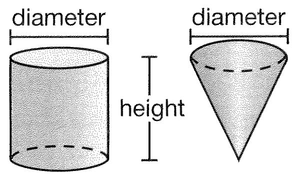
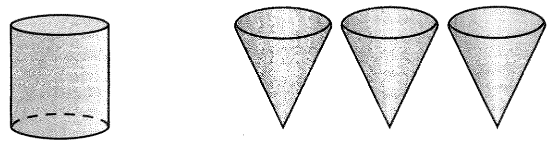
Suppose we have a cone-shaped container and a cylindrical can with the same diameters and heights.



To fill the can with water using the cone, we would have to fill and empty the cone three times.



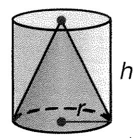
volume of cylinder = volume of 3 cones

The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base diameter and height.

Thinking Skill

Explain

Why are $V = \frac{1}{3}Bh$ and $V = \frac{1}{3}\pi r^2 h$ equivalent formulas for the volume of a cone?



volume of cone $= \frac{1}{3}\ B \cdot h$

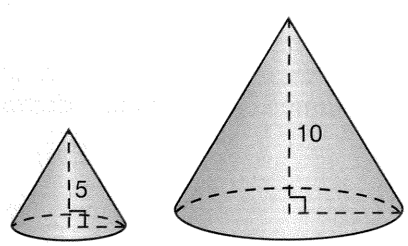
$V = \frac{1}{3}\pi r^2 h$

Example 1

Aidan built two cones out of sand on the beach. The diameter of the smaller cone is 6 in. and its height is 5 in. The diameter of the larger cone is 12 in. and its height is 10 in. How many cubic inches of sand did Aidan use for each cone? How many times greater is the volume of the larger cone than the volume of the smaller cone? (Express π as π.)

Solution

We draw diagrams of the two cones, one with a radius of 3 in. and a height of 5 in., the other with a radius of 6 in. and a height of 10 in.



$r = 3$ $r = 6$

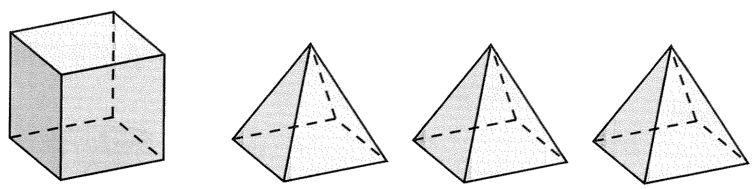
Smaller Cone Step:	Justification:	**Larger Cone** Step:
$V = \frac{1}{3}\pi r^2 h$	Formula	$V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}\pi(3)^2(5)$	Substituted for r and h	$V = \frac{1}{3}\pi(6)^2(10)$

$V = \frac{1}{3}\pi \cdot 9 \cdot 5$	Evaluated 3^2 and 6^2	$V = \frac{1}{3}\pi \cdot 36 \cdot 10$
$\boldsymbol{V = 15\pi}$ **in.^{3}**	**Simplified**	$\boldsymbol{V = 120\pi}$ **in.^{3}**

Dividing 120π in.^{3} by 15π in.^{3}, we find that the larger cone has a volume **8 times as large** as the smaller cone.

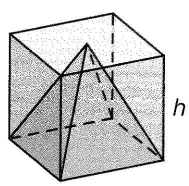
$$\frac{120 \not{\pi}^{1} \not{\text{in.}^3}}{15 \not{\pi}_{1} \not{\text{in.}^3}} = 8$$

The volumes of prisms and pyramids are similarly related. From a block of clay in the shape of a prism, we can form 3 pyramids with the same base and height as the original prism. This is shown with a square base below, but it is true for bases of any shape.



volume of 1 prism = volume of 3 pyramids

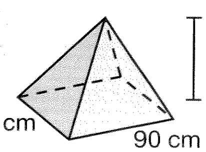
The volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height.



$V = \frac{1}{3}Bh$

Example 2

Find the volume of this square pyramid.



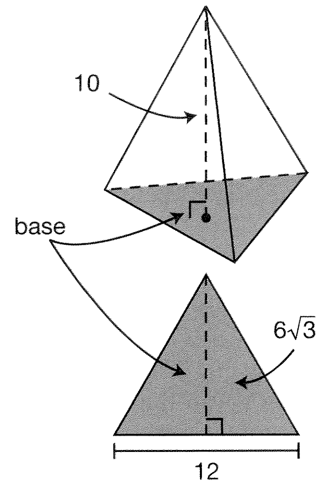
Solution

The volume is $\frac{1}{3}$ the product of the area of the square base and the height.

Step:	**Justification:**
$V = \frac{1}{3}Bh$	Given
$V = \frac{1}{3}(90 \text{ cm})^2(50 \text{ cm})$	Substituted for B and h
$V = \frac{1}{3}(8100 \text{ cm}^2)(50 \text{ cm})$	Squared 90 cm
$\boldsymbol{V = 135{,}000}$ **cm^3**	Simplified

Example 3

A triangular pyramid is shown. The area of its base is $36\sqrt{3}$ units. Its height is 10 units. What is its volume?



Solution

We are given the height and the area of the base. We substitute these values for B and h in the formula and perform the calculations.

Step:	Justification:
$V = \frac{1}{3}Bh$	Formula
$V = \frac{1}{3}36\sqrt{3} \cdot 10$	Substituted
$V = 120\sqrt{3}$	Simplified

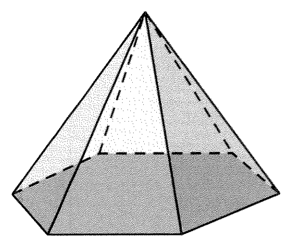
Practice Set

Find the volume.

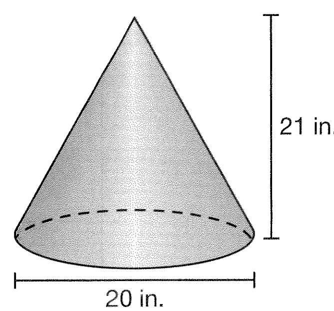
a. Refer to the pyramid in example 2. Find the volume of a pyramid with dimensions $\frac{1}{10}$ of the pyramid in example 2. The volume of the smaller pyramid is what fraction of the volume of the pyramid in example 2?

Connect The volume of the smaller pyramid is what percent of the volume of the pyramid in example 2?

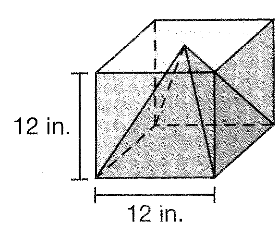
b. The area of the hexagonal base of a pyramid is $18\sqrt{3}$. The height of the pyramid is 12. What is the volume?



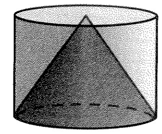
c. The diameter of the base of the cone is 20 inches. The height is 21 inches. What is the volume of the cone? Express the answer in terms of π and again rounded to the nearest cubic inch.



d. Using a 12 inch cube of clay, Lucian makes a 12-inch high pyramid with a 12-inch square base. What is the volume of the pyramid? Does Lucian have enough clay from the original cube to make another pyramid the same size as the first?



e. Connect A party hat arrived in a cylindrical container in which it fit perfectly. If the volume of the container is 99 in.^{3}, what is the volume of the cone-shaped hat?



Written Practice *Strengthening Concepts*

1. (80) In two months a puppy's weight increased from 5 lb 15 oz to 8 lb 5 oz. Find the amount of increase.

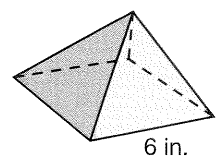
2. (71) The infant's weight increased from 15 to 21 pounds in three months. What was the percent increase?

* **3.** (81) Evaluate A 12-inch diameter wall clock is divided into twelfths by equally spaced numbers.

