

• Volumes of Prisms and Cylinders

Power Up

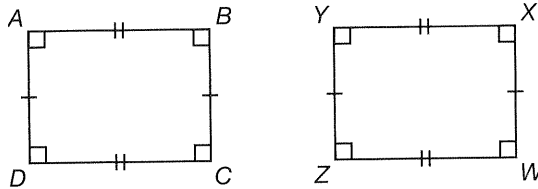
Building Power

facts

mental math

Power Up P

- a. **Algebra:** Simplify: $5 + 2x + 6x$
- b. **Sequences:** A rule of a sequence is $a_n = 5n$. What are the first four terms?
- c. **Rate:** Marvin loads the truck at a rate of 15 boxes per minute. Craig loads 12 boxes per minute. How many boxes can they load together in 3 minutes?
- d. **Number Sense:** Arrange A, B, C and D from least to greatest:
A 0.04 **B** 0.08 **C** 1.2 **D** 0.016
- e. **Powers/Roots:** $x^4 \cdot x^7 \cdot x^{-1}$
- f. **Geometry:** Rewrite and complete this congruence statement:
 $\square ABCD \cong \square YX___$



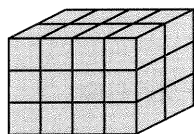
- g. **Select a Method:** Mr. Chen is completing his income tax forms. To perform the calculations, which method is he likely to use?
A mental math
B pencil and paper
C calculator
- h. **Calculation:** How many minutes is a quarter of an hour plus a third of an hour?

problem solving

Theodore counted by threes, saying one number every second. Franklin started 12 seconds later and counted by fives, saying one number every second. Will they ever say the same number at the same time? If so, what will that number be?

Hint: What number did Theodore say right before Franklin started?
 How much does Franklin gain on Theodore each second?

In Lesson 42 we found the volume of a rectangular solid by first calculating the area of the base and then multiplying the area of the base times the height.

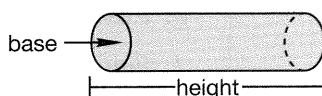
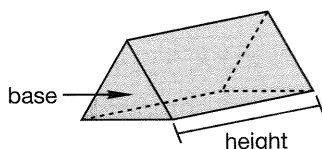


$$\text{Volume} = \text{Area of base} \cdot \text{height}$$

$$V = Bh$$

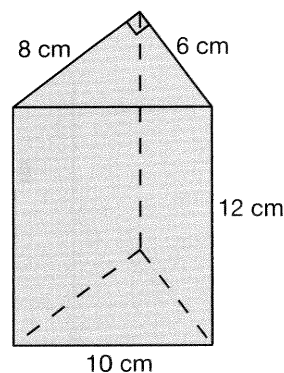
We may use the same process to find the volume of other prisms and cylinders. We find the area of one of the bases, whether it is a polygon or a circle, and multiply that area by the height.

Recall that the bases of a prism or a cylinder are the parallel surfaces at opposite ends of the figure. The height is the perpendicular distance between the bases, whether or not the prism or cylinder is upright.



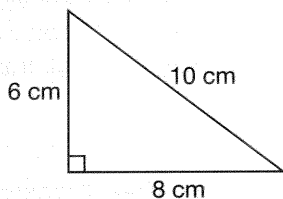
Example 1

Sketch the base of this triangular prism. Find the volume.



Solution

The base of the triangular prism is a right triangle.



The perpendicular sides are 6 cm and 8 cm, so the area of the triangular base is 24 cm^2 .

Thinking Skill

Discuss

How do units used for area differ from units used for volume?

Step:

$$A = \frac{1}{2}bh$$

Justification:

Area formula for triangle

$$A = \frac{1}{2}(8 \text{ cm})(6 \text{ cm})$$

Substituted

$$A = 24 \text{ cm}^2$$

Simplified

The height of the prism is the distance between the bases, which is 12 cm. We find the volume by multiplying the area of the base times the height.

Step:

$$V = Bh$$

Justification:

Volume formula

$$V = (24 \text{ cm}^2)(12 \text{ cm})$$

Substituted

$$V = 288 \text{ cm}^3$$

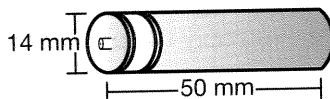
Simplified

The volume of the triangular prism is **288 cm³**.

Analyze Notice that in the formulas in example 1, the variable representing base appears in both lower-case form (b) and upper-case form (B). What is the difference in the meanings of b and B ?

Example 2

A double-A battery is about 50 mm long and has a diameter of about 14 mm. Find the approximate volume of a double-A battery. (Use $\frac{22}{7}$ for π .)



Solution

The base is a circle. We find the area of the circle substituting $\frac{22}{7}$ for π .

Step:

$$A = \pi r^2$$

Justification:

Area formula for circle

$$A \approx \frac{22}{7} (7 \text{ mm})^2$$

Substituted

$$A \approx 154 \text{ mm}^2$$

Simplified

We find the volume of the cylinder by multiplying the area of the base by the height.

Step:

$$V = Bh$$

Justification:

Volume formula

$$V \approx (154 \text{ mm}^2)(50 \text{ mm})$$

Substituted

$$V \approx 7700 \text{ mm}^3$$

Simplified

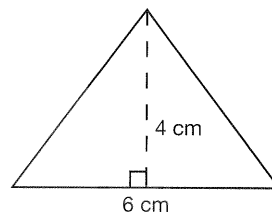
The volume of a double-A battery is about **7700 mm³**.

Reading Math

Recall that the symbol \approx means *is approximately equal to*.

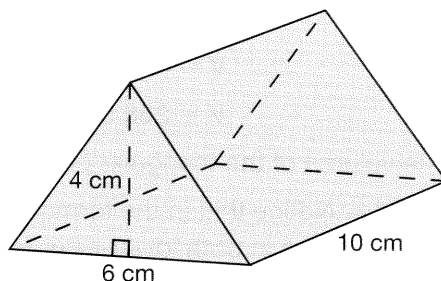
Example 3

The figure shows a triangle that is the base of a triangular prism. The height of the prism is 10 cm. Sketch the prism. Then find its volume.



Solution

We sketch the prism.



To find the volume we first find the area of the triangular base.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6 \text{ cm})(4 \text{ cm})$$

$$A = 12 \text{ cm}^2$$

Then we multiply the area of the base by the height.

$$V = bh$$

$$V = 12 \text{ cm}^2 \cdot 10 \text{ cm}$$

$$V = 120 \text{ cm}^3$$

The volume is **120 cm³**.

Example 4

Buster has two empty cylindrical soup cans. The smaller can is 2 in. in diameter and 3 in. high. The dimensions of the larger can are twice the dimensions of the smaller can.

- Find the volume of both cans in terms of π .
- What is the scale factor from the smaller can to the larger can?
- If the smaller can is filled with water and the contents are poured into the larger can and the process is repeated until the larger can is full, how many small cans of water would be used to fill the larger can?

Solution

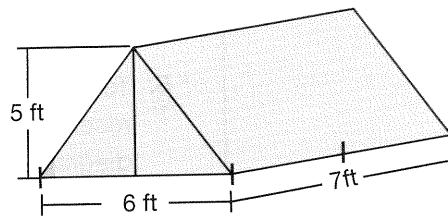
a. Smaller can: $3\pi \text{ in.}^3$

Larger can: $24\pi \text{ in.}^3$

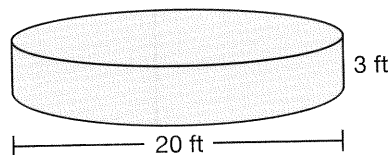
Practice Set

- b. The dimensions of the larger can are twice the dimensions of the smaller can, so the scale factor is **2**.
- c. We cube the scale factor to find the relationship between the volumes of the cans: $2 \cdot 2 \cdot 2 = 8$. **Eight** small cans will fill the larger can.

