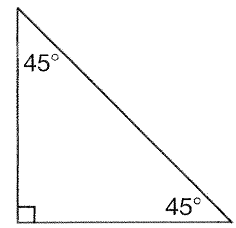
Example 1

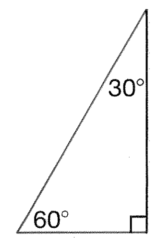
Draw the two special right triangles from the beginning of this lesson and on the drawing indicate the measure of each interior angle.

Solution

One of the right triangles has legs of equal length, like two sides of a square. So the triangle is an isosceles triangle.



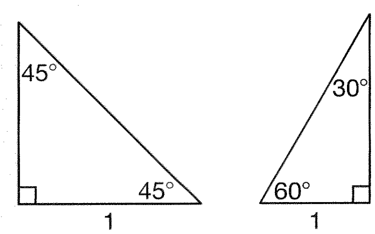
The other right triangle has one leg that measures half the hypotenuse. We begin sketching an equilateral triangle but stop halfway.



We may refer to these special triangles as 45-45-90 triangles and 30-60-90 triangles. Besides knowing the angle measures of these triangles, it is helpful to know their relative side lengths.

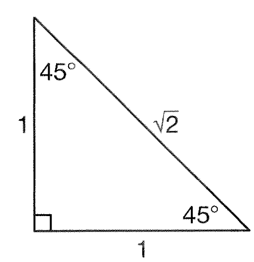
Example 2

The length of one leg of each special triangle is given. Find the lengths of the other sides.

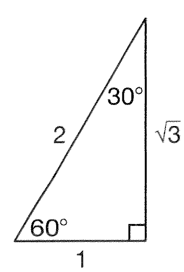


Solution

Since a 45-45-90 triangle is isosceles, we know that the other leg is **1 unit.** Using the Pythagorean Theorem we find that the hypotenuse is **$\sqrt{2}$ units.**



Since a 30-60-90 triangle is half an equilateral triangle, the shortest side is half the hypotenuse. Therefore, the hypotenuse is **2 units.** Using the Pythagorean Theorem we find the remaining side is **$\sqrt{3}$ units.**



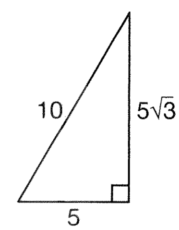
All 45-45-90 triangles have side lengths in the ratio of $1{:}1{:}\sqrt{2}$. All 30-60-90 triangles have side lengths in the ratio of $1{:}\sqrt{3}{:}2$. (Some students remember this ratio as 1, 2, $\sqrt{3}$ noting that the longest side is 2.)

Example 3

The shortest side of a 30-60-90 triangle is 5 inches. What are the lengths of the other sides?

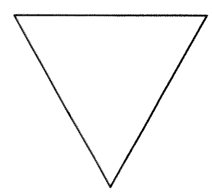
Solution

The side length ratio of all 30-60-90 triangles is $1{:}\sqrt{3}{:}2$. If the shortest side is 5, we simply apply the scale factor 5 to the other two sides. Since 5 times $\sqrt{3}$ is $5\sqrt{3}$, the other leg is **$5\sqrt{3}$ inches** and the hypotenuse is **10 inches.**



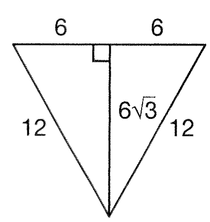
Example 4

A traffic sign in the shape of an equilateral triangle has sides 12 inches long. What is the area of the front of the sign?



Solution

To find the area we multiply the perpendicular base and height. Although we are not given the height, we know that half an equilateral triangle is a 30-60-90 triangle. We can use the side-length ratio $1{:}\sqrt{3}{:}2$ to find that the height is $6\sqrt{3}$ in.^{2}, because 6 times $\sqrt{3}$ is $6\sqrt{3}$. Now we find the area.



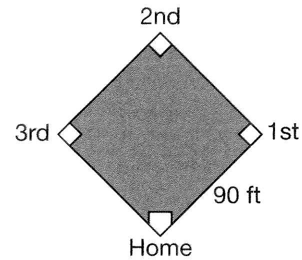
Step:	**Justification:**
$A = \frac{1}{2}bh$	Area formula for a triangle
$A = \frac{1}{2}(12 \text{ in.})(6\sqrt{3} \text{ in.})$	Substituted
$A = \mathbf{36\sqrt{3}\ in.^2}$	Simplified

To estimate measures involving $\sqrt{2}$ or $\sqrt{3}$, it is helpful to remember these approximations.

$$\sqrt{2} \approx 1.41$$
$$\sqrt{3} \approx 1.73$$

Example 5

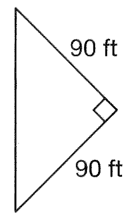
A major league baseball diamond is a square with sides 90 feet long. If a base runner tries to "steal" second base, the catcher throws the ball from home plate to second base. How far does the catcher throw the ball?



Solution

The throw from home plate to second base is a diagonal of the square, which is the hypotenuse of a 45-45-90 triangle. Since the legs are 90 feet long, the hypotenuse is $90\sqrt{2}$ feet. So the catcher throws the ball $\mathbf{90\sqrt{2}}$ **ft** or about **127 ft.**

$$90\sqrt{2} \approx 90 \cdot 1.41 \approx 126.9$$



Practice Set

a. Sketch a 45-45-90 triangle and indicate the angle measures and side lengths if one leg measures 1 inch.

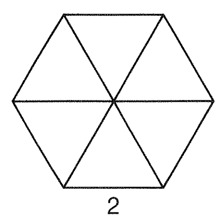
b. Sketch a 30-60-90 triangle and indicate the angle measures and side lengths if the shortest side is 1 inch.

c. What are the approximations for $\sqrt{2}$ and $\sqrt{3}$?

d. How many units is it from the origin to (2, 2) on the coordinate plane?

e. If each side of an equilateral triangle is 2 inches long, then what is the area of the triangle?

f. A regular hexagon has sides 2 feet long. What is the exact area of the hexagon? What is the approximate area of the hexagon?



Written Practice

Strengthening Concepts

1. (45) The multipacks of dried fruit contain three bags of banana chips and five bags of raisins. Several multipacks were purchased. If there were 104 bags in all, how many bags of banana chips were there?

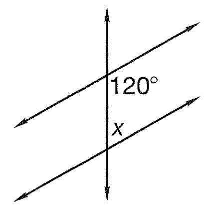
2. (58) Sixty percent of the students preferred raisins to banana chips. If there were 95 students in all, how many preferred banana chips?

3. (3, 4) The net weight of a bag of banana chips is 0.9 oz. A bag of raisins has a net weight of 1.1 oz. What is the net weight of a multipack that contains 3 bags of banana chips and 5 bags of raisins?