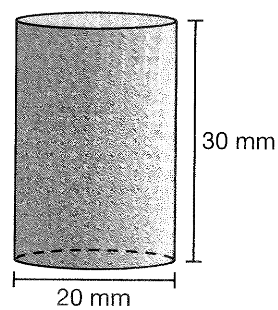
a. Find the measure of the central angle between consecutive numbers.

b. Find the length of the arc between two consecutive numbers on the circle.

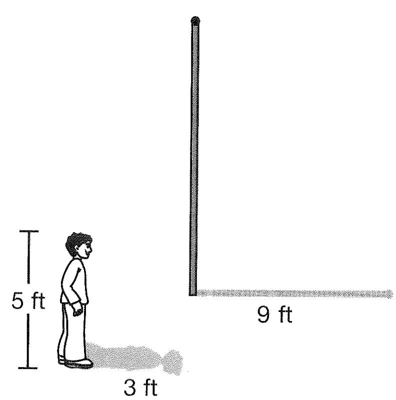
4. (86) A pyramid with a square base is 4 in. high. What is the volume of the pyramid?



5. (85) Find the lateral surface area of the cylinder to the nearest square *centimeter*.



* **6.** (65) Analyze The sun casts shadows as shown. How tall is the flag pole? (Use similar triangles.)



7. (80) Find the sum of the polynomials $3x - 2y - 4$ and $x + 2y - 7$.

*** 8.** (67) Errin bought stock in XYZ Corporation for $100 per share. One month later the stock was up 10%. After another month the stock fell 10%.

a. What was the price of the stock after the first month?

b. What was the price of the stock after the second month?

9. (58) There were 11 girls in a class of 25. What percent of the students were boys?

*** 10.** (48) If a car can travel 192 miles when its gas tank is 75% full, then how far can it travel at the same mileage rate on a full tank?

11. (73) Write the first four terms of the sequence that follows the rule $a_n = 5n - 2$.

Generalize Simplify.

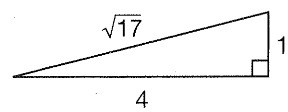
*** 12.** (74, 80) $\sqrt{5}\sqrt{15}$

13. (36) $3(-1) - 2\left(-\frac{1}{2}\right)$

*** 14.** (84) Find $33\frac{1}{3}\%$ of $\frac{4}{5}$ of $4.50.

*** 15.** (79) Solve for y: $y - 7 = 2x - 3$

16. (Inv. 2) **Evaluate** Demonstrate that a right triangle can have the dimensions shown.



17. (83) There are three winning cards out of twenty-one placed in a hat.

a. What is the probability that the first person to draw a card will select a winning card?

b. If the first person selects a winning card and does not replace it, what is the probability that the second person will select a winning card?

18. (64, 72) Use unit multipliers to convert 60 miles per hour to feet per minute.

19. (56, 82) **a.** Whatever number Xavier says, Yanos says the opposite. Graph $y = -x$.

b. The numbers Xena and Yolanda say always total 4. Graph $x + y = 4$.

c. Describe the relationship of the two graphed lines.

Solve.

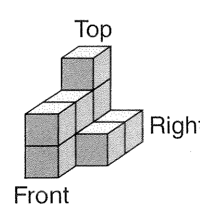
20. (25, 44) $\frac{2.7}{5.4} = \frac{3}{x}$

21. (25, 36) $1.2 - 3.4x = -5.6$

22. (22, 38) $\frac{4}{7} - \frac{3}{7}x = 1$

23. (78) $17 - 2x = x - 7$

*** 24.** (Inv. 4) Cubic boxes are stacked as shown. Sketch the front, top, and right side views.



25. (42) If each box in problem **24** has edges two feet long, then what is the volume of the stack of boxes?

LESSON 87

• Scale Drawing Word Problems

Power Up

Building Power

facts Power Up R

mental math

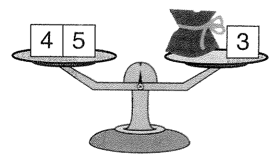
a. **Statistics:** Find the mean of this data: 11, 11, 17, 16, 15

b. **Intercepts:** The sum of two numbers is 7. Their product is 12. What are the two numbers?

c. **Fractional Parts:** $\frac{2}{5}$ of 65

d. **Proportion:** $\frac{42}{x} = \frac{6}{5}$

e. **Measurement:** Find the mass of the bag.



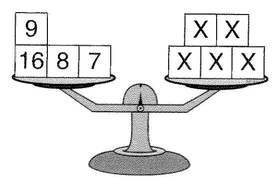
f. **Percent:** 100% more than $60

g. **Scientific Notation:** Write 0.00009 in scientific notation.

h. **Calculation:** What is the average of the first five positive odd numbers?

problem solving

Is it possible to remove a block from each side and still have a balanced scale? If so, which blocks can be removed?



New Concept

Increasing Knowledge

A map is a small representation of a larger physical region. If the map is **drawn to scale,** then small units of measure on the map correspond with larger units of measure in the actual region. Therefore, the lengths on the map are proportional to those in the actual region.

For example, if the key on a map shows the scale relationship 1 inch = 10 miles, then a one-inch length on the map corresponds with a 10-mile distance in the region represented by the map. Thus, a road 2.25 inches long on the map is really 22.5 miles long.

Most architectural drawings are also created to scale. People who look at architectural drawings or blueprints can easily calculate the actual sizes of the structures.

We use a scale factor to convert measures on the scale drawing to the actual measures they represent and vice versa. The scale factor is the ratio of a length on the drawing to the length it represents on the actual object. Since the scale factor is a ratio, we can solve scale drawing problems using proportions.

Example 1

Mary sketched a scale drawing of her dream garden using 1 inch to represent 2 feet.

a. If the drawing of the garden is 8 inches long, how long will Mary's garden be?

b. If Mary's garden is to be 12 feet wide, how wide should the drawing be?

Solution

We may find the answer to both questions by solving proportions. The scale of the drawing determines the ratio.

Thinking Skill

Explain

Explain two different methods we can use to solve a proportion.

a.

	Scale	Measure
Drawing, in inches	1	8
Garden, in feet	2	*L*

$$\frac{1}{2} = \frac{8}{L}$$

We solve the proportion and find that the length of the garden is **16 feet.**

b.

	Scale	Measure
Drawing, in inches	1	*W*
Garden, in feet	2	12

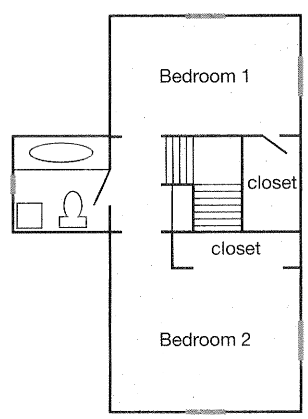
$$\frac{1}{2} = \frac{W}{12}$$

We solve the proportion and find that the drawing of the garden should be **6 inches wide.**

Represent Draw Mary's garden with a scale of 1 inch = 4 feet.

Example 2

Edmund wants to convert the attic to living space. He has made a scale drawing (1 inch = 16 feet) of his plans. What are the dimensions of Bedroom 2 excluding the closet?



Solution

Using a ruler to measure, we find that Bedroom 2 is 1 inch long and $\frac{3}{4}$ inch wide. Since 1 inch on the drawing represents 16 feet, the room is **16 feet long.** An alternative to writing and solving a proportion is multiplying by the scale factor. To compute the width of Bedroom 2, we multiply the width on the drawing by the scale factor.

$$\frac{3}{4}\cancel{\text{in.}} \times \frac{16\text{ ft}}{1\cancel{\text{in.}}} = 12\text{ ft}$$

$$\underbrace{\tfrac{3}{4}\text{ in.}}_{\text{drawing}} \times \underbrace{\tfrac{16\text{ ft}}{1\text{ in.}}}_{\text{scale factor}} = \underbrace{12\text{ ft}}_{\text{measure}}$$

We find the room is **12 ft wide.**

Predict Would a drawing of Bedroom 2 with a scale of 1 inch = 1 foot fit on this page? Explain.