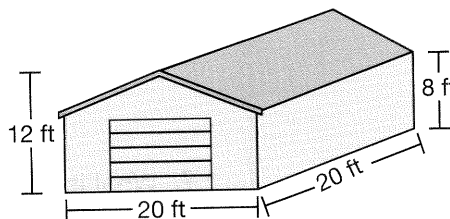
- a. **Analyze** A pup-tent has the dimensions shown. Sketch the base and then find the volume of the tent.



- b. A cylindrical backyard pool 3 feet high and 20 feet in diameter holds how many cubic feet of water when full? (Use 3.14 for π and round the answer to the nearest ten cubic feet.)



- c. The walls of a garage are 8 feet high, and the peak of the gable roof is 12 feet high. The floor of the garage is a 20 foot square. Sketch the base. Then find the volume of the garage.



- d. A cylindrical candle that is 15 cm high has a diameter of 8 cm. Sketch the shape. Find the volume of the candle (express in terms of π).
- e. Brenda is putting cubical boxes that are 6 inches on edge into a larger cubical box with inside dimensions 12 inches on edge. How many of the smaller boxes will fit in the larger box?

Written Practice

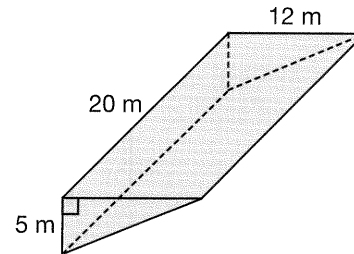
Strengthening Concepts

- * 1. **Analyze** A store purchases an item for \$80. It marks up the item by ⁽⁶⁷⁾ 50%. What is the customer's price?
2. ⁽⁴⁵⁾ There were 600 sheep and goats in the hills. If the ratio of sheep to goats is 1 to 2, how many sheep are there?
3. ⁽⁴⁸⁾ Seventy-six percent of the brass instruments in the band leading the parade were trombones. If there were 100 brass instruments, how many were trombones?

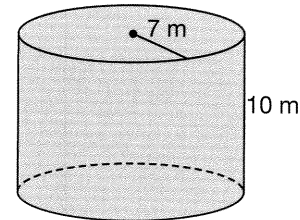
- * 4. **Explain** In the table are shown the number of hours Wendy works and the amount of pay she receives. Does this table show direct variation? If so, give the constant of variation. What does the constant mean in this situation?

Hours	Pay
3	27
4	36
5	45
6	54

- * 5. To roughly estimate the volume of the reservoir pool, Isabel sketches this diagram. What volume does she estimate?

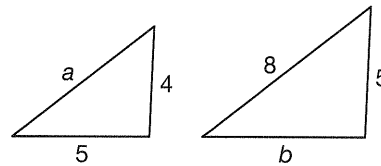


- * 6. **Estimate** Suppose a municipal water tank has these dimensions. Find the volume of the tank. First express the volume in terms of π . Then approximate the volume to the nearest whole unit.

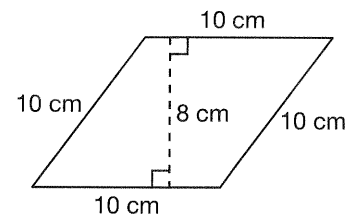


- * 7. What is the volume of one cubic yard in cubic feet? Use three unit multipliers.

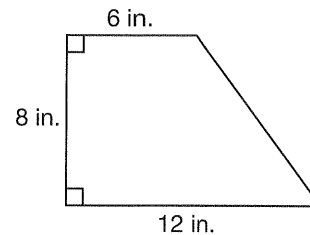
8. Find a and b in the similar triangles to the right.

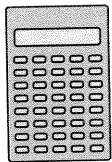


9. a. Find the area of the parallelogram.
b. Name the type of parallelogram.



- * 10. For a–c, refer to the figure.
a. Classify the quadrilateral.
b. Find its area.
c. Find its perimeter.





- * 11. When fractions with a denominator of 7 are converted to decimals, there are six digits that repeat. Write **a** $\frac{1}{7}$ as a decimal and **b** $\frac{2}{7}$ as a decimal.
c What do you notice about the two decimal numbers?

12. If a cube-shaped box with edges one foot long is stored in a cube-shaped box with edges two feet long, then what percent of the volume of the larger box is occupied by the smaller box?

- * 13. **Generalize** A rule of a sequence is $a_n = 7(2^n)$. Find the first three terms of the sequence.

14. Factor:

(21)

a. $-2x^2 - 2x - 2$

b. $5x^2 - 10x$

15. What is the equation of a line passing through the origin with a slope of positive one?

(56)

Generalize Simplify.

16. $(-3)(-2) - (-1)^2$

(36)

* 17. $\sqrt{27}$

(74)

* 18. $\sqrt{32}$

(74)

19. $\frac{12m^3}{18xm^{-2}}$

(27, 51)

Solve.

* 20. $\frac{4.5}{6.3} = \frac{x}{7}$

(25, 44)

* 21. $\frac{4}{3}x - \frac{2}{7} = -\frac{34}{21}$

(50)

* 22. $\frac{x}{4} = \frac{0.42}{0.14}$

(25, 44)

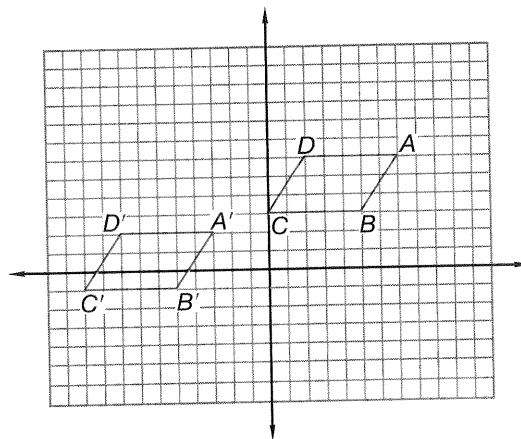
23. $0.03 + 0.011x = 0.36$

(50)

24. $\frac{m}{5} = 2.2$

(25, 38)

25. Describe the transformation applied to figure ABCD to create figure A'B'C'D'.



• Inequalities with Negative Coefficients

Power Up

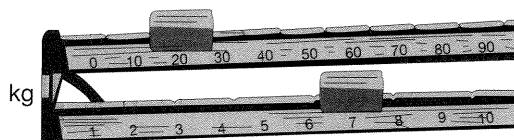
Building Power

facts

Power Up P

mental math

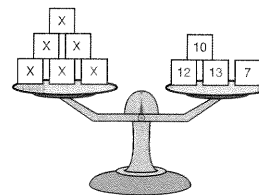
- a. **Algebra:** Simplify: $2x + 4 + 7x$
- b. **Fractional Parts:** $\frac{3}{8}$ of 56
- c. **Probability:** 90% of the students were right handed. What is the probability that one student selected at random will be left handed?
- d. **Percent:** 60% of \$10
- e. **Proportions:** The ratio of runners to cyclists was 9 to 4. If there were 26 all together, how many were cyclists?
- f. **Measurement:** What mass is indicated on this scale?



- g. **Select a Method:** Before depositing checks at the bank, Imelda fills out a deposit slip. If she is depositing three checks, she will probably use what method?
 - A mental math
 - B pencil and paper
 - C calculator
- h. **Calculation:** $100 - 1, \div 9, \times 4, + 1, \div 9, \times 2, - 1, \div 9$

problem solving

Is it possible to remove one block from each side and have the scale still be balanced? If so, which blocks should be removed?



