



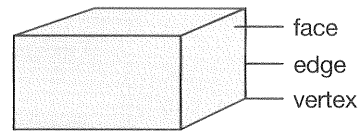
Focus on

• Drawing Geometric Solids

Closed figures that we draw on paper, like polygons and circles, are two-dimensional. They have length and width, but they are flat, lacking depth. Physical objects in the world around us are three-dimensional; they have length and width and depth.

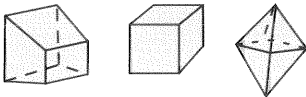
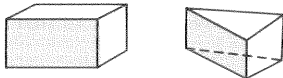
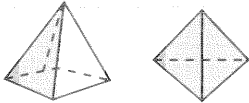
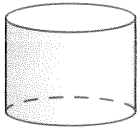
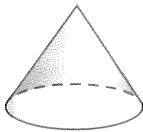

Two-dimensional figures are called plane figures because they are contained in a plane. Three-dimensional figures are sometimes called space figures, because they occupy space and cannot be contained in a plane. Space figures are also called **geometric solids**.

The terms **face**, **edge**, and **vertex** (pl. vertices) refer to specific features of solids. The face of a polyhedron is a polygon. Two faces meet at an edge and edges intersect at a vertex.

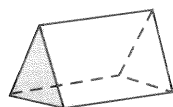


In the table below we name, illustrate, and describe some geometric solids.

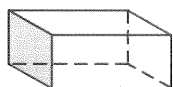
Geometric Solids

Polyhedron (pl. polyhedra)		A general term that identifies a solid with faces that are polygons. A polyhedron has no curved surfaces or edges.
Prism		A type of polyhedron with parallel congruent bases.
Pyramid		A type of polyhedron with lateral surfaces that narrow to a point (apex).
Cylinder		In this book we will use the term cylinder to refer to a right circular cylinder as illustrated.
Cone		In this book we will use the term cone to refer to a right circular cone as illustrated.
Sphere		A smooth curved solid every point of which is the same distance from its center.

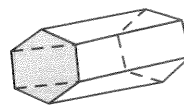
Prisms can be further classified by the shapes of their bases.



Triangular Prism



Rectangular Prism



Hexagonal Prism

One type of prism is a **rectangular prism** in which all faces are rectangles. One type of rectangular prism is a **cube** in which all faces are congruent squares.

Example

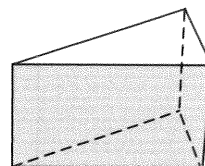
The pyramids of Egypt have square bases. A pyramid with a square base has how many faces, edges, and vertices?

Solution

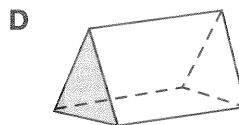
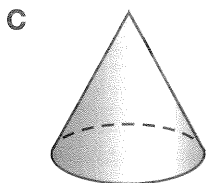
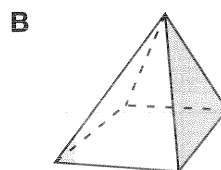
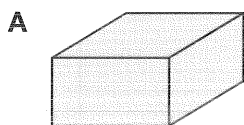
A pyramid with a square base has **5 faces** (one square and four triangles), **8 edges**, and **5 vertices**.

Conclude How do the number of faces of a pyramid relate to the shape of its base? How do the number of faces of a prism relate to the shape of its base?

1. A triangular prism has how many faces, edges, and vertices?

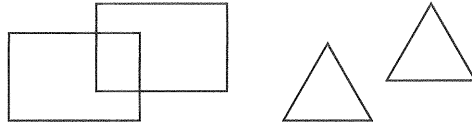


2. Name each figure. Which is not a polyhedron? Explain your choice.

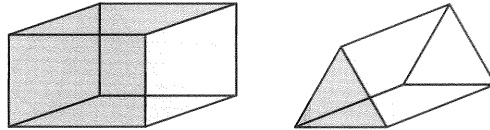


Geometric solids are three-dimensional figures. They have length, width, and depth (or height); they take up space. Flat surfaces like pages in a book, billboards, and artists' canvases are two-dimensional and lack depth. Thus, to represent a three-dimensional figure on a two-dimensional surface we need to create the illusion of depth. In this investigation we will practice ways to draw and represent three-dimensional figures in two dimensions.

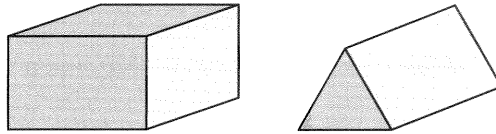
Parallel projection is one way to sketch prisms and cylinders. Recall that prisms and cylinders have two congruent bases, one at each end of the figure. Using parallel projection, we draw the base twice, one offset from the other. We draw each base the same size.



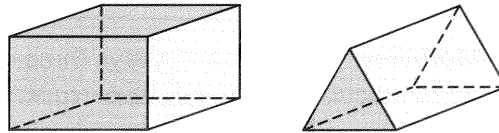
Then we draw parallel segments between the corresponding vertices (or the corresponding opposite sides of the circles).



The resulting figures show all the edges as though the prisms were made with wire. If we were drawing a picture of a box or tube, the hidden edges would be erased.