Practice Set

a. A map is drawn with a scale of 1 inch = 8 miles. Two towns $2\frac{3}{4}$ inches apart on the map are how many miles apart?

b. **Justify** Mariah is making a scale drawing of her apartment. Her apartment measures 36 feet long and 30 feet wide. She wants the drawing to fit on an 8.5 in.-by-11 in. piece of paper. Which of the following would be a good scale for Mariah to use? Why?

A 1 in. = 2 ft **B** 1 in. = 3 ft **C** 1 in. = 4 ft **D** 1 in. = 6 ft

c. Refer to the floor plan in Example 2 to find the actual length and width of Bedroom 1, excluding the closet.

d. **Model** Create a scale drawing of a building, classroom, playground, or landscape area. Specify the scale factor you used for your drawing.

Written Practice

Strengthening Concepts

* **1.** (87) **Analyze** Arthur is drawing the floor plan of his home. The scale is 1 inch equals 2 feet.

a. If he draws his hallway 5 inches long, how long is the actual hallway?

b. If the kitchen is 12 feet wide, how wide should the drawing of the kitchen be?

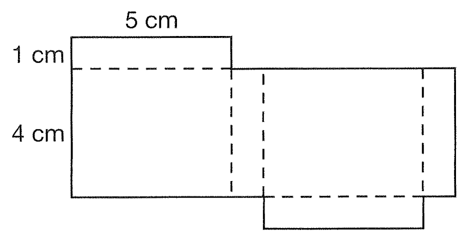
* **2.** (87) **Analyze** Kay is sketching her dream house. The scale is 1 inch equals 5 feet.

a. If her dining room is to be 25 feet long, how long should the drawing be?

b. If she draws the bookcase 2 inches long, how long is the actual bookcase?

3. (17) Mariya charged three purchases on her credit card. The charges were \$19.53, \$12.99, and \$26.90. She estimated to total by adding \$20, \$13, and \$27. Without performing the addition, explain how you know whether the actual total is greater than or less than her estimate.

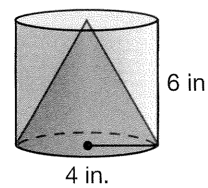
*** 4.** (55, 85) **Connect** If this net is folded to make a prism, what is its surface area?



A 20 cm^2 **B** 25 cm^2

C 50 cm^2 **D** 58 cm^2

*** 5.** (75, 86) In terms of π, what is the volume of the cylinder and the volume of the cone?



6. (85) In terms of π, what is the total surface area of the cylinder in problem **5?**

*** 7.** (87) **Model** Yoshi drew plans for a rectangular table that would measure 4 feet by 6 feet. On his drawing, $\frac{1}{2}$ in. represented 1 foot. Sketch the plan of the table top that Yoshi drew and label the side lengths.

*** 8.** (Inv. 1, Inv. 2) Three vertices of a square are (1, 1), (0, −1), and (−2, 0).

a. What are the coordinates of the fourth vertex?

b. What is the length of each side?

c. What is the area of the square?

9. (83) In an alphabet tile game there are six tiles face down on the table, and two of the tiles are A. If Gabbie selects two tiles randomly, what is the probability that both tiles will be A?

*** 10.** (80) A rectangle has sides with length $(x + 3)$ and $(x + 4)$. Find the perimeter by adding binomials.

*** 11.** (31, 36) **Generalize** Expand and add like terms: $x(x + 3) - 4(x + 3)$

Simplify.

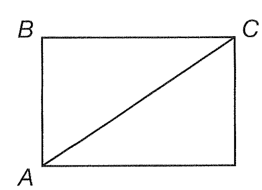
*** 12.** (51) $3x^{-1}y^2x^3y^{-1}$

*** 13.** (78) $\sqrt{8}\sqrt{10}$

14. (36) $\dfrac{(-3) - (2)(-4)}{(-2) - (-1)}$

*** 15.** (84) 50% of $33\frac{1}{3}$% of 1.2

16. (Inv. 2) Nigel takes a shortcut on his walk to school. Instead of going to the corner at *B,* he cuts across the field from *A* to *C.* If the distance from *A* to *B* is 60 yards, and from *B* to *C* is 80 yards, how many yards does he cut by taking the short cut?

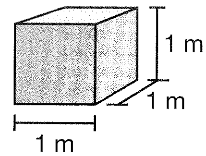


* **17.** A circle is divided into nine equal sectors.
(81)

a. Find the measure of the central angle for one of the sectors.

b. If the circumference of the circle is 27 in., find the length of the arc of one of the sectors.

* **18.** **a.** A meter is about 1.1 yards. A cubic meter is about how many cubic yards?
(72)



b. A cubic yard is 27 cubic feet, so a cubic meter is about how many cubic feet? Round to the nearest cubic foot.

19. Graph $y = x$. Then graph $x + 2y = 6$. What are the coordinates of the point where the lines intersect?
(56, 82)

Solve.

20. $\frac{2.1}{1.4} = \frac{x}{6}$
(25, 44)

21. $0.3x - 0.9 = 0.3$
(25, 50)

22. $\frac{4}{5} - \frac{3}{5}x = \frac{3}{5}$
(23, 50)

23. $\left(\frac{7}{2}\right)^{-1} = -\frac{1}{7}x$
(38, 63)

24. $4(x - 3) + 5 = x + 2$
(21, 78)

25. Klaudia can swim five laps in the city pool every two and a half minutes. If she swims for half an hour every day, how many laps does she swim every four days?
(49)

Early Finishers
Real-World Application

The Pantheon was built over 2000 years ago under the Roman Empire. It is the best-preserved of all Roman buildings due to its thickness and the quality of its materials. The main room of the Pantheon resembles a giant cylinder. The diameter of the cylindrical room is 43.3 meters and the height of the cylinder is 14 meters. What is the lateral surface area of the Pantheon's cylindrical room? (Use 3.14 for π.)

LESSON 88

Review of Proportional and Non-Proportional Relationships

Power Up

Building Power

facts Power Up R

mental math

a. **Algebra:** If x is 10, then what is the product of $x + 2$ and $x - 2$?

b. **Intercepts:** The sum of two numbers is 6. Their product is 8. What are the two numbers?

c. **Number Sense:** What digit is in the ones place in the product 34×2593?

d. **Powers/Roots:** $\frac{x^7}{x^5}$

e. **Function:** The mystery man made a marvelous machine. When Ricky typed 6, the machine output 35. When Albert typed 7, the machine output 48. When Jackie typed 8, the machine output 63. Wally's brother Timmy typed a number. The machine output 143. What did Timmy type?

f. **Geometry:** Angle B and $\angle C$ are supplementary. If $m\angle B$ is 20°, find $m\angle C$.

g. **Sequences:** What is the next term in this sequence?

penny, dime, dollar bill, ...

h. **Calculations:** $102 - 3, \div 3, \div 3, - 3, \div 2, \sqrt{}$

problem solving Brad and William start from the same point and fly in the same direction. Brad flies at 500 miles per hour, while William flies at 200 miles per hour. If William has a 150 mile head start, how long will it take Brad to catch up?

New Concept

Increasing Knowledge

Let us review three important ideas about proportional relationships.

1. Proportional relationships involve **two variables**—two quantities that can change values—such as the distance traveled and the time spent traveling or the quantity of items and the total cost. As one variable increases, the other changes by a constant factor k: $y = kx$.

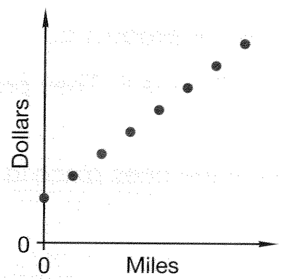
2. In a table of values for the two variables, all pairs form **equal ratios.** That is, in a proportional relationship, the ratio is constant. This idea is expressed by the equation $\frac{y}{x} = \boldsymbol{k}$, in which k (think k for "konstant") is the **constant of proportionality.**

3. A graph of a proportional relationship is **linear and aligns with the origin** (if one variable is zero, the other variable is zero). The graph may be a continuous line or may simply be aligned points.