New Concept

Increasing Knowledge

Solving inequalities is similar to solving equations. However, there is one important difference: when solving inequalities, we reverse the comparison symbol when multiplying or dividing by a negative number.

In the table below we summarize properties of inequalities. Notice the distinction in the table between positive and negative multipliers and divisors.

Properties of Inequalities

If $a > b$, then	
$a + c > b + c$ $a - c > b - c$	
$ac > bc$	if c is positive
$\frac{a}{c} > \frac{b}{c}$	if c is positive and $c \neq 0$
$ac < bc$	if c is negative
$\frac{a}{c} < \frac{b}{c}$	if c is negative and $c \neq 0$

Connect Why can c not equal zero in the inequalities $\frac{a}{c} > \frac{b}{c}$ and $\frac{a}{c} < \frac{b}{c}$?

Example 1

Multiply both sides of this inequality by negative one and explain how to preserve the truth of the inequality.

$$6 > 5$$

Solution

The given inequality is true. Six is greater than five. However, if we multiply both sides of the inequality by -1 , then the numbers become -6 and -5 .

Since -6 is less than -5 , we must reverse the sign of the inequality to preserve the truth of the statement.

$$-6 < -5$$

Example 2

Solve and graph $-3x - 10 > 8$.

Solution

Step:	Justification:
$-3x - 10 > 8$	Given inequality
$-3x > 18$	Added 10 to both sides
$x < -6$	Divided both sides by -3 and reversed comparison symbol

We graph the result.



Only numbers less than -6 satisfy the original inequality. To check the solution we can substitute a number less than -6 in place of x and simplify. The resulting inequality should be true. We can also substitute a number equal to or greater than -6 in place of x . The resulting inequality should be false.

Thinking Skill

Explain

What does the open circle on the graph mean? How would the inequality change if the circle were filled?

Substitute -7	Substitute -5
$-3(-7) - 10 > 8$	$-3(-5) - 10 > 8$
$21 - 10 > 8$	$15 - 10 > 8$
$11 > 8$ (true)	$5 > 8$ (false)

Justify Is -6 a solution? Explain why or why not.

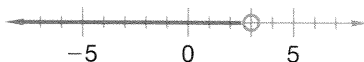
Example 3

Solve and graph.

$$-5(x - 3) > 5(x - 3)$$

Solution

Step:	Justification:
$-5(x - 3) > 5(x - 3)$	Given inequality
$-5x + 15 > 5x - 15$	Distributive Property
$-10x + 15 > -15$	Subtracted $5x$ from both sides
$-10x > -30$	Subtracted 15 from both sides
$x < 3$	Divided both sides by -10 and reversed the comparison symbol.



Example 4

Marci sees a shirt she likes that costs \$50. She will not buy it unless it is \$30 or less. To find the percent decrease in the price required, she solves the inequality:

$$50 - 50x \leq 30$$

Solve the inequality to find the minimum discount Marci requires.

Solution

First we solve the inequality.

Step:	Justification:
$50 - 50x \leq 30$	Given inequality
$-50x \leq -20$	Subtracted 50 from both sides
$x \geq \frac{20}{50}$	Divided both sides by -50 and reversed comparison symbol

Now we express the result as a percent.

$$x \geq 40\%$$

Marci will not buy the shirt unless it is discounted **40%** or more.

Practice Set

Model Solve. Then graph each inequality to show your work.

a. $-5x - 6 < -1$

b. $7x + 2 \leq 8x + 4$

c. $-5x + 25 > 5(x - 5)$

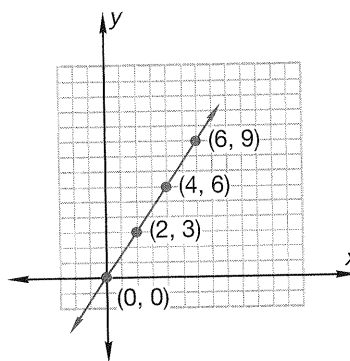
d. $-2x + 3 \geq -(x - 7)$

Written Practice

Strengthening Concepts

- ⁽⁴⁵⁾ The ratio of broken to working piano keys was 2 to 9. If there were 88 keys on the old piano, how many were broken?
- ⁽⁶⁷⁾ Last month the snake had a mass of 6.6 kilograms. Its mass is now 60% more than it was last month. What is the snake's mass now?
- ⁽⁵⁸⁾ * 3. Dena ordered a \$8.40 meal and Jules ordered a \$9.20 meal. If the sales tax is 8% and they leave a 20% tip on the price of the meal including tax, then what is the total?

- ⁽⁶⁹⁾ 4. Rosa watched the distance markers along the highway. She noticed that she had traveled 3 km in 2 min, 6 km in 4 min, and 9 km in 6 min. Does this graph show direct variation? If so, give the constant of proportionality. How far will she have traveled in 10 min?



- ^(41, 69) * 5. A calling card charges 50¢ for each call placed plus 3¢ per minute on the call.
 - Make a table that lists possible durations of phone calls and the corresponding charges.
 - Write an equation for the function.
 - Is the function an example of direct variation? Why or why not?
- ^(Inv. 6) * 6. **Connect** O'Grady averages 5 baskets for every 8 free throw attempts. If you were to construct a spinner to simulate O'Grady's free throw attempts, how many degrees would represent free throws made? Free throws missed?

Solve these inequalities and graph their solutions on a number line.

* 7. $-x + 15 > 0$
⁽⁷⁷⁾

* 8. $-2x \geq 2$
⁽⁷⁷⁾

Analyze Solve. If there is no solution, write, "no solution."

* 9. $\frac{4}{x} = \frac{0.22}{0.55}$
^(25, 44)

* 10. $\frac{3}{4} - \frac{2}{3}x = \frac{11}{12}$
⁽⁵⁰⁾

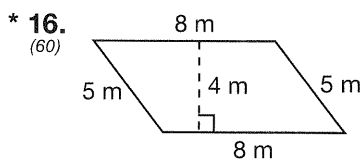
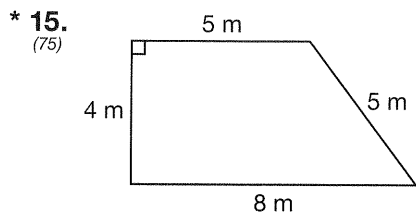
* 11. $12 = 16 - 4x$
⁽¹⁴⁾

* 12. $\frac{7}{8}x = 49$
⁽³⁸⁾

- ⁽⁷¹⁾ 13. If the dimensions of a 3-4-5 right triangle are increased 50%, by what percent is the area increased?

14. The rule for a sequence is $a_n = n^2 - n$. The first three terms are 0, 2, and 6. What is the tenth term?
(73)

Classify For problems 15 and 16, name the type of quadrilateral and find its area.



Simplify.

* 17. $\sqrt{54}$
(74)

18. $\frac{7x^{-3}y^3z^3}{14x^4y^3z^2}$
(27, 51)

19. Estimate each number between a range of two consecutive integers.
(16)

a. $\sqrt{90}$

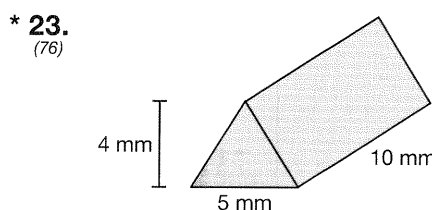
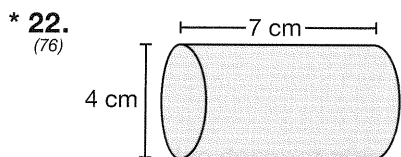
b. $\sqrt{80}$

c. $\sqrt{70}$

20. Geneka watched the bug scuttle 10 inches in 2 seconds. Use unit multipliers to convert 10 inches in 2 seconds to feet per minute.
(72)

21. Graph $y = -\frac{1}{2}x + 2$. Is (2, 1) a solution?
(56)

Find the volume of each solid. Express your answer in terms of π .