In geometric sketches we often show hidden edges with dashes.



Activity 1

Sketching Prisms and Cylinders Using Parallel Projection

Materials needed:

- unlined paper
- pencil

Description of activity:

Sketch a rectangular prism, a triangular prism, and a cylinder using parallel projection. Make the bases of each figure congruent. Show hidden edges with dashes. To sketch the figures from different viewpoints, shift the position of the more distant base.

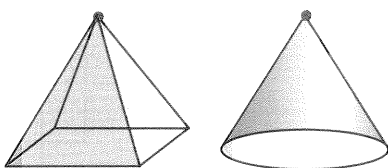
To draw pyramids and cones we can begin by sketching the base. To represent a square base of a pyramid we can draw a parallelogram. To represent the circular base of a cone we can draw an **ellipse**, which looks like a stretched or tilted circle.



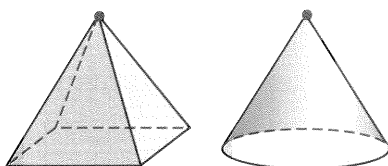
Math Language

A pyramid's **apex** is also one of its vertices.

Centered above the base we draw a dot for the **apex**, or peak, of the pyramid or cone. Then we draw segments to the vertices of the base of the pyramid (or opposite points of the circular base of the cone).



We erase hidden edges or represent them with dashes.



Activity 2

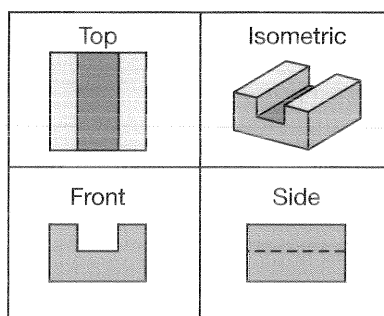
Sketching Pyramids and Cones

Sketch a few pyramids and cones from different viewpoints.

Model Sketch a pyramid that has a triangular base.

Discuss What steps will you follow to draw a pyramid with a triangular base?

Multiview projection displays three-dimensional objects from three perpendicular viewpoints, **top**, **front**, and **side** (usually the right side). An **isometric** view is often included, which is an angled view of the object, as shown in the projection below.



Notice that hidden edges do not appear in the isometric drawing but do appear in the other views as dashes. Notice that the width of the figure is the same in the top and front views. The height is the same in the front and side views, and the length is the same in the top and side views.

Activity 3

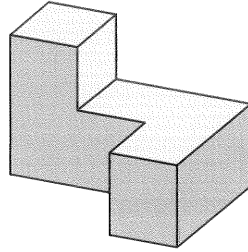
Create a Multiview Drawing

Materials needed:

- unlined paper
- pencil
- ruler

Description of activity:

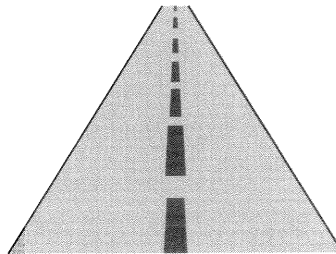
Divide your paper into four equal sections. In the isometric section copy the sketch below. Then draw the top, front, and side views.



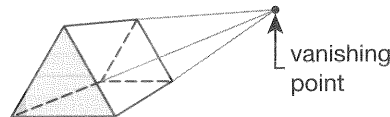
extensions

1. Build a figure from unit cubes and create a multiview projection for the figure.
2. Create a multiview projection for a building on your school's campus or in your school's neighborhood.

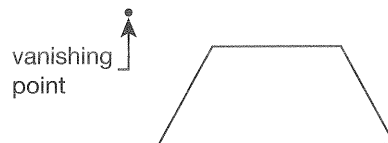
Parallel projection and isometric drawings do not provide the same perspective of depth our eyes perceive in the three-dimensional world. Objects appear to diminish in size the farther they are from us. For example, if we look down a straight road it may appear to nearly vanish at the horizon.



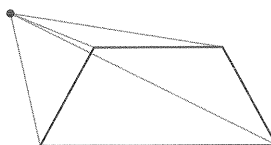
Using a **vanishing point** can provide a **perspective** to a figure as its more distant edges are shorter than corresponding nearer edges. Below we show a triangular prism drawn toward a vanishing point.



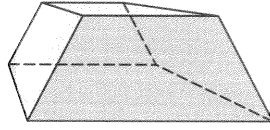
To create a one-point perspective drawing of a prism, draw a polygon in the foreground and pick a location for the vanishing point.



Then lightly draw segments from the vertices of the polygon to the vanishing point.



Determine the depth you want to portray and draw corresponding segments *parallel* to the sides of the polygon in the foreground. Darken the edges between the two bases. Use dashes to show hidden edges.