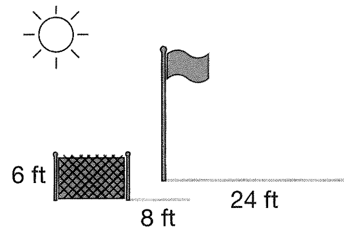
4. (53) **a.** Find the mean, median, mode and range of these measurements: 2.5 cm, 2.6 cm, 2.5 cm, 2.4 cm, 2.6 cm.

b. Display the data in a line plot.

5. (54) Two of the lines are parallel. Find *x*.



*** 6.** (65) **Evaluate** A flagpole casts a shadow 24 feet long while a six-foot fence pole casts a shadow 8 feet long. Find the height of the flagpole.



*** 7.** (56) Write the equation of the line with slope $\frac{1}{2}$ that intercepts the *y*-axis at 3.

*** 8.** (51) **Generalize** Simplify and express using positive exponents.

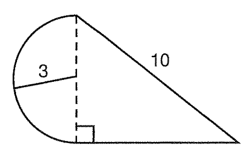
$$x^{-2}y^{-1}xy^2z$$

Solve.

9. (38) $\frac{4}{3}x = 28$

10. (14, 36) $\frac{x^2}{9} = 4$

11. (37, 40) Find the area of the figure to the right. (Use $\pi = 3.14$. Dimensions are mm.)



*** 12.** (63) Write $16\frac{2}{3}\%$ **a** as a decimal and **b** as a reduced fraction.

*** 13.** (63) **a.** Write $\frac{2}{3}$ as a decimal and percent.

b. Select one of these forms to state, "2 out of 3 band members are also in choir."

*** 14.** (15, 36) **Evaluate** Evaluate $-b - \sqrt{b^2 - 4ac}$ when $a = 1$, $b = -10$, and $c = 9$.

*** 15.** (64) Blair sees a highway sign that reads "speed limit 100 km/hr." Use a unit multiplier to convert 100 km/hr to miles per hour. (1 mi ≈ 1.6 km).

*** 16.** (64) Nora earns $12 per hour. Convert $12 per hour to cents per minute.

17. (21) Use the Distributive Property to

a. Factor: $15x^2 - 10x$

b. Expand: $7(x - 3)$.

Simplify.

18. (27) $(-3)^2 - 3^2 - 3^0$

19. (27) $\frac{z^3 b^4 r}{x^2 b^2 z^3}$

20. (21, 63) $\frac{9}{16} + \frac{1}{16} \cdot \left(\frac{2}{3}\right)^{-1}$

21. (33, 36) $(-6) - (-2) - (-6)(-2)$

Solve. If there is no solution, write *no solution.*

22. (50) $0.9 + 0.3x = 2.4$

23. (50) $3(-2x + 1) = 21$

* **24.** (66) **Model** Sketch a square and sketch an equilateral triangle. Draw a diagonal of the square to divide the square into two congruent right triangles. Then draw a segment in the equilateral triangle that divides it into two congruent right triangles. In all four right triangles, write each angle measure.

* **25.** (41) **Formulate** The eighth grade science class noticed that a faucet in the science lab was leaking. They gathered the following data using a watch with a second hand and a 100-millilter beaker to measure the amount of water that leaked during a specific time.

Time (Seconds)	10	20	30	40	50	60	70
Water Volume (ml)	8	16	24	32	40	48	56

Is the relationship between elapsed time and water volume proportional? Explain your answer. If the faucet continues to leak at the same rate, how many seconds will it take to fill the 100-milliliter beaker? Write and solve a proportion.

LESSON 67

Percent of Change

Power Up

Building Power

facts Power Up N

mental math

a. **Algebra:** Evalute: $\frac{1}{2}(10)(v^2)$ when $v = 3$

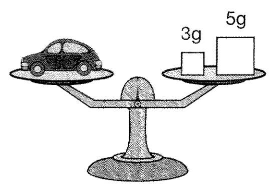
b. **Estimation:** $\sqrt{17} + \sqrt{24}$

c. **Fractional Parts:** $\frac{2}{3}$ of 99

d. **Percent:** 25% of $60

e. **Rate:** How long will it take to read 500 pages at 25 pages per day?

f. **Measurement:** Find the mass of 7 toy cars.



g. **Geometry:** A cube has how many vertices?

h. **Calculation:** How much money is half of a tenth of a dollar?

problem solving

Kurt rides his bike 18 miles per hour south. Andrew starts at the same place and rides 17 miles per hour north. If they keep on riding at these rates, about how long will it take them to be 210 miles apart?

New Concept

Increasing Knowledge

We have solved percent problems about part of a whole. Percents are also used to describe a change. A change may be a decrease or an increase.

- The dress was on sale for 30% off the regular price.
- The median price of a house in the county rose 20% last year.

We can use a ratio table to sort the numbers and help us write a proportion. We make three rows, the first for the original number, the second for the change, and the third for the new number. Here is the model.

	%	Actual count
Original	100	
Change (±)		
New		

Notice that the "original" percent is 100%. Also notice that we add or subtract the "change" number depending on whether the change is an increase (+) or a decrease (−).

Example 1

Donna bought a dress at a 30% off sale for $42. What was the original price of the dress?

Solution

We know the new price is $42. We know the percent of change, which is a 30% decrease. We can subtract 30% from the "original" 100% to find that the percent for the "new" price is 70%. We want to know the original price, so we use the "original" row in the proportion.

Thinking Skill
Verify
Why do we use the "New" row to complete the proportion?

	%	Actual count
Original	100	x
Change (−)	30	
New	70	42

$$\frac{100}{70} = \frac{x}{42}$$

$$70x = 100 \cdot 42$$

$$x = \frac{100 \cdot 42}{70} = \mathbf{60}$$

The original price was **$60.** The answer is reasonable because the original price should be somewhat more than the sale price. We can find 30% of $60 and subtract that result from $60 to check our answer. Notice that the sale price is **30% off** the regular price and **70% of** the regular price.

Analyze How much is the discount in dollars?

Example 2

The median price of a house in a neighborhood rose 20% in one year to $288,000. By how many dollars did the median price increase?

Solution

We record the given numbers in a ratio table. The original percent is 100%. The change is an increase of 20%. We add the increase to find the "new" percent is 120%. We know that the "new" price is $288,000. We are asked for the amount of change in dollars.

	%	Actual count
Original	100	
Change (+)	20	c
New	120	288,000

$$\frac{20}{120} = \frac{c}{288{,}000}$$

$$120c = 20 \cdot 288{,}000$$

$$c = \frac{20 \cdot 288{,}000}{120}$$

$$c = \mathbf{48{,}000}$$

The new price is **20% greater than** the original price and **120% of** the original price. We find that the median price increased **$48,000.**

Analyze Knowing the dollar increases, what are two ways we can find the original price?

Discuss Compare and contrast the percent columns in examples 1 and 2. How are they the same? How are they different?