24. Write $\frac{4}{11}$ as a **a** decimal and **b** percent.
(63)

25. The table shows the favorite fruit of the eighth-grade class at Canyon Vista Middle School.
(11, Inv. 6)

Favorite Fruit	Percent of Class
strawberries	13.3
oranges	23.3
bananas	36.7
blueberries	6.7
grapes	20.0

- a. What percent of the class preferred strawberries or bananas?
b. If there are 30 eighth-graders in Mrs. Li's class, what is a good estimate for the number of students who prefer blueberries?
A 21 B 18 C 7 D 2
c. What fraction of eighth-graders do not prefer grapes?

• Products of Square Roots

Power Up

Building Power

facts

Power Up P

mental
math

- a. **Statistics:** Find the median of these numbers: 13, 15, 15, 15, 12
- b. **Estimation:** $6 - \sqrt{15}$
- c. **Number Sense:** Arrange A, B, C, and D from least to greatest:
A 0.2 **B** 0.8 **C** $0.2 + 0.8$ **D** 0.2×0.8
- d. **Powers/Roots:** $x^4 \cdot x^{-3}$
- e. **Proportion:** $\frac{x}{12} = \frac{2}{3}$
- f. **Geometry:** Draw an equilateral triangle. Does it have a right angle?
- g. **Scientific Notation:** Write 4.01×10^8 in standard notation.
- h. **Calculation:** $16 \div 2, \div 2, \div 2, \div 2, \div 2, \div 2$

problem
solving

Karina and Kristina are facing each other. Karina turns around once every 3 seconds. Kristina turns around once every 4 seconds. How long will it take for Karina and Kristina to face each other again? If they turn at this rate for a minute, how many times will they face each other?

New Concept

Increasing Knowledge

Math Language

The number under the radical symbol is called the **radicand**. In the expression $\sqrt{12}$, 12 is the radicand.

Recall the product property of square roots:

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

This property means that square roots can be factored.

$$\sqrt{36} = \sqrt{4} \cdot \sqrt{9} = 4\sqrt{1}$$

This property also means square roots can be multiplied. Here is one example.

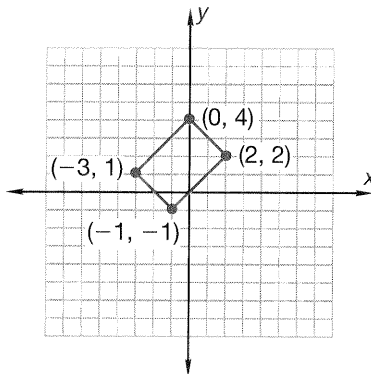
$$\sqrt{4} \cdot \sqrt{3} = \sqrt{12}$$

$$2 \cdot 3 = 6$$

In this lesson we will practice multiplying square roots.

Example 1

Find the length and width of this rectangle. Then find its area.

**Solution**

Using the Pythagorean Theorem we find that the length is $\sqrt{18}$, which simplifies to $3\sqrt{2}$, and the width is $\sqrt{8}$, which simplifies to $2\sqrt{2}$. To find the area we multiply the length and width.

$$\sqrt{18}\sqrt{8} = 3\sqrt{2} \cdot 2\sqrt{2}$$

$$\sqrt{144} = 6\sqrt{4}$$

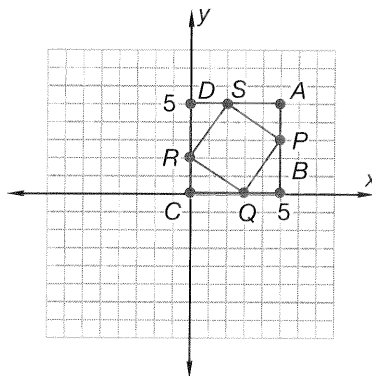
$$12 = 12$$

Using either the unsimplified or the simplified square roots, we find that the area is **12 square units**.

Verify How can we determine by the illustration that the area is 12 square units?

Example 2

The area of $\square PQRS$ is what percent of the area of $\square ABCD$?

**Solution**

Square ABCD is 25 square units. To find the area of square PQRS we square the length of its side. We use the Pythagorean Theorem to find the length.

$$s^2 = 2^2 + 3^2$$

$$s^2 = 4 + 9$$

$$s^2 = 13$$

$$s = \sqrt{13}$$

Each side of $\square PQRS$ is $\sqrt{13}$, so the area of $\square PQRS$ is

$$(\sqrt{13})^2 = \sqrt{13} \cdot \sqrt{13} = \sqrt{169} = 13$$

Square $PQRS$ is $\frac{13}{25}$ of the area of $\square ABCD$. We convert the fraction to a percent.

$$\frac{13}{25} \cdot \frac{4}{4} = \frac{52}{100} = 52\%$$

We find the area of $\square PQRS$ is **52%** of the area of square $ABCD$.

Model Was it necessary to find the length of one side of $\square PQRS$ in order to find its area? Sketch $\triangle PBQ$ and draw squares on each side to represent the squares of the legs and hypotenuse. Does square $PQRS$ appear in your sketch? Now look at the solution steps for finding the length of a side. Does the area of the square appear in the solution steps? Explain.

Example 3

Simplify:

a. $\sqrt{3} \cdot \sqrt{12}$

b. $\sqrt{42} \cdot \sqrt{24}$

c. $3\sqrt{6} \cdot \sqrt{15}$

Solution

a.

Step:

$$\sqrt{3} \cdot \sqrt{12}$$

$$\sqrt{36}$$

$$6$$

Justification:

Given

Multiplied $\sqrt{3}$ and $\sqrt{12}$

Simplified

b.

Step:

$$\sqrt{42} \cdot \sqrt{24}$$

$$\sqrt{42 \cdot 24}$$

$$\sqrt{2 \cdot 3 \cdot 7 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$$

$$\sqrt{2 \cdot 2} \sqrt{2 \cdot 2} \sqrt{3 \cdot 3} \sqrt{7}$$

$$2 \cdot 2 \cdot 3 \sqrt{7}$$

$$12\sqrt{7}$$

Justification:

Given

Multiplied $\sqrt{42}$ and $\sqrt{24}$

Factored 42 and 24

Grouped pairs

Simplified radicals

Multiplied $2 \cdot 2 \cdot 3$

c.

Step:

$$3\sqrt{6} \sqrt{15}$$

$$3\sqrt{6 \cdot 15}$$

$$3\sqrt{2 \cdot 3 \cdot 3 \cdot 5}$$

$$3\sqrt{3 \cdot 3} \sqrt{2 \cdot 5}$$

$$3 \cdot 3 \cdot \sqrt{10}$$

$$9\sqrt{10}$$

Justification:

Given

Multiplied $\sqrt{6}$ and $\sqrt{15}$

Factored 6 and 15

Grouped pair of 3s

Simplified radical

Multiplied $3 \cdot 3$

Practice Set

Simplify.

a. $\sqrt{2}\sqrt{14}$

b. $\sqrt{15}\sqrt{3}$

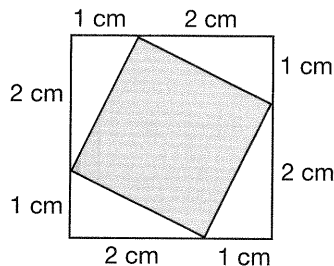
c. $3\sqrt{5}\sqrt{5}$

d. $5\sqrt{3} \cdot 2\sqrt{3}$

e. Find the length of a side of the square with vertices at (5, 4), (1, 5), (0, 1), and (4, 0). Then find the area of the square.

f. Find the area of a rectangle with vertices at (0, 5), (3, -1), (-1, -3), and (-4, 3).

g. **Explain** Describe two ways to find the area of the shaded square, one way using radicals and one way without using radicals.



