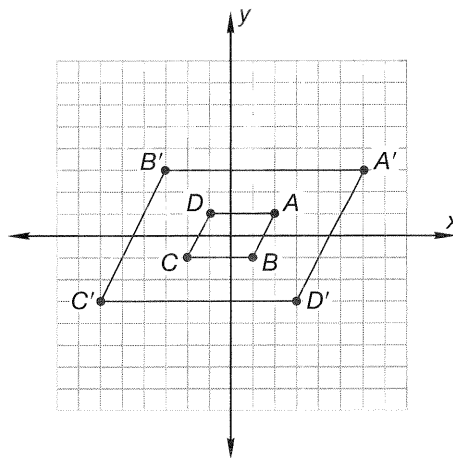
Model Draw a parallelogram with an area of 24 in.^2 . Label its dimensions.

Example 2

- On a coordinate plane sketch and find the area of parallelogram $ABCD$ with vertices $A(2, 1)$, $B(1, -1)$, $C(-2, -1)$ and $D(-1, 1)$.
- Graph the dilation of parallelogram $ABCD$ with scale factor 3. What is the area of the image?
- The area of the image is how many times the area of parallelogram $ABCD$? Why?

Solution

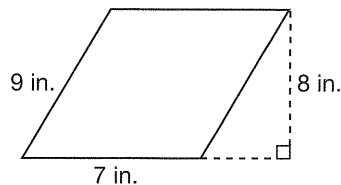
- The area of parallelogram $ABCD$ is **6 sq. units**.
- The area of the dilated image is **54 sq. units**.
- The area of the image is **9 times** the area of parallelogram $ABCD$ because the scale factor tripled the base and tripled the height. Therefore, when we multiply the base and height, the product (the area) is 9 times as great.



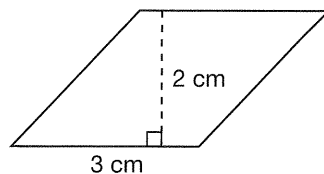
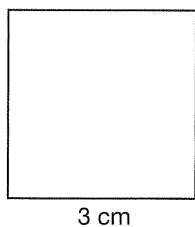
Practice Set

- a. The base and the height of a parallelogram are always
A perpendicular **B** parallel
C sides **D** congruent

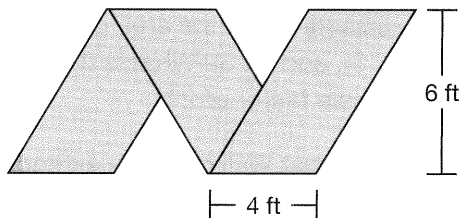
- b. What is the area of this parallelogram?



- c. The sides of a square and another rhombus are 3 cm long. Find the perimeter and area of each parallelogram.



- d. Find the area of a parallelogram with vertices at $(4, 2)$, $(2, -2)$, $(-2, -2)$ and $(0, 2)$. What would be the area of a dilation of this parallelogram with a scale factor of 3?
- e. **Evaluate** A brass sculpture of the letter N is formed from one long metal parallelogram folded into three congruent parallelograms. What is the area of one of the smaller parallelograms? What is the total area of the long metal parallelogram?



- f. **Justify** How do you know your answer to problem e is correct?

Written Practice

Strengthening Concepts

- * **1.** ⁽⁵⁸⁾ Gina has a coupon for 15% off at the video store. If she purchases a video marked \$16.80, how much will she pay using the coupon?
- * **2.** ⁽⁵¹⁾ Scientists know that the diameter of a normal cell is 0.000025 cm. How is that number written in scientific notation?
- * **3.** ^(Inv. 1, 20) A triangle has vertices at $(0, 5)$, $(-10, 1)$, and $(10, 1)$. What is the area of the triangle?

* 4. ⁽⁵⁸⁾ The bill for a family of four at a restaurant was \$62. Simon paid \$72 for the meal and tip. How much was the tip? What percent was the tip?

5. ⁽⁵¹⁾ Marshall knows that 0.01 is equal to 10^{-2} . How would he represent this using factors?

- A 8 B $\frac{1}{10} \times \frac{1}{10}$ C -20 D $10 \times (-2)$

* 6. ⁽⁵⁸⁾ **Analyze** Some stores have a layaway plan for customers. This means that the customer will pay for the item in installments and will be able to take the item home when it is fully paid for. One store requires that the person make a minimum down payment of 25%. Kayla wants to put a \$60 item on layaway at that store. What is the minimum down payment she must make?

Generalize Simplify.

7. ^(31, 36) $-13 + (-4)(-2)$

* 8. ⁽²¹⁾ $256 \div [2(6 - (-2))^2]$

9. ⁽⁴⁶⁾ $(1.6 \times 10^4)(2.0 \times 10^5)$

* 10. ^(25, 36) $(1.5)^2 (-2)^4$

* 11. ⁽²⁷⁾ $\frac{(5^3)^2}{5^3}$

Solve.