Practice Set

a. An item with a regular price of $15.00 is discounted 30%. What is the sale price?

b. A shopkeeper buys an item for $25.00 and marks up the price 80% to sell in the store. What is the store price?

c. Lillian saves $12 buying a rug on sale for $36.00. The sale price of the rug was what percent of the regular price?

d. The town's population increased from 80,000 to 90,000 in ten years. What was the percent of increase?

e. A $15 item on sale for $12 is marked down what percent?

f. **Analyze** If Jenna buys an item for **40% off** the regular price, will she pay more or less than half price?

g. **Analyze** If Jenna buys an item for **40% of** the regular price, will she pay more or less than half price?

h. If Jenna saves $20 buying an item for 40 percent off the regular price, then what were the regular price and the sales price? Make a ratio table and fill in all six cells (spaces for numbers in the table).

i. Using the following information, write two questions that can be answered using the methods taught in this lesson. Evan uses a 25% off coupon to buy a shirt regularly priced at $24.

Written Practice

Strengthening Concepts

1. (45) At the university, the student to teacher ratio is 12 to 1. If there are 3000 students, how many teachers are there?

2. (58) Ten percent of the class period was spent taking roll and making announcements. If there were 45 minutes that remained, how long is the class period?

3. (3, 4) Nine dimes and nine nickels are equal to two nickels and how many quarters?

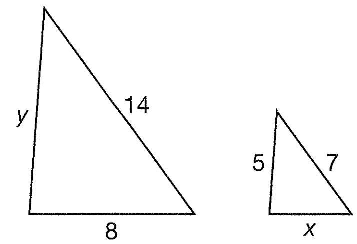
4. (7) Find the mean, median, mode and range of these diving scores: 9.6, 9.5, 9.5, 9.7, 9.8.

*** 5.** (67) **Analyze** The airline advertised lower fares. The new fare for flying to Austin is \$320. If this fare is 20% lower than the original fare, what was the original fare?

6. (Inv. 2, 35) The two triangles are similar.

a. Find the lengths x and y.

b. Are the triangles right triangles? How do you know?



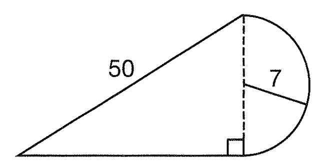
*** 7.** (56) Write an equation for a line that has a slope of $\frac{2}{3}$ and passes through the origin.

*** 8.** (56) **Represent** Gwen drew a line with a slope of –3 passing through the point (0, –8). Write an equation for the line Gwen drew in slope-intercept form.

9. (41) Graph the line $y = -2x$. Is the point $(-4, 2)$ on the line?

10. (Inv. 3) True or false: All parallelograms have two pairs of congruent angles.

11. (Inv. 2, 39) Find the perimeter of the figure to the right. Round to the nearest meter. (Dimensions are in m. Figure is not to scale.)



12. (48) The bank offered a 0.6% interest rate. Find 0.6% of \$1300.

*** 13.** (63) Write $\frac{5}{9}$ as a **a** decimal and **b** percent.
c Order these numbers from least to greatest: 0.55, 0.56, $\frac{5}{9}$.

14. (Inv. 1, 20) The vertices of a triangle are $(-6, 1)$, $(6, 1)$, and $(6, 6)$. Find **a** the perimeter and **b** the area of the triangle.

15. (21) Use the Distributive Property:

a. Factor $5x^2 + 5x + 10$

b. Expand $-3(x + 3)$

16. (32) Jenny has a deck of 52 alphabet cards (26 uppercase and 26 lower case). Jenny selects one card.

a. What is the probability that she selects a vowel?

b. If on the first draw Jenny selects a vowel and does not replace it, how many cards remain? How many are vowels?

c. Supposing Jenny selected a vowel on her first draw, what is the probability she selects a vowel on her second draw?

* **17.** (64) Ryun runs in a race at a rate of 400 meters per minute. Use a unit multiplier to convert 400 meters per minute to miles per minute. (1 mi ≈ 1600 m.)

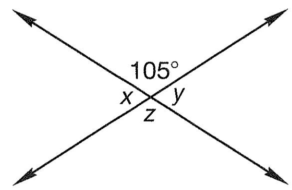
Simplify.

18. (15) **a.** $(-2)^2 - 4(3) + (\sqrt{11})^2$

b. (27) $\frac{m^2b^4r^3}{r^4b^4m}$

19. (22, 63) $\frac{5}{12} - \frac{3}{4} \cdot \left(\frac{9}{5}\right)^{-1}$

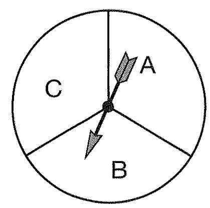
20. (54) Find x, y, and z.



21. (32) A coin is flipped and the spinner is spun.

a. What is the sample space of the experiment?

b. What is the probability of tails and a consonant (B or C)?



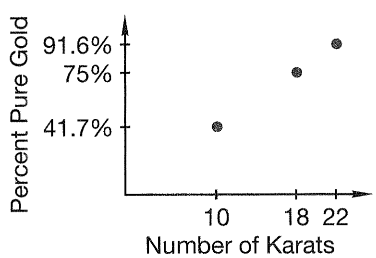
Solve.

22. (50) $5(x - 6) = 40$

23. (50) $8x + 3x - 2 = 75$

24. (50) $0.007x + 0.28 = 0.7$

* **25.** (41, 48) The purity of gold is described in *karats*. The graph at right shows the relationship between the number of karats a piece of gold has and its purity. Is the relationship proportional? How many karats is 100% pure gold? Each karat represents what fraction of pure gold? Determine the purity of a piece of 14 karat gold jewelry.



LESSON 68

• Probability Multiplication Rule

Power Up — *Building Power*

facts Power Up N

mental math

a. **Algebra:** Evaluate: $\sqrt{a^2 + 4^2}$ when $a = 3$

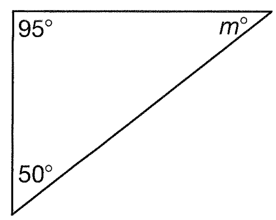
b. **Number Sense:** Compare $\frac{1}{5} + \frac{2}{5} \bigcirc \frac{1}{5} \times \frac{2}{5}$

c. **Powers/Roots:** $x^7 \cdot x$

d. **Probability:** Jackson flipped a coin 20 times and the outcome was tails twelve times. What is the probability that the coin lands tails-up on the next flip?

e. **Ratio:** Every three students shared two calculators. If there are 30 students in the class how many calculators are there?

f. **Geometry:** Find the measure of $\angle m$.



g. **Measurement:** How many inches is 5 ft 2 in.?

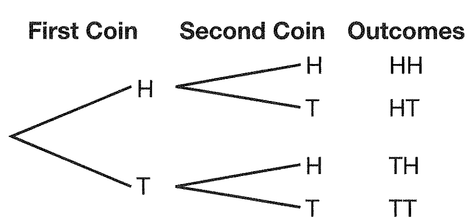
h. **Calculation:** How much money is one fifth of one fourth of a dollar?

problem solving Theo is thinking of two numbers. He gives these two hints: the sum of the numbers is 6, and one number is ten more than the other number. What are the numbers?

New Concept — *Increasing Knowledge*

When an experiment occurs in stages or has more than one part, we can use a tree diagram to help us find the sample space and count outcomes.