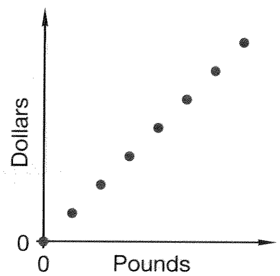
Example 1

State whether each graph below depicts a proportional relationship. Justify your decision.

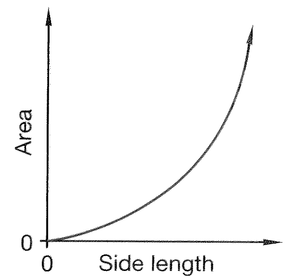
a. Taxi Fare



b. Cost of Oranges



c. Area of Square



Solution

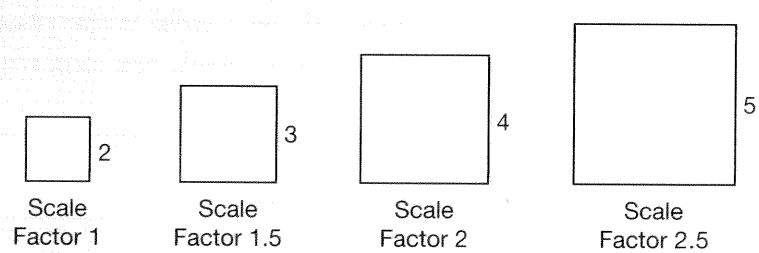
a. The data points are aligned but not with the origin, thus the relationship is **not proportional.**

b. The data points are aligned and include the origin, thus the relationship is **proportional.**

c. The origin is included but the graph is not linear, thus the relationship is **not proportional.**

Example 2

The following tables record the effect of dilating on the perimeter and area of a square.



Side Length	Perimeter
2	8
3	12
4	16
5	20

Side Length	Area
2	4
3	9
4	16
5	25

State whether each table displays a proportional relationship and justify your decision.

Solution

Thinking Skill

Analyze

Why is the word "proportional" used to describe a relationship between two variables whose ratios are constant?

To determine if paired numbers represent a proportional relationship, we check the ratios formed by the pairs. If the ratio is constant, the relationship is proportional.

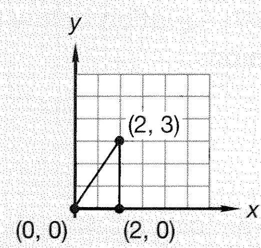
Side Length	Perimeter	*P/s*
2	8	$\frac{8}{2} = 4$
3	12	$\frac{12}{3} = 4$
4	16	$\frac{16}{4} = 4$
5	20	$\frac{20}{5} = 4$

Side Length	Area	*A/s*
2	4	$\frac{4}{2} = 2$
3	9	$\frac{9}{3} = 3$
4	16	$\frac{16}{4} = 4$
5	25	$\frac{25}{5} = 5$

We see that as a square is dilated, the ratio of side length to perimeter does not change. However, the ratio of side length to area does change. **Thus, side length and perimeter are proportional, but side length and area are not proportional.**

Example 3

The heights of similar triangles are proportional to their bases. As a triangle is dilated, its base and height increase proportionally.



Base	Height
2	3
3	4.5
4	6
5	7.5

Predict the height of the triangle shown when its base is 15.

Solution

We will show two methods for finding the height. Since the relationship is proportional, one way to find the unknown height is to write and solve a proportion. We select any row of numbers from the table to write one ratio for the proportion. We use 15 as the base for the second ratio and h for the unknown height.

$$\begin{matrix}\text{base} \\ \text{height}\end{matrix} \quad \frac{2}{3} = \frac{15}{h}$$

We solve the proportion using cross products.

$$\frac{2}{3} = \frac{15}{h}$$

$$2h = 3 \cdot 15$$

$$h = \frac{45}{2}$$

$$\mathbf{h = 22.5}$$

Another way to find the height is to multiply the base by the constant of proportionality. Notice that the ratio of height to base is constant.

Therefore, we can multiply the base (15) by the constant of proportionality (1.5) to find the height.

$$\text{height} = 1.5 \times \text{base}$$
$$h = 1.5 \times 15$$
$$\boldsymbol{h = 22.5}$$

Base	Height	$\frac{\text{height}}{\text{base}}$
2	3	$\frac{3}{2} = 1.5$
3	4.5	$\frac{4.5}{3} = 1.5$
4	6	$\frac{6}{4} = 1.5$
5	7.5	$\frac{7.5}{5} = 1.5$

Notice the equation we wrote at the end of example 3.

$$\text{height} = 1.5 \times \text{base}$$
$$h = 1.5b$$

This form of equation is typical of proportional relationships. Here we show the general form of the equation. Look at it carefully.

$$y = kx$$

We see two variables (x and y) on either side of the equals sign with a constant (k) multiplying one variable. We do not see addition, subtraction, or exponents.

Example 4

State whether each equation below indicates a proportional relationship. Justify your decisions.

a. $c = \pi d$ **b.** $A = \pi r^2$ **c.** $y = 2x + 1$

Solution

a. We see two variables on either side of the equals sign. The constant, π, multiplies one variable. The relationship is **proportional.**

b. We see two variables on either side of the equals sign. The constant, π, multiplies one variable, but the variable r has an exponent of 2. The relationship is **not proportional.**

c. We see two variables on either side of the equals sign. The constant, 2, multiplies one variable. The number 1 is added to one side of the equation. That means if one variable is zero, the other is not zero. The relationship is **not proportional.**

Example 5

Which of these problems can be solved with proportional thinking?

A It takes 2 workers 5 hours to complete the job. How long would it take 4 workers to complete the job?

B The shipping charge is \$2 per item plus a \$3 flat fee, so it costs \$13 to ship 5 items. What is the shipping charge on 10 items?

C Each student read the same number of books. Thirty students read a combined total of 150 books. How many books were read by 60 students?

Solution

Only in **C** are the quantities related by a constant factor. We can find the number of books read by 60 students by solving the proportion:

$$\frac{30}{150} = \frac{60}{x}$$

$$x = 300$$

Practice Set

a. *Explain* In a proportional relationship, there are how many variables? The graph of the relationship is what shape? When one variable is zero, what number is the other variable?

b. An equation in the form $y = kx$ indicates a direct proportion. As x changes by 1, y changes by a constant factor, k. Which of these equations indicates a direct proportion?

A $y = 5 + x$ **B** $P = 4s$ **C** $A = bh$

c. The equation $y = kx$ transforms to $\frac{y}{x} = k$, which means that the ratio of y to x is constant. If $\frac{y}{x} = 3$, then which pair of numbers does not fit the relationship?

A (4, 12) **B** (6, 18) **C** (9, 3)

d. *Conclude* Which of the following is not an example of a proportional relationship?

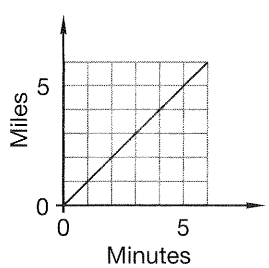
A Distance and time when driving at a constant speed

B Quantity and price when the unit price remains the same

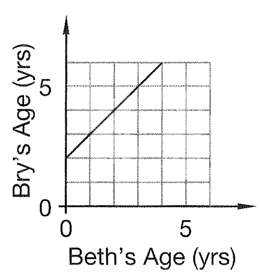
C Hours awake and hours asleep when the number of hours in a day remains the same

e. *Classify* Which graph below shows a proportional relationship?