Written Practice

Strengthening Concepts

* 1. **Evaluate** ⁽⁷¹⁾ A perpendicular cut through a water pipe reveals a circular cross section. The diameter of a $\frac{3}{4}$ -inch pipe is 50% greater than the diameter of a $\frac{1}{2}$ -inch pipe. The cross-sectional area of the larger pipe is what percent greater than the cross-sectional area of the smaller pipe?

* 2. **Analyze** ⁽⁶⁷⁾ This year there was a thirty-five percent reduction in spending from last year's budget. If last year's budget was \$1,200,000, what was this year's spending?

3. ⁽⁴⁸⁾ About twelve percent of the 4000 roses had already opened. About how many of the roses had opened?

4. ⁽⁷⁰⁾ The number of apples on display varies directly as the number of boxes the produce manager unloads. If the manager unloads 2 boxes there are 60 apples. How many apples are there when he unloads 3 boxes?

Simplify.

* 5. ^(74, 78) $\sqrt{8}\sqrt{6}$

* 6. ^(74, 78) $\sqrt{21}\sqrt{3}$

* 7. ^(74, 78) $\sqrt{6}\sqrt{12}$

8. ^(27, 51) $\frac{8m^4b^3}{4m^2b^{-1}}$

Solve.

* 9. ^(25, 44) $\frac{0.32}{0.56} = \frac{8}{m}$

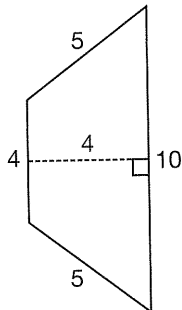
10. ⁽⁵⁰⁾ $0.5 - 0.02x = 0.1$

11. ⁽⁵⁰⁾ $\frac{6}{7} - \frac{1}{2}x = -\frac{1}{7}$

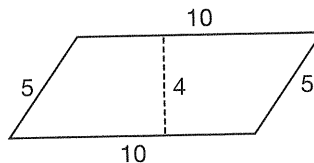
12. ⁽³⁸⁾ $\frac{2}{3}m = 6$

For problems **13** and **14**, name the type of quadrilateral and find its area. Dimensions are in cm.

* **13.**
(75)



* **14.**
(60)



15. Estimate each number between a range of two consecutive integers.
(16)

a. $\sqrt{60}$

b. $\sqrt{50}$

c. $\sqrt{40}$

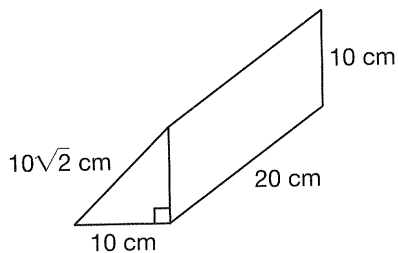
16. Use unit multipliers to convert \$15 per hour to cents per minute.
(72)

17. Graph $y = -\frac{1}{2}x - 5$. Is $(-10, 0)$ a solution?
(56)

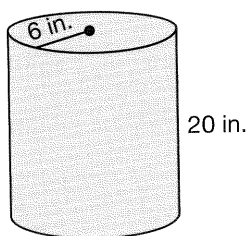
18. A five-foot tall person casts a six-foot shadow when a nearby tree casts a 30 foot shadow. Sketch a diagram for the description and find the approximate height of the tree.
(65)

Analyze For problems **19** and **20** find the volume of each solid. For problem **20** leave in terms of π .

* **19.**
(76)



* **20.**
(76)



21. A machine shrink-wraps 100 packages every 4 minutes. Which equation shows how many packages the machine can shrink-wrap in 10 minutes?
(34)

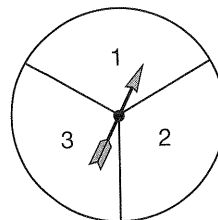
A $\frac{100}{4} = \frac{x}{10}$

B $\frac{100}{x} = \frac{10}{4}$

C $\frac{100}{10} = \frac{x}{4}$

D $100 \cdot 4 = 10 \cdot x$

22. A number cube is rolled and the spinner is spun. What is the probability that both outcomes will be even numbers? Express the probability as a ratio, as a percent, and as a decimal number rounded to the nearest hundredth.
(63, 68)

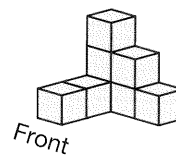


Solve these inequalities and graph their solutions on a number line.

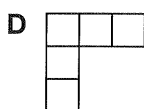
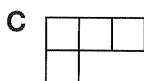
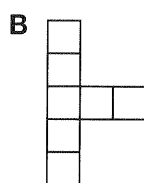
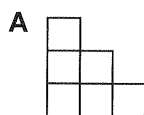
23. $4(x + 1) > 0$
(77)

24. $2(3 - x) \leq 2$
(77)

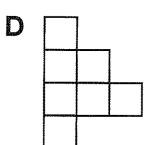
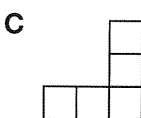
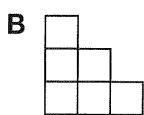
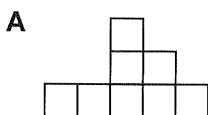
25. **Analyze** Use this figure to answer the following questions.
(Inv. 4)



a. Which of the following is the top view of the figure?



b. Which of the following is the front view of this figure?



• Transforming Formulas

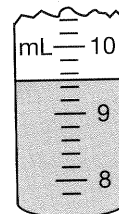
Power Up

Building Power

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mental
math

Power Up P

- Algebra:** Simplify: $12x - x - 9x$
- Sequences:** A rule of a sequence is $a_n = 6n$. What are the first four terms?
- Fractional Parts:** $\frac{3}{7}$ of 63
- Percent:** 0% of \$400
- Proportions:** 10 push-ups in 20 seconds is a rate of how many per minute?
- Measurement:** Find the volume indicated by this figure.



- Geometry:** What is the specific name for an equilateral quadrilateral that is not a square?
- Calculation:** Square 5, $\times 2$, $- 2$, $\div 2$, $+ 1$, $\sqrt{\quad}$

problem
solving

Patty, Laura and Barbara traveled by plane, train, and automobile. Patty did not take the train, and Barbara didn't travel on the ground. Which form of transportation did each woman use?

New Concept

Increasing Knowledge

We have used formulas to solve problems about rates and about geometric figures. Standard formulas are expressed with one variable isolated. For example, a rate equation states that distance traveled equals rate of travel times the amount of time traveled.

$$\text{distance} = \text{rate} \cdot \text{time}$$

$$d = rt$$

If we know the rate and time we substitute those values for r and t and then simplify to find d .

$$d = (60 \text{ mph})(2 \text{ hours})$$

However, if we know distance and rate, then we need to solve the equation to find the time.

$$240 \text{ mi} = (60 \text{ mph})t$$

Instead of solving the equation *after* substituting values, we can rearrange the formula *before* we substitute. Since we know we want to find time, we transform the formula to isolate t .

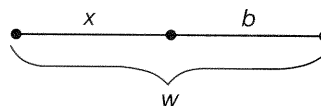
Step:	Justification:
$d = rt$	Distance formula
$\frac{d}{r} = \frac{rt}{r}$	Divided both sides by r
$\frac{d}{r} = t$	Simplified
$t = \frac{d}{r}$	Symmetric property of equality

Example 1

The illustration shows that w equals the sum of x and b .

$$w = x + b$$

Solve this equation for x .