In the experiment of flipping a coin twice (or flipping two coins), there are four outcomes.



In the first flip, there are two possible outcomes. For each of these outcomes there are two possible outcomes in the second flip. Therefore the total number of outcomes is the product $2 \cdot 2 = 4$.

This experiment illustrates the fundamental counting principle.

Fundamental Counting Principle

> If an experiment has two parts, the first part with m possible outcomes and the second part with n possible outcomes, then the total number of possible outcomes for the experiment is the product $m \cdot n$.

Example 1

How many possible outcomes are there for an experiment in which two number cubes are rolled together?

Solution

The first roll has 6 possible outcomes, and the second roll has 6 possible outcomes. Altogether, there are $6 \cdot 6$, or **36 possible outcomes.**

Represent Use a tree diagram to illustrate the number of outcomes when two number cubes are rolled together.

A tree diagram can be used to display and count outcomes of experiments with multiple stages. If an experiment has only two stages, we can use a rectangular table or area model. For example, we can display the sample space of rolling two number cubes this way.

First Cube

Second Cube	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

We see that the 6 by 6 rectangle has $6 \cdot 6$ possible outcomes, even though there are only 11 different sums. This sample space display is preferable to listing the 11 different outcomes because the table shows equally-likely outcomes. If we were to simply list the outcomes from 2 to 12, we would not be able to tell which outcomes are more/less likely.

The multiplication counting principle also helps us count ways that an event can occur.

Example 2

Two number cubes are rolled once. How many possible outcomes are there for which the first roll is even and the second roll is even?

Solution

An even outcome (2, 4, or 6) can occur in 3 ways for each number cube, so the total number of possible outcomes for which both outcomes are even is $3 \cdot 3$, which is **9.**

Connect If two number cubes are rolled, what is the probability of rolling an even number on each cube?

To find probabilities of events in experiments with multiple parts, we can count outcomes by multiplying:

$$P(\text{even and even}) = \frac{3 \cdot 3}{6 \cdot 6} = \frac{1}{4}$$

Math Language

In an experiment, two events are **independent** if the occurrence of one does not change the probability of the other.

We can also consider the probability of each part: For the first roll, $\frac{1}{2}$ of the outcomes are even, and for each of these, $\frac{1}{2}$ of the second roll are even. In all, $\frac{1}{4}$ (which is $\frac{1}{2}$ of $\frac{1}{2}$) of the outcomes are two even numbers. That is,

$$P(\text{even and even}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

This is an example of a multiplication rule for probability.

Multiplication Rule for Probability

If events A and B are independent, then $P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$

Example 3

Suppose a bag contains 2 red and 3 blue marbles. Robert selects a marble from the bag, looks at it, then replaces it. Then he selects a second marble from the bag. What is the probability he selects a red marble and then a blue marble?

Solution

$$P(\text{red and blue}) = P(\text{red}) \cdot P(\text{blue})$$
$$= \frac{2}{5} \cdot \frac{3}{5}$$
$$= \mathbf{\frac{6}{25}}$$

Verify What is the probability Robert selects a red and blue marble in either order? Calculate the probability then verify the results by drawing a 5 by 5 table.

Example 4

Maribel holds a stack of 52 alphabet cards: 26 uppercase cards (A, B, C, … Z) and 26 lowercase cards (a, b, c, … z). She shuffles the stack, and selects one card. What is the probability that it is an uppercase vowel?

Solution

There are five uppercase vowel cards (A, E, I, O, U) in the 52 cards.

$$P(\text{uppercase vowel}) = \frac{5}{52}$$

We can also find this probability by considering the independent events "uppercase" and "vowel". (Even if the letter on the card is uppercase, the probability it is a vowel remains $\frac{5}{26}$.) Since one-half of the cards are uppercase cards, and there are 10 total vowel cards, the probability can be computed:

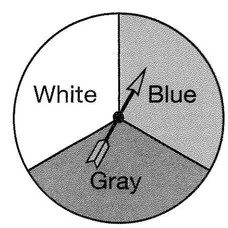
$$P(\text{uppercase vowel}) = P(\text{uppercase}) \cdot P(\text{vowel})$$

$$P(\text{uppercase vowel}) = \frac{1}{2} \cdot \frac{5}{26}$$

$$P(\text{uppercase vowel}) = \frac{5}{52}$$

Practice Set

In a certain experiment, a coin is tossed and this spinner is spun. Compute the probability of the events in **a** and **b.**



a. *P*(heads and blue)

b. *P*(heads and not blue)

c. Two number cubes are rolled. What is the probability of rolling a number greater than four on each number cube?

d. A coin will be flipped four times. What is the probability the coin will land heads-up all four times?

Written Practice

Strengthening Concepts

1. (45) In Mateo's aquarium, silver fish outnumber red fish nine to two. If there were twenty-two silver and red fish in all, how many were silver?

2. (48) Fifteen percent of the teachers wore ties. If 51 teachers did not wear a tie, how many teachers were there in all?

3. (3, 4) Kurt collected quarters while Robert collected dimes. Together they had 30 coins. If ten were quarters, how much money did they have?

* **4.** (58) **Analyze** By the middle of the ninth inning, 9% of the fans had left the stadium. If 18,200 fans remained, how many had left?

5. (65) Imelda held a 6-inch long pencil at arm's length so that it matched the height of a tree 100 feet away. If the pencil was 24 inches from her eyes, about how tall was the tree?

6. (41) Mr. Martinez parks at a lot where he is charged a fee as shown in the table.

Months	1	2	3	4	5
Fee ($)	17	34	51	68	85

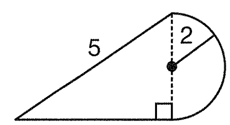
If he parks at the lot for 9 months, what will he be charged?

7. (32) If a number cube is rolled once, which is the more likely outcome, a prime number or a composite number?

8. (41) Graph $y = \frac{2}{5}x - 1$. Is the point (10, 4) on the line?

* **9.** (56) **Model** Two points on the graph of a linear equation are (−2, 3) and (1, 0). Graph the points and draw a line through the points. Then write the equation of the line.

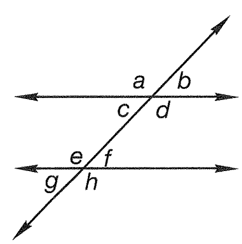
10. (Inv. 2, 40) Find the area of the figure to the right. Round to the nearest square inch. (Units are in inches.)



* **11.** (63) **Evaluate** Convert $\frac{7}{9}$ to **a** a percent and **b** a decimal, then **c** write these numbers from least to greatest:

$$\frac{6}{8}, \frac{7}{8}, \frac{7}{9}$$

12. (54) Which two angles are adjacent to $\angle f$?



13. (21) Use the Distributive Property to **a** factor: $3x^2 + 3x + 9$, **b** expand $x(x - 5)$.

* **14.** (64) Use a unit multiplier to convert 18 inches per minute to inches per hour.