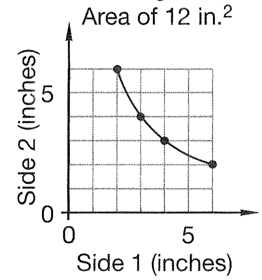
A Distance Traveled at 60 mph



B Ages of Bry and Beth



C Length and Width of Triangle with Area of 12 in.^{2}



f. Which equation is an example of a proportional relationship?

A $d = 30t$ **B** $A = s^2$ **C** $P = 2l + 2w$

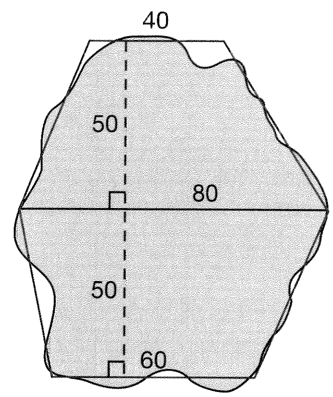
Written Practice *Strengthening Concepts*

* **1.** (87) **Analyze** Michael drew a scale drawing of a building. The scale is 0.5 inches equals 1 foot.

a. If he drew the hallway 2.5 inches wide, how wide is the actual hallway?

b. The building itself is 60 feet wide. How wide is it in Michael's drawing?

* **2.** (75) **Estimate** To estimate the area of a pond Gerard added the areas of two trapezoids. What is the approximate area of the pond? (Dimensions are in yards.)

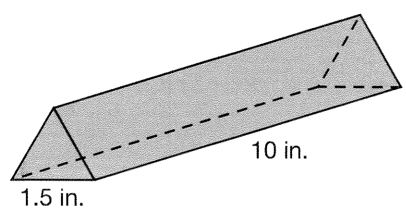


3. (67) When the teacher returned the tests, the number of smiles increased by forty percent. If there had been 15 smiles in the room before the tests were returned, how many were there after the tests were returned?

* **4.** (80) **Analyze** A rectangle has sides with lengths $(x + 5)$ and $(x + 6)$. Find the perimeter.

* **5.** (36, 80) Expand and write the product in descending order. $-2x(3 + x - 3x^2)$

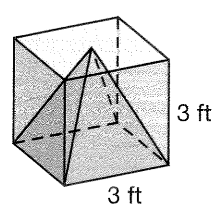
6. (85) Find the lateral surface area of this triangular prism with an equilateral base.



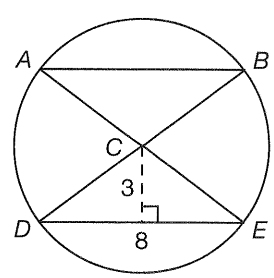
7. (86) The illustration shows a pyramid with the same base and height as the cube.

a. What is the volume of the cube?

b. What is the volume of the pyramid?



For **8–11** refer to the circle. Segment AE and BD are diameters. Units are in cm.



* **8.** (81) **Classify** **a.** Name two obtuse central angles

b. Name two acute central angles.

9. (20) **a.** What is the area of each triangle?

b. Classify $\triangle ABC$ by sides.

10. (Inv. 2, 39) The dashes showing the height divide $\triangle CDE$ into two congruent right triangles.

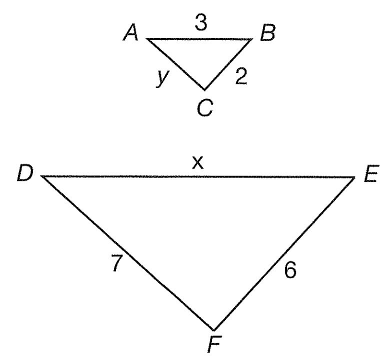
a. Use the Pythagorean Theorem to find *CD*.

b. What is the diameter of the circle?

11. (20, 40) In terms of π, the two triangles occupy what fraction of the area of the circle? Is the fraction more or less than $\frac{1}{3}$? How do you know?

12. (40, 45) **a.** Find *x* and *y* in the similar triangles to the right.

b. Find the ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle DEF$.



Simplify.

* **13.** (74) $\sqrt{98}$

* **14.** (78) $4\sqrt{10}\sqrt{20}$

15. (22, 63) $\left(\frac{2}{3}\right)^{-2} + \left(\frac{3}{4}\right)^{-1}$

16. (84) 150% of $\frac{3}{10}$ of 0.2

* **17.** (88) **Evaluate** Joshua completed tasks and recorded the time it took to complete them. Does this table show a proportional relationship? If so, give the constant of proportionality. Use words to describe the relationship between the number of tasks and the time necessary to complete them.

Tasks	Time
3	18 sec.
4	24 sec.
5	30 sec.
6	36 sec.

18. (64, 72) Siti registered to run in a 10k (a 10 km race). Use a unit multiplier to write the distance in miles. (One mile is just over 1.6 km.) Round down to the tenth of a mile.

19. (63) Write $\frac{5}{9}$ as **a** a percent, **b** a decimal, and **c** a decimal rounded to the nearest hundredth.

20. (41, 56) Graph the equations $y = x$ and $y = x + 2$. Describe the relationship of the two lines.

Solve.

21. (23, 50) $\frac{2}{3}x + \frac{3}{8} = \frac{3}{4}$

22. (25, 44) $\frac{2.7}{1.8} = \frac{x}{2}$

23. (25, 50) $0.4 - 0.02x = 1.2$

24. (24, 38) $\frac{2}{5}n = -18$

25. (69) The table below show the amount of Mighty Clean Detergent needed to wash loads of laundry.

Amount of Detergent	$\frac{3}{4}$ C	$1\frac{1}{2}$ C	$2\frac{1}{4}$ C	3 C	$3\frac{3}{4}$ C
Number of Laundry Loads	1	2	3	4	5

Graph the data. Is the number of cups of detergent proportional to the number of loads of laundry? If so, write an equation that describes the graph and state the constant of proportionality. If not, explain why.

Early Finishers
Real-World Application

The Great Pyramid of Giza was built as a tomb for the Egyptian king Khufu nearly 5000 years ago. The pyramid is made from blocks of limestone and granite, with each block weighing between 2 and 2.5 tons. If the side length of the base of this square pyramid is about 745 feet and the height of the pyramid is about 449 feet, what is the approximate volume of the Great Pyramid of Giza? (Round your answer to the nearest thousand cubic feet.)

LESSON 89

Solving Problems with Two Unknowns by Graphing

Power Up

Building Power

facts Power Up R

mental math

a. Algebra: If $3x = 9$, then what is the product of x^3 equal?

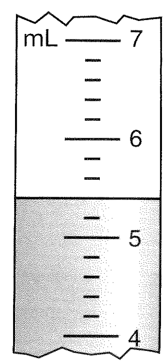
b. Fractional Parts: $\frac{9}{8}$ of 480

c. Probability: Ten of the 11 girls completed the assignment. Twelve of 13 boys completed the assignment. If the teacher calls on a student at random, what is the probability that the student has the assignment completed?

d. Percent: What number is 75% more than $40?

e. Ratio: The ratio of lions to tigers to bears was 2 to 3 to 5. If there were 45 bears, how many tigers were there?

f. Measurement: Find the volume of liquid in this graduated cylinder:



g. Select a Method: With a tape measure you find the dimensions of a room. To find its perimeter you would probably use

A mental math.

B pencil and paper.

C a calculator.

h. Calculation: 6×5, $+ 2$, $\div 2$, $\sqrt{\ }$, $\times 7$, $- 1$, $\div 3$, $\sqrt{\ }$

problem solving When $\frac{85}{99}$ is written as a decimal, what is the 237th digit to the right of the decimal?

New Concept *Increasing Knowledge*

Theo is thinking of two numbers. He says that the sum of the two numbers is 6. He also says that one number is 10 more than the other number. What are the two numbers?

We could solve this problem by guessing and checking, but there are other strategies we can use.