Solution

Step:	Justification:
$w = x + b$	Given equation
$w - b = x$	Subtracted b from both sides
$x = w - b$	Symmetric property of equality

The equation, solved for x , is $x = w - b$.

Referring to the diagram, we find x by subtracting length b from length w .

Example 2

If we measure the circumference of a basketball, then we can calculate its diameter using the formula $C = \pi d$. Transform the formula to solve for d . Then describe the meaning of the transformed formula.

Solution

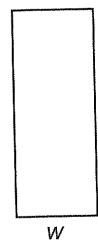
We isolate d by dividing both sides by π .

Step:	Justification:
$C = \pi d$	Given equation
$\frac{C}{\pi} = \frac{\pi d}{\pi}$	Divided both sides by π
$\frac{C}{\pi} = d$	Simplified
or $d = \frac{C}{\pi}$	Symmetric property of equality

The formula means that **to find the diameter of a circle (or of the basketball), we divide the circumference by π .**

Example 3

The formula for the area of a rectangle is $A = lw$. Transform the formula to solve for l . Then use the transformed formula to find the length of a rectangle with area 42 and width 3.

**Solution**

We isolate l by dividing both sides by w .

Step: **Justification:**

$$A = lw \quad \text{Given equation}$$

$$\frac{A}{w} = \frac{lw}{w} \quad \text{Divided both sides by } w$$

$$l = \frac{A}{w} \quad \text{Symmetric property of equality}$$

If $A = 42$ and $w = 3$, then $l = \frac{A}{w} = \frac{42}{3} = 14$ units.

Example 4

Solve for r : $A = \pi r^2$. Then find the radius of a circle with an area of 154 cm^2 . Use $\frac{22}{7}$ for π .

Solution

Isolate r by dividing both sides by π , then taking the square root.

Step: **Justification:**

$$A = \pi r^2 \quad \text{Given equation}$$

$$\frac{A}{\pi} = \frac{\pi r^2}{\pi} \quad \text{Divided both sides by } \pi$$

$$\frac{A}{\pi} = r^2 \quad \text{Simplified}$$

$$\sqrt{\frac{A}{\pi}} = r \quad \text{Square root of both sides}$$

$$r = \sqrt{\frac{A}{\pi}} \quad \text{Symmetric property of equality}$$

We can use this transformed formula to find the radius. We substitute 154 for area and $\frac{22}{7}$ for π .

$$r \approx \sqrt{\frac{154}{\frac{22}{7}}}$$

$$r \approx \sqrt{154 \cdot \frac{7}{22}}$$

$$r \approx \sqrt{49} = 7$$

The radius is about **7 cm**.

Thinking Skill**Explain**

Explain the formula $r = \sqrt{\frac{A}{\pi}}$ in words.

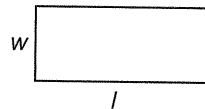
Practice Set

- The formula for the area of a rectangle is $A = lw$. Solve the formula for w . Then describe the meaning of the transformed formula.
- Solve the equation $P = a + b + c$ for a .
- The Pythagorean Theorem is

$$a^2 + b^2 = c^2$$

Solve for c .

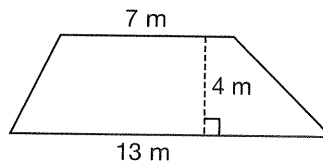
- The formula for the perimeter of a rectangle is $P = 2l + 2w$. Solve the formula for l . Then use your formula to find the length of a rectangular field with a perimeter of 620 feet and a width of 140 feet.



Written Practice

Strengthening Concepts

- ⁽⁶⁷⁾ Last year's meeting had 18,000 attendees. This year 20,700 attended. This was an increase of what percent?
- ⁽⁴⁸⁾ If 46% of the 150 people responded to the survey, how many did not?
- ^(43, 71) * **Analyze** One cube has edges 2 ft long. Another cube has edges 50% longer. The surface area of the large cube is what percent greater than the surface area of the smaller cube?
- ⁽⁷⁹⁾ * 4. Solve for r . $C = 2\pi r$
- ⁽³²⁾ * 5. Two red marbles and one blue marble are in a bag. One marble is selected then placed back in the bag, then a marble is selected again.
 - Find the sample space.
 - What is the probability of selecting the blue marble twice?
- ⁽⁶³⁾ 6. Write $\frac{5}{9}$ as a **a** percent and **b** decimal. **c** If 5 out of 9 people surveyed favor an initiative, is the fraction of those who favor the initiative closer to one half or two thirds?
- ⁽⁷⁵⁾ * 7. A portion of a wall that runs along a highway has this shape and will be painted on one side. Name the shape to be painted and find its area.



Solve.

8. $0.03x + 0.1 = 0.7$
(50)

9. $\frac{2}{3}x - \frac{1}{2} = \frac{1}{6}$
(50)

* 10. $3(x - 2) = x + 6$
(38)

* 11. $2x + 3 = x - 5$
(79)

Generalize Simplify.

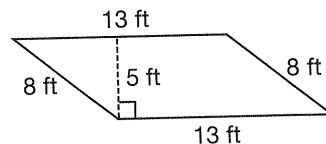
* 12. $\sqrt{30} \sqrt{3}$
(74, 78)

* 13. $\sqrt{8} \sqrt{5}$
(74, 78)

* 14. $(-2)(-3) + (-2) - (-3)$
(31, 36)

15. $\frac{32r^2m^3}{16r^2m^{-1}}$
(27, 51)

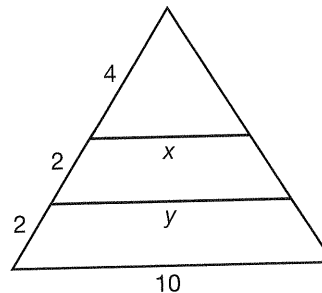
- * 16. Name the shape and find its area.
(60)



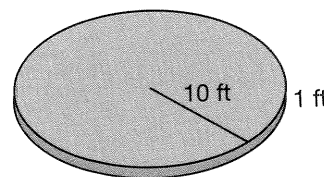
17. The collect call will cost Trini 50¢ per minute. Use unit multipliers to convert 50¢ per minute to dollars per hour.
(72)

18. Graph $y = \frac{1}{3}x$. Is $(15, 3)$ a solution?
(41)

- * 19. Carmen designed the company's logo using similar triangles like these. Find x and y . (Hint: Draw the three triangles separately.)
(65)

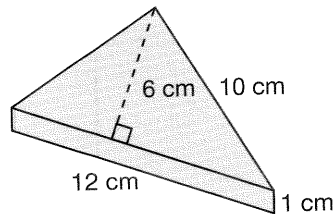


20. A slab of concrete with these dimensions will support a gazebo at the park. What volume of concrete is needed? (Use $\pi \approx 3.14$)
(76)



- * 21. Concrete is ordered by the cubic yard. How many whole cubic yards should be ordered for the slab in problem 20?
(72)

- * 22. Find the volume of the triangular prism.
(76)

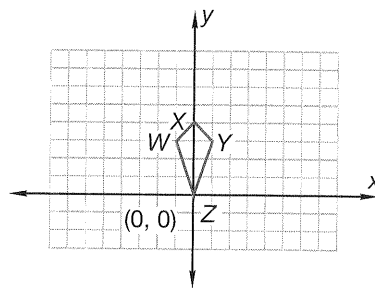


- * 23. The weight of an object on the moon varies directly with the object's weight on earth. If an astronaut who weighs 150 pounds on earth weighs 25 pounds on the moon, what is the weight on the moon of an astronaut who weighs 180 pounds on earth?
(51)

- * 24. Solve the inequality and graph the solution.
(77, 79)

$$2x + 2 \geq x + 6$$

25. a. Figure $WXYZ$ is rotated 90° clockwise about point Z . Which are the coordinates of the vertices of its image?
(Inv. 5)



- A $W' (-3, -1), X' (-4, 0), Y' (-3, 1), Z' (0, 0)$
- B $W' (3, 1), X' (4, 0), Y' (3, -1), Z' (0, 0)$
- C $W' (1, 3), X' (3, 3), Y' (-1, 3), Z' (0, 0)$
- D $W' (1, -3), X' (0, -4), Y' (1, -3), Z' (0, 0)$
- b. The coordinates for points Z and Z' are $(0, 0)$. Explain why.