* **15.** (64) Songhee leisurely rides her bicycle at 10 miles per hour. In about 675 revolutions of the wheels, Songhee travels one mile. Use a unit multiplier to convert 10 miles per hour to revolutions per hour.

Simplify.

16. (27) $\frac{2m^2ac^3}{ca^2m^2}$

17. (36) $(-3)(-4) - (-5)(6)$

Solve.

* **18.** (50) $\frac{3}{5}x + \frac{7}{10} = \frac{19}{10}$

* **19.** (50) $\frac{1}{4} + \frac{3}{8}x = 1$

20. (50) $0.003x - 0.02 = 0.07$

21. (50) $0.03x - 0.02 = 0.07$

22. (50) $5x + 2x - 3 = 18$

Graph the solutions on a number line.

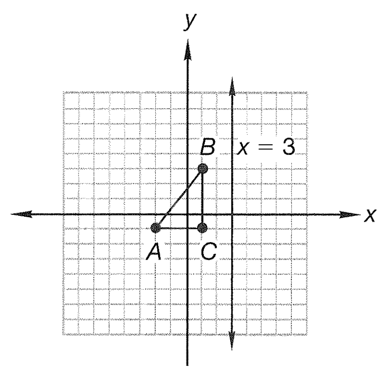
23. (62) $x - 3 < 1$

24. (62) $3x - 2 \geq 4$

25. (Inv. 5) If $\triangle ABC$ is reflected across the line $x = 3$, which of the following will be the coordinates of the image of point A (A')?

A (1, −1) **B** (2, −1)

C (5, −1) **D** (8, −1)



Early Finishers

Real-World Application

The United States Census Bureau provides data about the nation's people and economy. The 2004 U.S. Census reported 1,108 of 4,000 adults aged 25 years and older have a bachelor's degree or higher level of education. What is the chance that the next adult you meet 25-years-old or older has a bachelor's degree or higher level of education?

LESSON 69

• Direct Variation

Power Up

Building Power

facts Power Up N

mental math

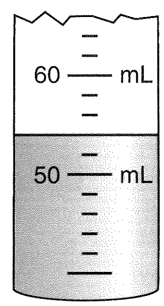
a. Statistics: Find the mode of this data set: 12.5, 13.1, 12.5, 13.0, 12.9

b. Estimation: $5\sqrt{17}$

c. Sequences: Find the next three terms: $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \ldots$

d. Proportion: $\frac{3}{7} = \frac{21}{x}$

e. Measurement: Find the volume of liquid in this container.



f. Percent: 20% of $50

g. Scientific Notation: Write 7×10^{-5} in standard notation.

h. Calculation: Square 10, − 1, ÷ 11, square it, − 1, ÷ 10, square it, − 1, ÷ 9

problem solving Some pulsars spin at a rate of 30,000 rotations per minute (rpm). How many rotations per second is that? How long does it take for each rotation?

New Concept

Increasing Knowledge

Math Language
Recall that a **constant** is a number whose value does not change.

Recall that when two variables are proportional the value of one variable can be found by multiplying the other by a constant factor. We call this relationship between the variables **direct variation.**

Direct Variation

$$y = kx$$

In this equation x and y are the variables, and k is the constant multiplier, called the **constant of proportionality.** The constant of proportionality might be a unit rate, a scale factor, a unit multiplier, or an unchanging number that relates the two variables.

Here are some examples of direct variation. Can you find the variables and the constant of proportionality in each example?

- total pay = $12 × hours worked
- miles traveled = 60 mph × hours traveled
- perimeter = 4 × side length

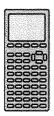
Characteristics of Direct Variation

1. When one variable is zero, the other is zero (the graph of the equation intersects the origin).
2. As one variable changes, the other variable changes by a constant factor (the graph is a line).

We can solve the equation $y = kx$ for k by dividing both sides of the equation by x.

$y = kx$	direct variation equation
$\frac{y}{x} = k$	divided by x

This alternate equation shows that the ratio of the two variables in direct variation equals the constant of proportionality. We use the word proportionality because quantities that vary directly are proportional—their ratio is constant. Knowing this we can determine from a table if a relationship is an example of direct variation.



Visit www.SaxonPublishers.com/ActivitiesC3 *for a graphing calculator activity.*

Example 1

Which of the following is an example of direct variation? (The variables are underlined.)

A A taxi company charges three dollars to start the ride plus two dollars per mile.

B The area of a square is the square of the length of its side.

C The perimeter of a square is four times the length of its side.

Solution

We make a table for the three relationships and check for a constant ratio.

$D = 2m + 3$

Miles	Dollars	$\frac{\text{Dollars}}{\text{mile}}$
1	5	$\frac{5}{1}$
2	7	$\frac{7}{2}$
3	9	$\frac{9}{3}$
4	11	$\frac{11}{4}$

$A = s^2$

s	A	$\frac{A}{s}$
1	1	$\frac{1}{1}$
2	4	$\frac{4}{2}$
3	9	$\frac{9}{3}$
4	16	$\frac{16}{4}$

$P = 4s$

s	P	$\frac{P}{s}$
1	4	$\frac{4}{1}$
2	8	$\frac{8}{2}$
3	12	$\frac{12}{3}$
4	16	$\frac{16}{4}$

Of these three, the only example of direct variation is **the relationship between the perimeter of a square and its side length.** In every number pair, the ratio reduces to 4. The relationship is proportional. In the other examples, the ratios vary. The relationships are not proportional.

We can also identify direct variation from a graph. The three relationships from example 1 are graphed below.

A Taxi

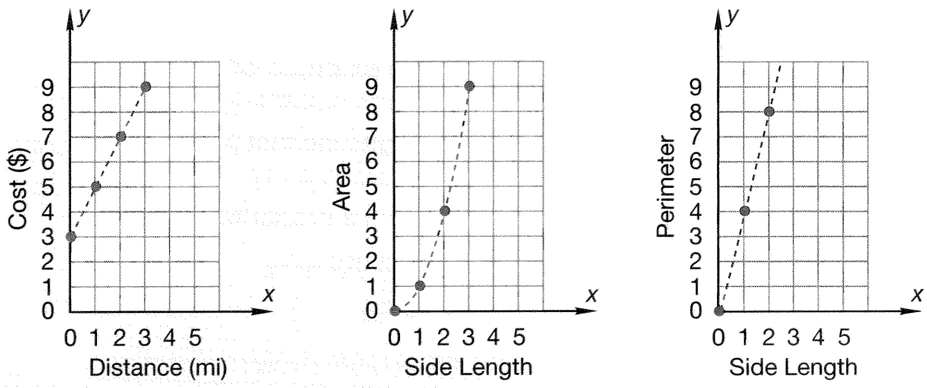
$D = 2m + 3$

B Area

$A = s^2$

C Perimeter

$P = 4s$



Thinking Skill

Analyze

How do $A = s^2$ and $D = 2m + 3$ differ from $y = kx$?

A graph indicates a proportional relationship only if all the points fall along a straight line that intersects the origin. Graph **B** is not linear. Graphs **A** and **C** are linear, but graph **A** does not align with the origin.