Notice we are given two pieces of information about two numbers. One piece of information is that their sum is 6. We can write that information as an equation. We will use x and y to represent the two numbers.

$$x + y = 6$$

The other piece of information is that one number is ten more than the other number. We can write an equation for this information also. We let y represent the larger number.

$$y = x + 10$$

We have written two equations using the same two variables. Together the equations form a **system of equations,** which is two or more equations with common variables. We often use a brace to indicate a system of equations:

$$\begin{cases} x + y = 6 \\ y = x + 10 \end{cases}$$

Systems of equations are sometimes called **simultaneous equations.** We solve a system of equations by finding the values that satisfy both equations simultaneously. There are several ways to solve a system of equations. The method we will practice in this lesson is solving by graphing.

Visit www.SaxonPublishers.com/ActivitiesC3 *for a graphing calculator activity.*

Example

Theo is thinking of two numbers. He says that the sum of the numbers is 6. He also says that one number is 10 more than the other number. What are the two numbers? Solve this system of equations by graphing.

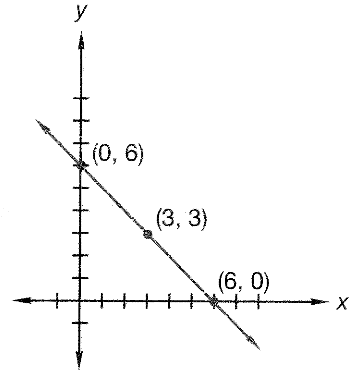
$$\begin{cases} \mathbf{x + y = 6} \\ \mathbf{y = x + 10} \end{cases}$$

Solution

By graphing the two equations, we will "see" the answer to the problem. We will graph a line for each equation. There are many pairs of numbers that satisfy $x + y = 6$ such as $6 + 0$, $3 + 3$, and $0 + 6$. We can make a table of ordered pairs, plot the points, and draw a line to represent all possible pairs of numbers whose sum is 6.

x + y = 6

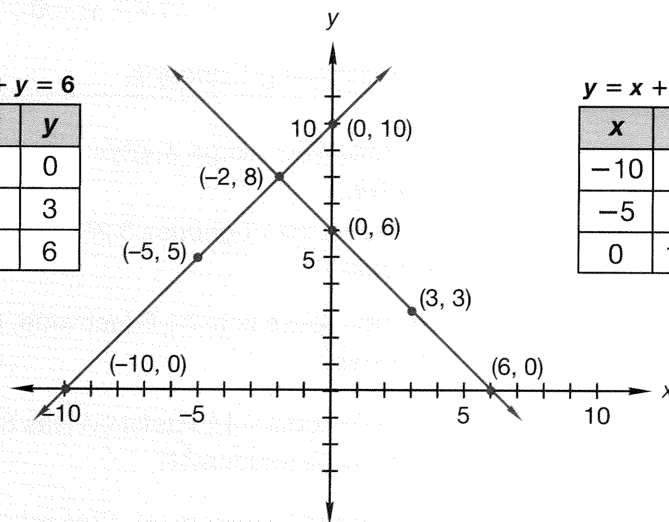
x	y
6	0
3	3
0	6



Now we consider the second equation, $y = x + 10$. Some (x, y) pairs that satisfy this equation are $(-10, 0)$, $(-5, 5)$, and $(0, -10)$. We graph this equation on the same coordinate plane as the first equation.

x + y = 6

x	y
6	0
3	3
0	6



y = x + 10

x	y
−10	0
−5	5
0	10

Thinking Skill

Infer

Each equation in this example has an infinite number of solutions. How do we know that the system of equations has only one solution?

This line represents all pairs of numbers in which one number is 10 more than the other number. We see that there is one point, or one pair of numbers, that is a solution to both equations. The lines intersect at $(-2, 8)$. This means that $x = -2$ and $y = 8$ is the one pair of values that satisfies both equations and both conditions of the problem. Therefore, **Theo's two numbers are –2 and 8.**

Verify Look back. Do these two numbers satisfy the condition of the original problem?

Practice Set

Solve each problem by graphing the system of equations and checking the solution.

a. Together Xena and Yolanda have \$12. Yolanda has 6 dollars more than Xena. How much money does each person have? Graph this system of equations to find the answer.

$$\begin{cases} x + y = 12 \\ y = x + 6 \end{cases}$$

b. Nikki is thinking of two numbers. Their sum is 12. The greater number is double the lesser number. What are the two numbers? Graph this system of equations to find the numbers.

$$\begin{cases} x + y = 12 \\ y = 2x \end{cases}$$

c. **Formulate** Use inspection to predict the solutions to this system of equations. Then graph the equations to verify your prediction.

$$\begin{cases} x + y = 6 \\ y = x \end{cases}$$

d. Describe two numbers represented by this system of equations. Then graph to find the two numbers.

$$\begin{cases} y = x - 6 \\ x + y = 0 \end{cases}$$

Written Practice

Strengthening Concepts

* **1.** (87) Andy is designing a large doghouse. His drawing has the scale 1 inch equals 4 feet.

a. If he draws the doghouse 3.25 inches long, how long is the actual doghouse?

b. If the doghouse is to be 6 feet wide, how wide must Andy draw the doghouse?

2. (45) The ratio of exuberant to deflated was 6 to 1. If there were 567 in all, how many were exuberant?

3. (67) If value of a $100 stock drops 10% one year, then increases 10% the next year, what is the value of the stock after the increase?

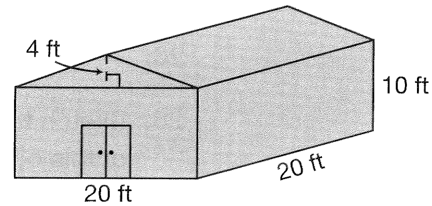
4. (46) An hour is 3.6×10^3 seconds. A common year is 8.76×10^3 hours. Find the number of seconds in a common year. Express the answer in scientific notation.

* **5.** (8, 9) Using whole numbers of inches only, give the widths and lengths of six different rectangles that have an area of 90 square inches.

6. (72) A one cubic meter shipping container can hold how many 1-cm cubes? Use unit multipliers.

7. (80) Add the trinomials $x + 2y - 4$ and $-x - y + 2$.

Analyze The illustration shows a building with a gabled roof. Refer to the illustration for problems **8–10.**



* **8.** (76) What is the volume of the building?

* **9.** (43) Enough paint must be purchased to cover the four sides of the building. What is the total surface area of the four sides?

10. (Inv. 2, 85) The roof overhangs the building by about one foot on all four sides. Estimate the total area of the two sloping sections of roof. (Hint: first use the Pythagorean Theorem to estimate the distance from the peak to the edge of the roof.)

11. (40, 45) **Evaluate** On a sunny day Jason sees that the shadow of a flag pole is about four times as long as his own shadow. If Jason is about $5\frac{1}{2}$ feet tall, about how tall is the flagpole?

Simplify.

* **12.** (78) $2\sqrt{6}\sqrt{15}$

13. (54) $\sqrt{720}$

14. (36) $(-12) + (-3)(-4) - (-5)$

15. (84) $\frac{2}{3} \times 0.15$

* **16.** (Inv. 2, 78) Can a right triangle have sides of length 1, 7, and $5\sqrt{2}$? Justify your answer.

* **17.** (88) Alex's phone company charges these rates for collect calls. Does this table show a proportional relationship? If so, state the constant of proportionality.

Minutes	Charge
1	$1.10
2	$1.20
3	$1.30
4	$1.40

18. (41) Graph the lines $y = -x$ and $y = x$. Describe the relationship between the two lines. Where do the lines intersect?

19. (89) **Model** Melissa is thinking of two numbers. She gives two hints: their sum is 1, and their difference is 5. Solve this system of equations by graphing to find her two numbers.

$$\begin{cases} x + y = 1 \\ x - y = 5 \end{cases}$$

20. (63) Write $\frac{8}{9}$ as **a** a percent, **b** a decimal, and **c** a decimal rounded to two places.