Early Finishers

*Real-World
Application*

The local fuel plant has cylindrical holding tanks that are each 85 feet tall with a diameter of 32 feet. One million cubic feet of oil is being shipped to the plant for storage. How many holding tanks will the plant need to store all the oil?

- Adding and Subtracting Mixed Measures
- Polynomials

Power Up

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facts

Power Up P

mental math

a. **Algebra:** Simplify: $x^2 + 3x - 2x + 1$ b. **Number Sense:** Arrange from least to greatest:

$$\frac{1}{5} \quad \frac{2}{5} \quad \frac{1}{5} + \frac{2}{5} \quad \frac{1}{5} \times \frac{2}{5}$$

c. **Powers/Roots:** $x^{10} \cdot x^{10}$ d. **Probability:** A spinner has eight congruent sections labeled 1–8. Find the probability that the spinner stops on the number 3 or 4.e. **Proportions:** The ratio of ducks to geese was 12 to 5. If there were 48 ducks, how many geese were there?f. **Geometry:** What is the measure of each angle of an equilateral triangle?g. **Select a Method:** The parade officials want to know how many spectators came to watch the parade. The officials will probably

A estimate an appropriate number.

B count the exact number.

h. **Calculation:** $12 \times 3, \sqrt{\quad}, \times 5, -10, \times 5, \sqrt{\quad}, -1, \sqrt{\quad}$

problem solving

The first three terms of the arithmetic sequence $a_n = 10n$ are 10, 20, and 30.The first three terms of the sequence of squares, $a_n = n^2$, are 1, 4, and 9.

The first term of the first sequence is 10 times the first term of the second sequence (10 to 1). The second term of the first sequence is 5 times the second term of the second sequence (20 to 4).

- Which numbered term of the two sequences is equal?
- For which numbered term is the number in the second sequence twice the number in the first sequence?
- For which numbered term is the number in the second sequence three times the number in the first sequence?

adding and subtracting mixed measures

Mixed measures are measurements with different units combined by addition. We combine the units separately and then make adjustments to the totals as we see in the following examples.

Example 1

Competing in a triathlon Regina swam for 27 minutes 15 seconds, she biked for 48 minutes 28 seconds, and she ran for 42 minutes 53 seconds. What was her total time for the triathlon?

Solution

Each length of time is expressed with two units, minutes and seconds. We add the units separately.

$$\begin{array}{r} 27 \text{ min } 15 \text{ sec} \\ 48 \text{ min } 28 \text{ sec} \\ + 42 \text{ min } 53 \text{ sec} \\ \hline 117 \text{ min } 96 \text{ sec} \end{array}$$

Now we adjust the units. Since 96 seconds is 1 min 36 sec, we add 1 minute to the total leaving 36 in the seconds column.

$$118 \text{ min } 36 \text{ sec}$$

Since 118 minutes is 1 hr 58 min, we express Regina's total time as

$$\mathbf{1 \text{ hr } 58 \text{ min } 36 \text{ sec}}$$

When we subtract we often need to regroup units.

Example 2

Junior weighed 7 pounds 12 ounces when he was born. Now he weighs 12 pounds 5 ounces. How much weight has Junior gained since he was born?

Solution

In order to subtract ounces we rename 1 pound as 16 ounces. Combining the 16 ounces and 5 ounces, we get 21 ounces. Then we subtract.

$$\begin{array}{r} \text{ pounds } \text{ ounces} \\ \text{ pounds } \text{ ounces} \\ - 7 \text{ pounds } 12 \text{ ounces} \\ \hline 4 \text{ pounds } 9 \text{ ounces} \end{array}$$

$\left. \begin{array}{c} 11 \\ 21 \end{array} \right\} 1 \text{ pound}$

Junior has gained **4 pounds 9 ounces**.

Example 3

Last year during physical fitness testing, Marissa completed a mile in 12 minutes 15 seconds. This year she finished in 9 minutes 48 seconds. Marissa improved her time by how many minutes and seconds?

Solution

We compare the times by subtracting. We need to rename 1 minute as 60 seconds. Combining 60 seconds and 15 seconds totals 75 seconds. Then we subtract.

$$\begin{array}{r}
 \overset{\curvearrowright 1 \text{ min}}{11} \quad 15 \\
 \text{12 min } 15 \text{ sec} \\
 - 9 \text{ min } 48 \text{ sec} \\
 \hline
 2 \text{ min } 27 \text{ sec}
 \end{array}$$

Marissa improved her time by **2 minutes and 27 seconds**.

polynomials

A **monomial** is a constant or the product of constants and/or variables with exponents that are whole numbers. Examples of monomials are:

$$2x^3 \quad -5 \quad x^2y \quad 7xyz \quad z$$

A monomial consists of one term. A **binomial** has two terms. Examples of binomials are:

$$x + 5 \quad xy - 7y \quad x^2 - 4$$

A **trinomial** has three terms. Examples of trinomials are:

$$x^2 + 7x + 10 \quad x + 2xy + y$$

Monomials, binomials, and trinomials are all types of **polynomials** and can have any number of terms. The **degree of a polynomial** is the highest degree of any of its terms. If a term has one variable, the **degree of the term** is the exponent of the variable.

$$y \quad \text{exponent is 1, first degree term (linear)}$$

$$b^2 \quad \text{exponent is 2, second degree term (quadratic)}$$

$$p^3 \quad \text{exponent is 3, third degree term (cubic)}$$

If a term has more than one variable, the degree of the term is the sum of the exponents.

$$4ac \quad \text{exponents are } 1 + 1 = 2, \text{ the degree of the term is 2}$$

$$6xyz \quad \text{exponents are } 1 + 1 + 1 = 3, \text{ the degree of the term is 3}$$

Classify What is the degree of these terms: $a^2 - 4$, $m + 3$, and $5abc + 2$?

Thus we find the degree of each of the following polynomials by looking at the highest degree of the terms in the polynomial.

Math Language

The prefixes in the words *monomial*, *binomial*, and *trinomial* indicate the number of terms in algebraic expressions. *Mono-* means one, *bi-* means two, and *tri-* means three.

Example 4

Name the degree of each polynomial.

- a. $4x^3 + 2x^2 + x - 3$
- b. $x^2 - 2x + 3$
- c. $x + y + 3$

Solution

For each polynomial we find the term with the highest degree by finding the sum of the exponents of variables.

- a. The term of highest degree is $4x^3$, which is **3rd degree**.
- b. The term of the highest degree is x^2 , so the polynomial is **2nd degree**.
- c. Both x and y are first degree terms and the polynomial is **1st degree**.

Notice the order of the three terms in this polynomial.

$$x^2 - 2x + 3$$

We say that the terms are arranged in **descending order** because the degrees of the terms are 2 then 1 then 0.

Example 5

Arrange the terms of this polynomial in descending order.

$$x + 2x^3 + 2 + x^2$$

Solution

We focus our attention on the variables and their exponents.

$$2x^3 + x^2 + x + 2$$

Example 6

Add the binomials $x + 6$ and $2x - 3$.

Solution

We collect like terms. One way to do this is to align like terms and add.