Characteristics of Graphs of Direct Variation

Quantities which vary directly have pairs of points which lie on the same line, and the line intersects the origin.

Connect Look at the perimeter graph and follow the line. How many units does the line rise for each unit it moves to the right?

Analyze Are paired quantities whose points are aligned always proportional?

In a direct variation relationship, the value of one variable *depends* on the value of the other. One variable is **independent,** like the number of hours a person works, and the other is **dependent,** like the total pay the person earns. In a table, the independent variable is typically placed in the first column and the dependent variable in the second column. On a graph, the independent variable is typically plotted on the horizontal axis and the dependent variable on the vertical axis. The constant of proportionality is the ratio of the dependent variable to the independent variable.

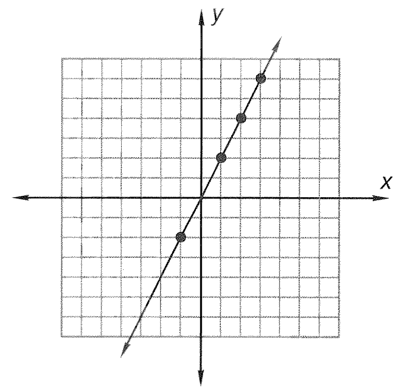
Example 2

Doubleday doubled every number Eunice said. The table shows four numbers Eunice said and Doubleday's replies. Sketch a graph of all the numbers Eunice and Doubleday could say. Is the relationship of numbers they say an example of direct variation? Write an equation for the relationship. What is the constant of proportionality?

E	D
2	4
1	2
−1	−2
−2	−4

Solution

Eunice and Doubleday could go on endlessly saying numbers. The graphed line shows all the numbers they could say that fit their rule. The arrowheads show that the possible pairs of numbers extend without end.



This relationship is an example of direct variation. We see that the possible pairs of numbers form a straight line that passes through the origin. Any two pairs of numbers from the relationship form a proportion.

An equation for the relationship is

$$d = 2e$$

in which e is the number Eunice says and d is the number Doubleday says. The constant of proportionality is **2** because every value of d is 2 times the given value of e. Every d-to-e ratio equals 2.

e	d	$\frac{d}{e}$
2	4	$\frac{4}{2} = 2$
1	2	$\frac{2}{1} = 2$
−1	−2	$\frac{-2}{-1} = 2$
−2	−4	$\frac{-4}{-2} = 2$

Example 3

Jarrod earns $12 an hour helping a painter. Create a table that shows his pay for 1, 2, 3, and 4 hours of work. Is Jarrod's pay for hours worked an example of direct variation? If so, what is the constant of proportionality?

Solution

We make a table for hours worked and pay. We calculate the ratio of pay to hours worked for each row of our table.

The ratios are equal because they all reduce to $\frac{12}{1}$. The relationship **is an example of direct variation,** and the constant of proportionality is **12.**

Hours	Pay	$\frac{P}{H}$
1	12	$\frac{12}{1}$
2	24	$\frac{24}{2}$
3	36	$\frac{36}{3}$
4	48	$\frac{48}{4}$

Represent Write an equation for the relationship in example 3.

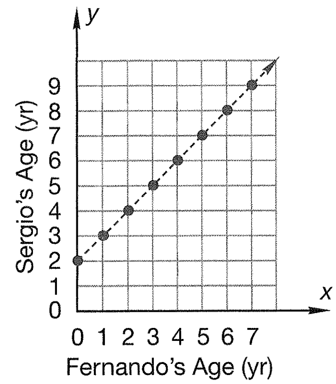
Practice Set

a. Trinny drew some equilateral triangles and measured their side lengths and perimeters. She decided that the perimeter of an equilateral triangle varies directly with its side length. Write an equation for the relationship and identify the constant of proportionality.

s	*p*
2	6
3	9
4	12
7	21

p = perimeter
s = side length

b. *Analyze* The relationship between Sergio's age and Fernando's age is illustrated in the graph. Are their ages proportional? Is the relationship an example of direct variation? Why or why not?



c. *Formulate* The weight of water in a trough is directly proportional to the quantity of water in the trough. If 20 gallons of water weigh 166 pounds, what is the weight of 30 gallons of water? Find the constant of proportionality in pounds per gallon. Then write an equation for the relationship using p for pounds and g for gallons. Start the equation $p =$.

Written Practice

Strengthening Concepts

1. (45) The number of home-team fans in a crowd of 980 fans was 630. What was the ratio of home-team to visiting-team fans?

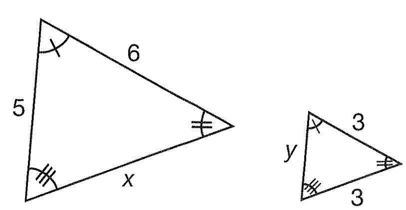
2. (48) Lani recognized thirty-two percent of the melodies. If she heard 25 melodies in all, how many did she not recognize?

*** 3.** (67) The soccer league had 30% more participants this year than last year. If there are 1170 participants this year, how many participated last year?

*** 4.** (69) *Analyze* Does the table to the right show direct variation? If so, state the constant of variation.

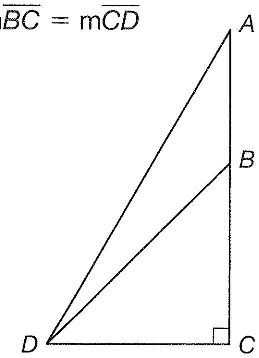
x	2	18	30
y	5	45	80

5. (35) Find the lengths of x and y in the similar triangles to the right.



* **6.** (26) *Classify* Specifically describe the two triangles in problem **5** as a transformation.

7. (66) In the figure, m$\angle ADC = 60°$. Segment $CD = 1$ in. m$\overline{BC}$ = m$\overline{CD}$



a. Find m$\angle DAC$.

b. Find m$\angle DBC$.

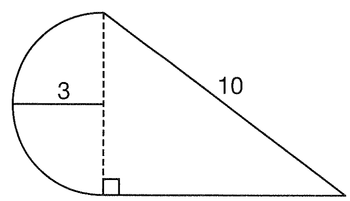
c. Find DB.

d. Find DA.

8. (56) Write the equation of the line with slope $\frac{1}{4}$ that passes through the point $(0, -1)$.

9. (56) Graph the points $(-2, -4)$ and $(6, 0)$ and draw a line through the points. Then write the equation of the line.

10. (37) An atrium was built in the shape shown to the right. Find the **a** area and **b** perimeter of the atrium. Round to the nearest whole unit. (Dimensions are in m.)



11. (63) Write $\frac{4}{9}$ as **a** a decimal and **b** a percent. **c** Write these three numbers in order from least to greatest: 0.4, 0.5, $\frac{4}{9}$.

12. (32) A coin is tossed three times.

a. Find the sample space of the experiment.

b. What is the probability of "at least two heads?"

c. Name a different event with the same probability as "at least two heads."