Solve.

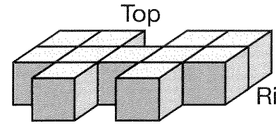
21. $\frac{0.5}{2.5} = \frac{x}{10}$ *(25, 44)*

22. $0.007x - 0.07 = 0.7$ *(25, 50)*

23. $x - 4 = 3x + 2$ *(14, 36)*

24. $\frac{5}{9}x - \frac{1}{3} = \frac{2}{3}$ *(23, 50)*

25. Sketch the top, front, and right side views of this object. *(Inv. 4)*



Early Finishers
Real-World Application

Cartography is the art of making maps. Cartographers create large-scale maps and small-scale maps. A large-scale map shows a small area with a large amount of detail. A small-scale map shows a large area with a small amount of detail. To understand the relationship between the distances on a map and the corresponding distances on the Earth's surface, you must understand the map's scale factor. On an certain globe, one inch represents five hundred miles on the Earth's surface. If a region is 660 miles wide, how many inches wide is the region on the globe?

LESSON 90

• Sets

Power Up — *Building Power*

facts Power Up R

mental math

a. **Statistics:** Find the median of this data: 11, 11, 17, 16, 15

b. **Estimation:** \$19.89 + \$21.89

c. **Number Sense:** What digit is in the ones place in the product 1528×432?

d. **Powers/Roots:** $\frac{m^5}{m}$

e. **Proportion:** $\frac{3}{2} = \frac{x}{4}$

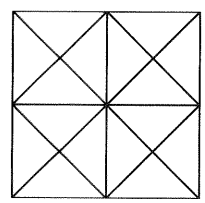
f. **Geometry:** Angle P and $\angle Q$ are supplementary. If $m\angle P = 95°$, find $m\angle Q$.

g. **Scientific Notation:** Write 6.02×10^{-2} in standard notation.

h. **Calculation:** 8×7, $-\ 20$, $\sqrt{\ }$, $\times\ 8$, $+\ 1$, $\sqrt{\ }$, $\div\ 2$

problem solving

How many different sizes of triangles are there in this figure? How many triangles of each size are there? How many triangles all together?



New Concept — *Increasing Knowledge*

Thinking Skill

Explain

Why does the set of whole numbers not begin with an ellipsis?

A set is a collection of elements. Almost anything can be an element, for example, numbers, ordered pairs, variables, or geometric figures. The elements of a set are often listed in braces, { }. The set of whole numbers can be expressed this way:

$$\{0, 1, 2, 3, \ldots\}$$

The ellipsis (...) indicates that the elements of the set continue in the same pattern without end. The set of integers can be expressed this way:

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Example 1

Use set notation to denote the set of even integers.

Solution

$$\mathbf{\{\ldots, -4, -2, 0, 2, 4, \ldots\}}$$

To indicate an element of a set we use a symbol like a curved E.

$3 \in$ {integers}	$\frac{1}{2} \notin$ {integers}
Read: 3 is an element of the set of integers	Read: $\frac{1}{2}$ is not an element of the set of integers

Example 2

Use set notation to indicate whether each number is an element of the set of rational numbers.

a. 2 **b.** $\frac{1}{2}$ **c.** $\sqrt{2}$

Solution

a. $2 \in$ **{rational numbers}**

b. $\frac{1}{2} \in$ **{rational numbers}**

c. $\sqrt{2} \notin$ **{rational numbers}**

Connect What other math symbol uses a slash mark to indicate the word *not?*

Example 3

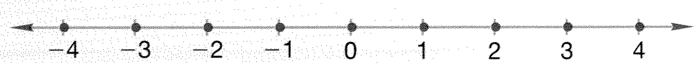
Graph the elements of each set on a number line.

a. the set of integers

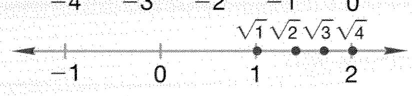
b. $\{\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}\}$

Solution

a.



b.



Sets can be related to each other in various ways. If all of set A is contained in set B, then set A is a **subset** of set B. The subset symbol is similar to a ∪ rotated 90° clockwise.

$$A \subset B$$

"A is a subset of B"

For example, the set of rectangles is a subset of the set of quadrilaterals, because every rectangle is a quadrilateral.

$$\text{rectangles} \subset \text{quadrilaterals}$$

Example 4

Use the subset symbol to show the relationship between the set of natural numbers $\mathbb{N}$ and the set of integers $\mathbb{Z}$.

Solution

Recall from Lesson 1 that natural numbers are the numbers we say when we count by ones starting from 1: {1, 2, 3, . . .}. Integers include all the natural numbers, their opposites, and zero. Thus, the set of natural numbers is a subset of the set of integers.

$$\mathbb{N} \subset \mathbb{Z}$$

Symbols like the N and Z in example 4 are used in some math books to indicate certain sets of numbers. Here are the symbols that are used.

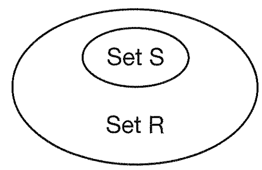
$\mathbb{N}$ Natural (counting) numbers

$\mathbb{Z}$ Integers

$\mathbb{Q}$ Rational numbers (quotients)

$\mathbb{R}$ Real numbers

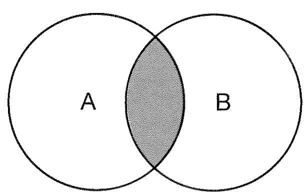
We can use **Venn diagrams** to illustrate the relationships among sets. This diagram shows that set S (squares) is a subset of set R (rectangles).



Sets A and B overlap if some elements of set A are also elements of set B. The shaded region of the Venn diagram illustrates the common elements. This region is called the **intersection** of sets A and B and is shown with the symbol ∩. The intersection of A and B is the set of elements that are in both sets.

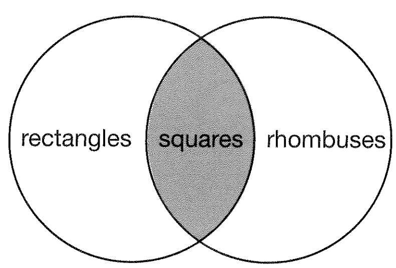
$$A \cap B$$

"The intersection of sets A and B."

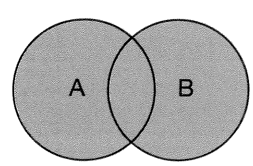


A ∩ B

For example, the intersection of the set of rectangles and the set of rhombuses is the set of squares.



We use the word **union** and the symbol ∪ to indicate the combining of sets. The shaded Venn diagram below illustrates the union of sets A and B. The union is the set of all the elements of sets A and B combined.



A ∪ B

For example, the union of the set of boys in the classroom and the set of girls in the classroom is the set of students in the classroom.

Example 5

Sets A and B are shown below. Find A ∩ B and A ∪ B.

$$A = \{1, 2, 3, 4\} \qquad B = \{2, 4, 6, 8\}$$

Solution

$$A \cap B = \{2, 4\}$$
$$A \cup B = \{1, 2, 3, 4, 6, 8\}$$

If there are no elements in the intersection of two sets, we say the intersection is an **empty set:** ∅.

Formulate Name two sets in which the intersection is the empty set.

In the table below we list some symbols commonly used to refer to relationships among sets.

Set Notation Symbols

{ }	The set of
∈	Is an element of
⊂	Is a subset of
∩	The intersection of
∪	The union of
∅	Empty set

Example 6

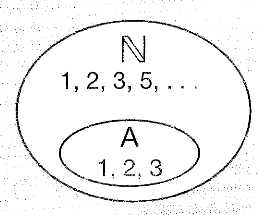
Use a Venn Diagram to illustrate the sets. Find A ∩ B.

a. $A = \{1, 2, 3\}$
$B = \mathbb{N}$

b. $A = \{2, 4, 6\}$
$B = \{1, 3, 5\}$

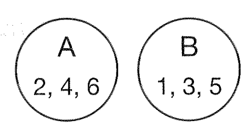
Solution

a.