12. ⁽⁵⁰⁾ $\frac{1}{3}x - 1 = 4$

13. ⁽⁵⁰⁾ $-5k - 11 = 14$

14. ⁽⁵⁰⁾ $\frac{x}{7} = -2$

15. ⁽⁵⁰⁾ $5(2t + 4) = 140$

16. ⁽⁵⁰⁾ $-4m + 1.8 = -4.2$

17. ⁽⁵⁰⁾ $27 = 8b - 5$

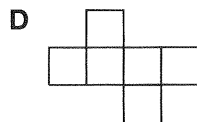
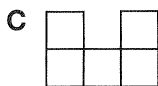
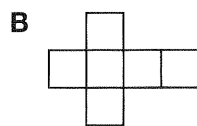
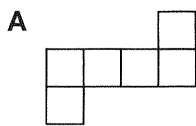
* 18. ^(Inv. 5, 60) **Evaluate** Find the area of a parallelogram with vertices at $(-2, 0)$, $(6, 0)$, $(1, 3)$, and $(9, 3)$. What is the area of a dilation of this parallelogram with a scale factor of 2?

* 19. ^(25, 52) The Great Wall of China extends approximately 4163 miles long. A mile is approximately 1.61 kilometers. Find the length of the Great Wall to the nearest kilometer.

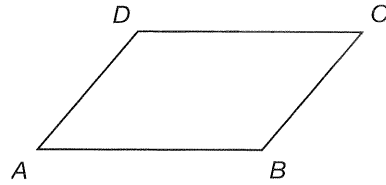
20. ⁽⁵⁰⁾ Jodi is joining a fitness center. The application fee is \$75 and the monthly rate is \$25. She wants to pay \$125 now. Solve this equation to find out how many months (m) she will be able to go to the fitness center for this amount of money.

$$25m + 75 = 125$$

* 21. ⁽⁵⁵⁾ **Analyze** Which of these nets is not a net of a cube?



22. In parallelogram $ABCD$, $m\angle B$ is 128° .
(54) What is the measure of $\angle C$?



23. What is the median of this data?
(53)

24. The pet store sells guppies for 8 for
(49) \$1.50. How much would 36 guppies cost? Write a proportion and solve.

25. At Washington Middle School, 64% of
(48) the students play a sport. If 192 play a sport, how many students attend Washington Middle School?

26. Will a $25\frac{1}{2}$ -inch umbrella fit into a gift
(Inv. 2) box that is 10 inches wide by 24 inches long? (Hint: Will it fit if it is placed diagonally? Use the Pythagorean Theorem.)

27. A bakery sells two kinds of bread, whole wheat and white, in a ratio of
(45) 3 whole wheat to 2 white. If the bakery made 60 loaves of bread, how many were whole wheat?

28. Lou makes pancakes at a pancake house. He makes 12 pancakes
(44) in 4 minutes. At that rate, how long would it take him to make 54 pancakes?

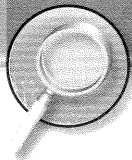
29. Wallpaper comes in rolls that cover 80 square feet. Vicki wants to
(43) wallpaper the walls and ceiling of her living room. The room is 15 feet wide by 18 feet long by 9 feet high. How many rolls of wallpaper will she need?

- * 30. **Conclude** Tran earns \$7.50 per hour at
(41) his part-time job. Copy and complete the function table for 1, 2, 3, and 4 hours of work. Then write an equation for the function. Start the equation with $P =$. Is the relationship between the number of hours worked and amount of pay proportional?

**Average Annual Precipitation
for Selected Texas Cities**

City	Inches
Brownsville	26
Dallas-Fort Worth	32
El Paso	9
Houston	47
Midland-Odessa	14
Port Arthur	55

Number of Hours (h)	Amount of Pay (P)
1	
2	
3	
4	



Focus on

• Collect, Display, and Interpret Data

Statistics is the science of collecting data and interpreting the data in order to draw conclusions and make predictions.

Suppose the managers of a theme park plan to build a new restaurant on its grounds, and the managers want to know the preference of the park attendees. The managers can collect this information by conducting a survey.

The group of people that researchers want to study is called the **population** of the study. In the case of the theme park, the population would be all park visitors. Often it is not practical to survey an entire population, so researchers select a smaller **sample** of the population to survey. Researchers use sampling methods to ensure that the sample is representative of the larger population and to avoid **bias** or slant toward a particular point of view. For example, the managers of the theme park might ask every 100th person who walks through the gates to participate in the survey.

1. Consider the survey that the theme park managers wish to conduct. Discuss the difficulties of surveying the entire population, that is, all park visitors.
2. If the survey were conducted only among people in one of the other restaurants at the park, would the sample be representative of the entire population?
3. Conduct a **closed-option** survey (multiple choice, not free response) among your classmates. As a class, choose three restaurant options for the theme park. Ask each student to write their preference on a piece of paper, then collect and tally the results. Display the data in a **bar graph**.
4. **Evaluate** Which type of restaurant is most popular among your classmates? Do you think the survey results would differ if it were conducted among a sample of park attendees?

In the survey, we collected **qualitative data** (data that falls into categories), in this case, types of restaurants. Data that is numerical is called **quantitative data**.