13. (21) Use the Distributive Property to **a** factor: $5x^2 + 10x + 15$, **b** expand: $-4(2x + 3)$.

14. (69) The water leaks from the faucet at a rate of $\frac{1}{2}$ gal/hr.

a. Create a table to show how many gallons of water would leak in 1, 2, 3, and 4 hours.

b. Is the relationship between the number of hours that elapse and the amount of water that leaks an example of direct variation? Explain.

15. (64) Use a unit multiplier to convert the rate described in Exercise **14** to gallons per day.

Simplify.

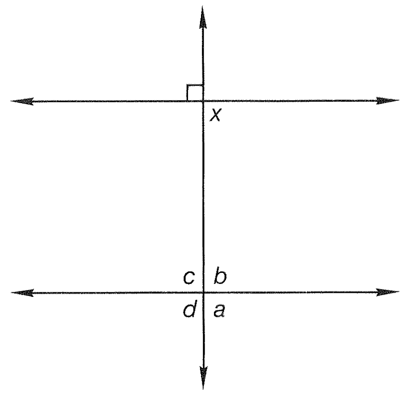
16. (27) $\frac{8x^3yz^2}{4xyz^2}$

17. (36, 63) $\left(-\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^{-1}$

18. (54) Two of these lines are parallel.

a. Compare: m∠x ◯ m∠a.

b. If it is uncertain whether or not the lines are parallel, can the comparison be made? Explain.



Solve.

19. (50) $\frac{3}{8}x + \frac{3}{4} = \frac{9}{8}$

20. (50) $\frac{1}{3}x - \frac{3}{4} = \frac{5}{12}$

21. (50) $0.007x + 0.03 = 0.1$

22. (44) $\frac{x}{5} = \frac{1}{2}$

23. (44) $\frac{2}{3} = \frac{5}{x}$

24. (64) Use a unit multiplier to convert 120 miles per hour to miles per minute.

* **25.** (41) *Conclude* The table shows the altitude of a hot-air balloon during the first 5 minutes and 45 seconds of flight. Does the table indicate a proportional relationship? Explain. How can you determine the balloon's altitude 5 minutes into the flight?

Hot-Air Balloon Altitude

Time (Minutes)	Altitude (Feet)
0	0
1	200
2	400
2.5	500
3	600
4	800
5.75	1150

LESSON 70

Solving Direct Variation Problems

Power Up

Building Power

facts Power Up N

mental math

a. **Algebra:** Evaluate: $\sqrt{6^2 + b^2}$ when $b = 8$

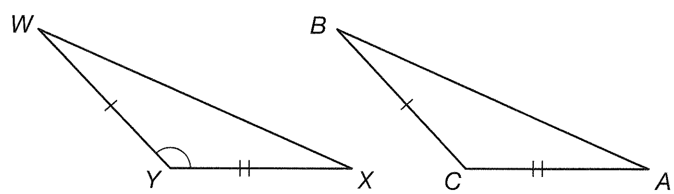
b. **Estimation:** $2\sqrt{26}$

c. **Number Sense:** Compare $0.9 \times 0.8 \bigcirc 0.9 + 0.8$

d. **Powers/Roots:** $x \cdot x \cdot x^2$

e. **Rate:** Hector sails north at 20 miles per hour. Skip sails south from the same starting location at 25 miles per hour. How far apart are they after 2 hours?

f. **Geometry:** The triangles are congruent. Use correct symbols to complete this congruence statement: $\triangle WXY \cong \triangle B$__



g. **Select a Method:** To calculate the area of a photo that is $2\frac{1}{2}$ inches long and $1\frac{1}{2}$ inches wide, which would you use?
A mental math
B pencil and paper
C calculator

h. **Calculation:** Half of half of a dollar is how much more than half of a tenth of a dollar?

problem solving

Find the missing square root. (Hint: What number should be in the tens place of the answer? Since 1521 has a 1 in the ones place, what options do you have for the ones place of the answer?)

$$\sqrt{1600} = 40$$
$$\sqrt{900} = 30$$
$$\sqrt{1521} = \_\_$$

New Concept

Increasing Knowledge

Recall that quantities which vary directly are proportional. The table below shows that the total price of CDs varies directly with the number of CDs a customer purchases.

Number of CDs	Price ($)
1	15
2	30
3	45
4	60

This means their ratio is constant. We often use the letter k for the constant of proportionality.

$$\frac{\text{price}}{\text{number}} = k$$

Every price/number ratio in the table equals the constant, which is 15.

$$\frac{15}{1} = \frac{30}{2} = \frac{45}{3} = \frac{60}{4} = 15$$

One quantity (the price) is determined by multiplying the other variable (the number of CDs) by the constant.

$$\text{price} = 15 \times \text{number}$$

Predict How much will it cost to purchase 6 CDs?

When you know that two quantities vary directly, knowing just one set of paired numbers allows us to solve for missing numbers in other pairs.

Example 1

The amount Ellen charges for banquet food varies directly as the number of people that attend. If Ellen charges $780 for 60 people, how much does she charge for 100 people?

Solution

We know that the ratio of the amount charged to the number of people is constant:

$$k = \frac{\text{charge}}{\text{number}}$$

$$k = \frac{\$780}{60}$$

$$k = \$13$$

The amount Ellen charges is $13 times the number of people who attend:

$$\text{charge} = \$13 \times \text{number}$$

When 100 people attend, Ellen charges **$1300.**

$$\text{charge} = \$13 \times 100 = \$1300$$

An alternate method for solving this problem is to write a proportion. In one case we know the charge and number, $780 and 60. In the second case, the number is known and the charge is unknown. For each case we write a ratio. We know the ratios are equal because we are told the quantities vary directly.

$$\frac{\text{charge}}{\text{number}} \quad \frac{\$780}{60} = \frac{x}{100}$$

Now we can solve using cross products.

$$\$780(100) = 60x$$
$$\$78000 = 60x$$
$$\$1300 = x$$

Example 2

Driving at a constant speed, the distance Maggie travels varies directly with the amount of time she drives. If Maggie drives 75 miles in one and one-half hours, how long does it take her to drive 100 miles?

Solution

Maggie's driving speed is constant. Using k for the constant speed, we write two equivalent equations.

$$k = \frac{\text{distance}}{\text{time}} \qquad \text{distance} = k \times \text{time}$$

The first equation gives us the constant of proportionality, in this situation, Maggie's speed. We express her driving time as 1.5 hours.

$$k = \frac{75}{1.5}$$
$$k = 50 \text{ mph}$$

We have found her speed. We use the second equation to solve for the driving time to drive 100 miles:

$$\text{distance} = 50 \times \text{time}$$
$$d = 50t$$
$$100 = 50t$$
$$2 = t$$

It takes Maggie **2 hours** to drive 100 miles.

Thinking Skill

Connect

Write a proportion to solve example 2.

Example 3

There were 23 people in line in front of Delia waiting to be helped. Three minutes later there were 18 people waiting in front of Delia. Predict how much longer Delia will wait to be helped.