$A \cap \mathbb{N} = 1, 2, 3$

b.



$A \cap B = \varnothing$

Practice Set

Indicate if the number is an element of the set of integers.

a. −1 **b.** 2 **c.** $\sqrt{3}$

Use the subset symbol to show the relationship between A and B.

d. A = {all triangles}; B = {all polygons}

e. $A = \mathbb{R}; B = \mathbb{Q}$

f. A = {vowels}; B = {letters in the alphabet}

Use a Venn Diagram to illustrate the relationship between the sets. Find $A \cap B$.

g. A = {3, 6, 9, 12}
B = {5, 12, 15, 20}

h. A = {all parallelograms}
B = {all trapezoids}

Use a Venn Diagram to illustrate the relationship between the sets. Then indicate the union of the sets.

i. $\{10, 20, 30\} \cup \{20, 30, 40\}$

j. $\{2, 4, 6, 8, \ldots\} \cup \{4, 8, 12, \ldots\}$

Written Practice *Strengthening Concepts*

* **1.** *(87)* James drew a village to the scale 1 inch equals 10 feet. **a** If he drew one building 2.1 inches long, how long was the actual building? **b** If a building was actually 18 feet wide, how wide was the building on James' drawing?

* **2.** *(46)* **Formulate** One atom of mercury has a mass of 3.33×10^{-22}g. What is the mass of 6.02×10^{23} atoms of mercury? Express your answer in standard form.

3. *(67)* The $45 price was increased by 8% for sales tax. What was the total price including tax?

* **4.** *(81)* What is the measure of the central angle formed by the hour and minute hands on the face of a clock at the following times?
a. 6:00 **b.** 3:00 **c.** 4:30 **d.** 1:30

5. *(68)* If a coin is flipped five times, what is the probability of getting heads all five times?

6. *(73)* Describe the rule of this sequence and express the rule with an equation. Then use your rule to find the 20th term.

5, 10, 15, 20, . . .

7. *(Inv. 2)* On a coordinate plane draw a rectangle with these vertices: (3, 1), (2, −2), (−1, −1), and (0, 2). Then find the length of each side.

8. *(Inv. 2, 78)* Find the area of the rectangle you drew in exercise **9.**

* **9.** *(64)* Some pulsars spin at 60,000 rotations per minute (rpm). How many rotations per second is that? How long does it take for each rotation?

Generalize Simplify.

10. *(33, 51)* $(-3)^1 - 3^2 - (-3)^3$

11. *(84)* 75% of $\frac{2}{3}$ of $1.44

12. *(74)* $\sqrt{20{,}000}$

13. *(51)* $2x^{-2}y^{-3}xy$

14. (53) At the airport, Andre surveyed passengers and asked how many trips in the last year they had taken by air. The data he collected is shown below:

3, 1, 2, 1, 5, 6, 2, 2, 3, 4

a. Find the mean, median, mode, and range of the data.

b. Which of these measures would you report to someone interested in knowing the most common number of trips people at this airport take in a year?

15. (41) Rene recorded the time and average speed she traveled on different trips from her home to work.

a. Graph the pairs of points on coordinate axes.

b. Use the graph to predict how long the trip might take Rene if she travels 55 mph.

Rate (mph)	Time (min)
30	90
45	60
50	54
60	45

* **16.** (89) *Model* Bingham and Gridley each thought of a number. Bingham's number was 5 less than Gridley's number. The sum of their numbers was 3. Find their two numbers by graphing this system of equations.

$$\begin{cases} y = x - 5 \\ x + y = 3 \end{cases}$$

* **17.** (90) *Explain* Adam counted by twos to twelve. Beverlee counted by threes to twelve.

$$A = \{2, 4, 6, 8, 10, 12\}$$
$$B = \{3, 6, 9, 12\}$$

a. Find $A \cap B$ and describe what the answer means.

b. Find $A \cup B$ and describe what the answer means.

18. (64, 72) The driving speed limit of some highways is 60 mph. Use unit multipliers to convert 60 miles per hour to feet per second. (1 mi = 5280 ft)

19. (63) Write $\frac{2}{11}$ **a** as a percent and **b** as a decimal, and **c** as a decimal rounded to three decimal places.

20. (21) **a.** Factor: $5m^2 - 40m + 5$ **b.** Expand: $7x(x - 3)$

Solve.

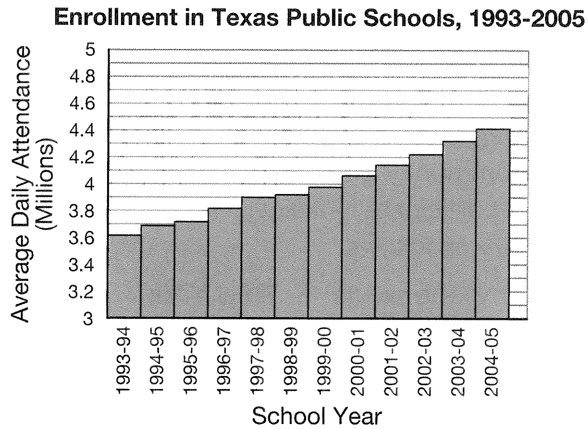
21. (25, 44) $\frac{1.5}{2.5} = \frac{3}{x}$

22. (25, 50) $0.05 + 0.02x = 1.95$

23. (23, 50) $\frac{4}{5} - \frac{2}{3}x = \frac{14}{15}$

24. (23, 38) $-\frac{m}{3} = 3$

25. *(Inv. 6)* The graph below shows the change in the total average daily attendance in all public schools in one state over a period of time. Use the graph to answer the following questions.



a. What kind of graph is this?

b. The data for which state is displayed in the graph?

c. What is the interval used for data categories on the x-axis?

d. In which school year was enrollment about 15% greater than the enrollment in 1993-94?

e. Describe the trend of the data, if one exists.

Early Finishers
Real-World Application

Mr. Tham needs to rent a car for an upcoming vacation. Cars R Us charges an initial fee of $10.00 plus $15.00 a day. Roger's Rental charges an initial fee of $15.00 plus $10.00 a day.

a. If d represents the number of days one car is rented and y represents the total cost of renting the car, write one equation for each rental car company to represent the cost of renting a car.

b. Graph both equations for values of d from 1–5. Which rental car company offers the best price for a five-day rental?

c. On which day do both shops charge the same amount?

INVESTIGATION 9

Focus on

Sampling Methods

The U.S. Census Bureau reported that for a one-way trip to work, the average nationwide commute time was 24.3 minutes in 2003. The American Community Survey randomly selected households from across the nation to participate in a similar commute study. Individual state and city averages vary as follows.

Nationwide	New York	Nebraska	Chicago	Tulsa
24.3 min	30.4 min	16.5 min	33.2 min	17.1 min

Every ten years, the U.S. Census Bureau conducts a study, the intent of which is to survey every resident in the nation. This study, in which every member of the population is included, is called a *census study*. With a census study, it is possible to accurately describe characteristics of the population.

A census of an entire population requires a large amount of time and money, so it is generally not practical to conduct a census study. Instead we study a **sample,** or small group, of the population. A sample should be **representative** of, or very similar to, the larger population. If a large enough sample is selected **randomly** from the entire population, we can be reasonably sure that the sample is representative.

Math Language
In a **random sample,** each member of the population has an equal probability of being selected.

Analyze How might the results of the survey above differ if the sample was selected from only one city or state? How representative would the results be for the entire nation?

When we conduct a study of a population, we randomly select a sample of sufficiently large size. To do this, we can assign a number to each member of the population to be studied and randomly select numbers to choose sample subjects.