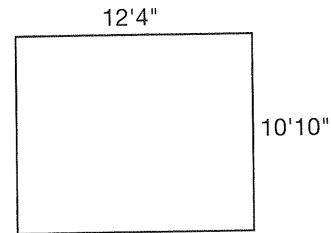
$$\begin{array}{r} x + 6 \\ 2x - 3 \\ \hline 3x + 3 \end{array}$$

Practice Set

Add.

- a. $\begin{array}{r} 3 \text{ hr } 32 \text{ min } 45 \text{ sec} \\ + 1 \text{ hr } 43 \text{ min } 27 \text{ sec} \\ \hline \end{array}$
- b. $\begin{array}{r} 5 \text{ lb } 8 \text{ oz} \\ + 8 \text{ lb } 9 \text{ oz} \\ \hline \end{array}$

- c. **Model** Neil needs to buy molding to mount around the room, so he makes a sketch of the room recording its length and width. What is the perimeter of the room?



Perform each indicated subtraction.

d.
$$\begin{array}{r} 3 \text{ hr } 15 \text{ min } 12 \text{ sec} \\ - 1 \text{ hr } 42 \text{ min } 30 \text{ sec} \\ \hline \end{array}$$

e.
$$\begin{array}{r} 6 \text{ ft } 2 \text{ in.} \\ - 4 \text{ ft } 7 \text{ in.} \\ \hline \end{array}$$

- f. Tony cut $2 \text{ ft } 7\frac{1}{2} \text{ in.}$ length from an 8 ft long board. What is the remaining length of the board?

Classify Identify each polynomial below as a monomial, binomial, or trinomial and name its degree.

g. $x^2 - 25$

h. $x^2 - 6x + 7$

i. $2x + 3y - 4$

- j. Arrange the terms in descending order.

$$5 - 4x^2 + x^3 - 6x$$

- k. Add the trinomials $x + y - 2$ and $x - y + 4$.

Written Practice

Strengthening Concepts

1. The item was discounted 20% from the original price of \$90.
 - (67) a. What was the sale price?
 - b. If the sale price was reduced 30% for closeout, what was the closeout price?
2. The farmer planted crops on 84% of his land. If 12 acres were left unplanted, how many acres of land did the farmer have?

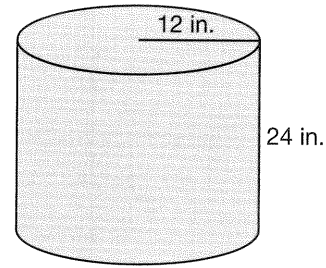
(48)
3. The bag contained only red and green marbles in the ratio of 2 to 7. If one marble is drawn from the bag, what is the probability the marble will be red?

(29, 32)
4. The sides of a triangle are 5 in., 7 in., and 9 in. Is the triangle a right triangle? Explain your answer.

(Inv. 2)
- * 5. **Analyze** Solve for r : $A = \pi r^2$

(79)

- * 6. Find the volume of this cylindrical container to the nearest cubic foot.
(76)



Analyze Solve.

* 7. $7x - 3 = 2x - 33$
(79)

* 8. $0.003x - 0.2 = 0.01$
(50)

* 9. $\frac{4}{5}x - \frac{2}{3} = \frac{1}{3}$
(50)

* 10. $3(x - 2) = 2(2x - 1)$
(79)

11. a. $1 \text{ yd } 2 \text{ ft } 7 \text{ in.}$
(80) $+ 2 \text{ yd } 1 \text{ ft } 10 \text{ in.}$

b. $2 \text{ hr } 15 \text{ min}$
 $- 1 \text{ hr } 37 \text{ min}$

12. a. What type of polynomial is $3x + 2$?
(80)

b. What is the sum of $3x + 2$ and $x - 1$?

Simplify.

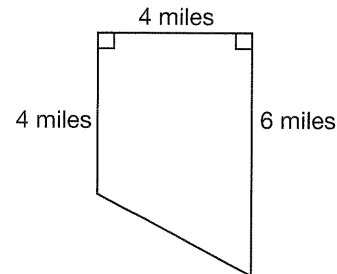
* 13. $\sqrt{1000}$
(74)

* 14. $\sqrt{40} \sqrt{10}$
(74, 78)

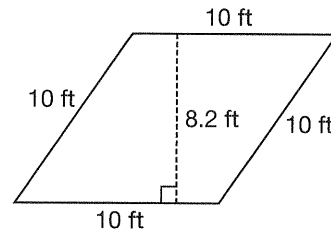
* 15. $\sqrt{3} \sqrt{24}$
(74, 78)

* 16. $\frac{12x^3y^4}{3x^4y^{-3}}$
(27, 51)

- * 17. **Analyze** The figure shows the city limits of a town. The town covers how many square miles?
(75)



18. Find the area of the parallelogram.
(60)



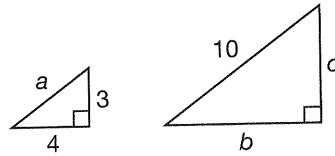
19. Solve this inequality. Graph its solutions.
(77)

$$2x + 1 \leq x + 2$$

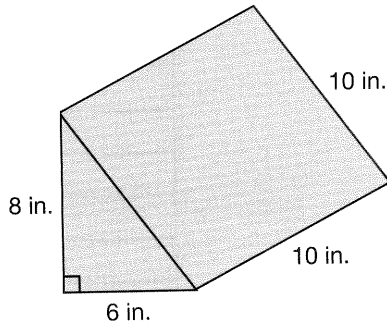
20. Aziel ordered carpeting installed for \$27 per square yard. Use unit multipliers to convert \$27 per square yard to dollars per square foot.
(64, 72)

21. Graph $y = -2x + 6$. Is $(3, 0)$ a solution?
(56)

22. Tedman plans a larger version of his model airplane. A diagram of the tail of the airplane and the larger version are shown here. Find a , b , and c .
(65)

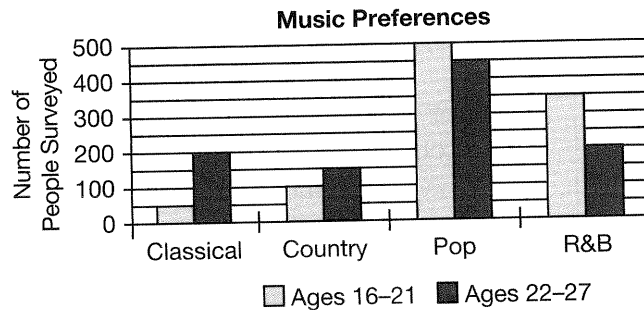


* 23. **Model** Find the volume of the triangular prism, then sketch the net of the prism.
(76)



24. Write $\frac{2}{9}$ **a** as a decimal and **b** as a percent.
(63)

25. The bar graph shows the music preferences of two different age groups.
(Inv. 6)



- How many people in the survey are in the 16-21 age group?
- How many more people in the 22-27 age group prefer classical music than in the 16-21 age group?
- How many people in the survey preferred R&B?



Focus on

• Scatterplots

Mr. Lopez’s physical education class did laps around the track, and each student recorded the total time exercising and the number of laps he or she completed. The students’ data are shown below.

Number of Laps	Time (Minutes)
3	9
4	8
2	3
6	12
5	15
4	7
2	4
6	15
3	7
5	11
6	7

We make a **scatterplot** of these paired data by plotting these data points on the coordinate plane. The number of laps is the independent variable and is marked on the horizontal axis. Elapsed time is the dependent variable and is marked on the vertical axis. Thus, the first set of numbers, “3 laps in 9 minutes” is plotted as the point (3, 9) on the graph.

Number of Laps	Time (min)
3	9
4	8
2	3
6	12
5	15
4	7
2	4
6	15
3	7
5	11
6	7