Solution

In three minutes, five people ($23 - 18 = 5$) were helped, so we can find the average length of time it took to help each person.

$$\frac{3 \text{ min}}{5 \text{ people}} = 0.6 \text{ min/person}$$

There is no certainty that the next 18 people will be helped at the same rate, but Delia can use this number to predict how much longer she will wait (w) in line.

$$\begin{aligned} w &= 0.6 \text{ min/person} \times 18 \text{ people} \\ &= 10.8 \text{ min} \\ &\approx \mathbf{11\ min} \end{aligned}$$

We estimate that Delia will need to wait about 11 more minutes to be helped.

Practice Set

a. The distance a coiled spring stretches varies directly with the amount of weight hanging on the spring. If a spring stretches 2 cm when a 30 gram weight is hung from it, how far will it stretch when a 75 gram weight is hung from the spring?

b. The number of words Tom types varies directly with the amount of time he spends typing. If Tom types 100 words in 2.5 minutes, how many words does he type in 15 minutes?

c. Traffic was dense for the evening commute. Mitchell traveled three miles on the highway in 15 minutes. Predict how much longer it will take Mitchell to reach his exit five miles away.

Written Practice

Strengthening Concepts

1. (45) Insects outnumbered humans 1001 to 2 at the campsite. If there were 10 humans at the campsite, how many insects were there?

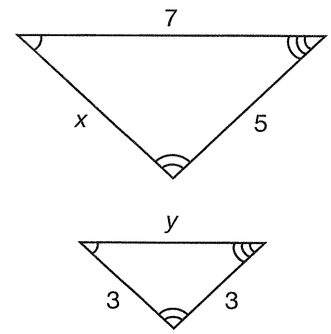
2. (48) Sixty-four percent of the coins in the money box were foreign currency. If 45 coins were not foreign currency, how many coins were in the money box?

*** 3.** (67) Robert bought a component for his computer at 15% off. If the sale price was $42.50, what was the price before the reduction?

*** 4.** (69) Does the table to the right show direct variation? If so, give the constant of variation. If not, explain why.

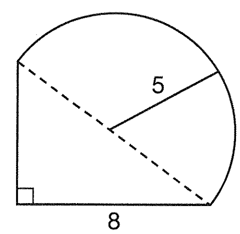
L	W
6	4
10	8
14	12

5. (35) Find the lengths of x and y in the similar triangles to the right. What is the scale factor from the larger to smaller triangle?



6. (56) Graph $y = \frac{1}{3}x - 1$. Is point (6, 1) a solution?

*** 7.** (37) *Analyze* Dale plans to build a balcony similar to the figure. Find the **a** area and **b** perimeter of the balcony. Round to the nearest whole unit. (Dimensions are in feet.)



*** 8.** (63) Write $\frac{1}{6}$ as **a** a decimal and **b** a percent.

Model Graph each solution on a number line.

*** 9.** (62) $x - 3 \geq -1$

*** 10.** (62) $2x + 3 < 1$

Simplify.

11. (27) $\dfrac{12w^2x^2y^3}{8x^2y^2}$

12. (51) $\frac{4}{5} - \frac{3}{5} \cdot \left(\frac{1}{3}\right)^{-1}$

13. (31) $3x + 3 - x - 3$

14. (24, 25) $0.2 + 0.3(0.4)$

Solve.

*** 15.** (50) $\frac{1}{3}x + \frac{4}{9} = \frac{7}{9}$

*** 16.** (50) $\frac{4}{5} + \frac{2}{15}x = \frac{14}{15}$

17. (50) $0.005x - 0.03 = 0.01$

18. (64) Use a unit multiplier to convert 30 inches per second to feet per second.

19. (21) Factor:

a. $2x^2 + 6x + 10$

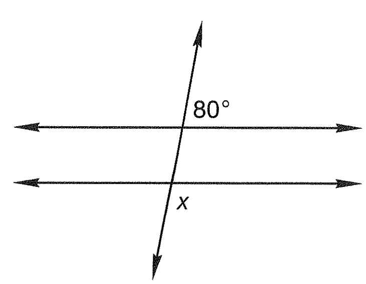
b. $x^3 - x^2$

20. (32) There are two black socks and one gray sock in a drawer. Without looking, Phil selects two socks.

a. Find the sample space of the experiment

b. What is the probability that Phil selects two black socks?

21. (54) Two of the lines are parallel. Find x.



22. (68) A number cube is rolled three times. What is the probability of rolling 6 all three times?

23. (66) Sketch a 30-60-90 triangle. If the longest side of a similar triangle is one foot, then how many inches long is the shortest side?

24. (44) Solve: $\frac{3}{x} = \frac{5}{7}$

25. (69) The cost of renting a riding lawnmower is $17.50, plus $2.50 for every hour that the mower is out. Create a table for the cost of renting a lawnmower for the first 5 hours a lawnmower is out. Is the relationship proportional? Explain why or why not.

Early Finishers
Real-World Application

The middle-school band is trying to raise money for concert uniforms. Rodney set up a booth outside the library each day for five days to raise money. The results of his efforts are recorded in the table below.

a. On average, how much money did Rodney make per day?

b. On which day did Rodney make the most dollars per hour?

Day	Donations	Hours
Sunday	$28.55	5
Monday	$26.40	3
Thursday	$22.35	3
Friday	$38.76	4
Saturday	$76.32	8

INVESTIGATION 7

Focus on

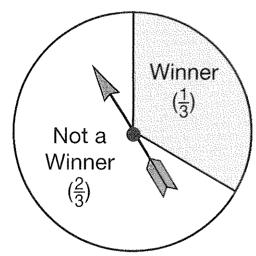
• Probability Simulation

A cereal company claims on its box, "One in three win a free movie pass!" If you decide to purchase this cereal, what is the probability it contains a movie pass?

The cereal box indicates that the probability of selecting a winner is $\frac{1}{3}$. This **theoretical probability** means that, out of all the cereal boxes produced for the contest, one-third contain a free pass. Thus, on average, one in three boxes is a winner.

Does this mean that if you buy three boxes you are guaranteed a winner? We can simulate the purchase of 3 boxes to determine the probability of buying at least one winner.

For the simulation we will need a spinner that looks like this:



Each spin will represent the purchase of one cereal box. The probability of landing on a "winner" is $\frac{1}{3}$, since the winning region fills $\frac{1}{3}$ of the circle.

Activity 1

Probability Simulation

Construct the face of a spinner like the one shown above. Use a compass to draw a circle. Then compute the central angle that intercepts $\frac{1}{3}$ of the circle:

$$\frac{1}{3}(360°) = 120°$$

Use a protractor to help make the central angle. Shade the interior of the 120° central angle, and label the region "winner." You may label the rest of the spinner "not a winner." Place the spinner on a level surface for the simulation.

To keep track of our results, we make and use a table that looks like this:

	Spin 1	Spin 2	Spin 3	At least 1 Winner?
Trial 1				
Trial 2				
Trial 3				
⋮				

Make rows for at least six trials on the table.