Activity 4

One-Point Perspective Drawing

Materials needed:

- unlined paper
- pencil
- ruler

Description of activity:

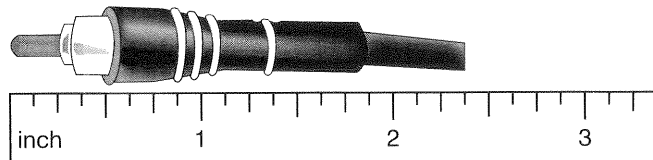
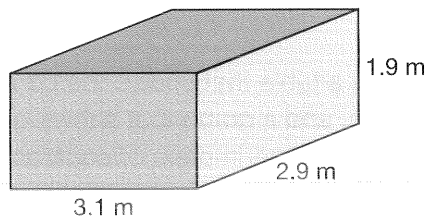
Create one-point perspective drawings of some prisms. Place the vanishing point in different locations to change the point of view. Try different polygons for the bases of the prisms.

Power Up

Building Power

facts

Power Up I

mental
matha. **Number Sense:** 19×20 b. **Probability:** The probability of rolling a number less than 3 on a number cube is $\frac{2}{6}$, because there are 2 numbers less than 3 (1 and 2) out of 6 possible outcomes. Find the probability of rolling a number less than 6.c. **Fractional Parts:** 75% of \$36d. **Measurement:** Find the length of this piece of cable.e. **Scientific Notation:** Write 4.06×10^6 in standard notation.f. **Rate:** The town's garbage trucks dump 7.7 tons of garbage per week. On average, how many tons per day is that?g. **Geometry:** To find volume of a rectangular prism, we multiply $l \times w \times h$. Approximate the volume of this rectangular prism.h. **Calculation:** $41 + 14, \div 11, \times 7, + 1, \sqrt{\quad}, + 3, \text{ square it}, + 19, \sqrt{\quad}, + 3$ problem
solving

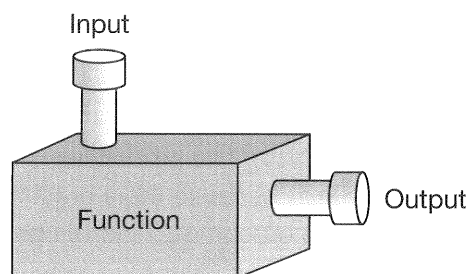
Find the sum of the whole numbers from 1 to 15.

New Concept

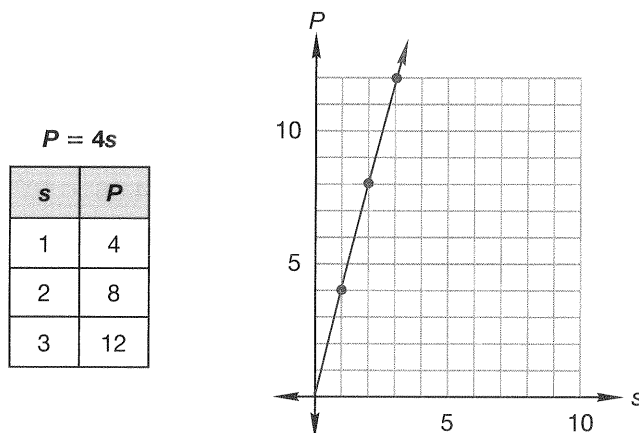
Increasing Knowledge

The formula for the perimeter of a square, $P = 4s$, identifies the relationship between two variables, the length of a side of a square (s) and its perimeter (P). This formula is an example of a function.

A **function** is a mathematical rule that identifies a relationship between two sets of numbers. A function's rule is applied to an input number to generate an output number. **For each input number there is one and only one output number.** In the formula $P = 4s$, the input is the side length and the output is the perimeter.



A function may be described with words or expressed with an equation, as in the formula for the perimeter of a square. We may illustrate functional relationships in tables or in graphs.



Here is a table that shows the perimeter of a square for some given side lengths, and a graph that shows other pairs of numbers that represent side lengths and perimeters of squares.

In this lesson we will consider function tables and graph some functions.

Example 1

The perimeter of an equilateral triangle is three times the length of one of its sides.

- Write an equation that expresses this relationship. Use P for perimeter and s for side length.
- Choose four different side lengths. Make a function table that shows the side lengths and the perimeters of the equilateral triangles.
- Graph the pairs of numbers from the table on a coordinate plane. Let the horizontal axis represent side length (s) and the vertical axis represent perimeter (P).

Solution

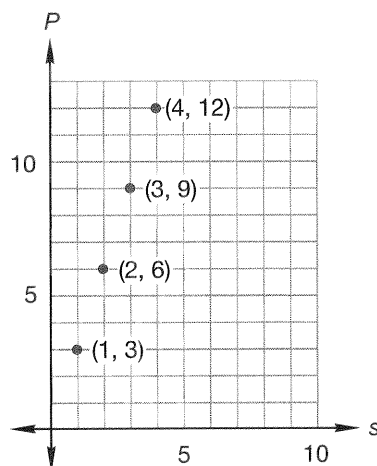
- a. To find the perimeter (P) of an equilateral triangle we multiply the length of a side (s) by 3.

$$P = 3s$$

- b. We can use the equation to help us fill in the table. The input number is on the left; the output number is on the right. We choose four side lengths such as 1, 2, 3, and 4. We apply the rule (multiply the side length by 3) to find each perimeter.

$P = 3s$

s	P
1	3
2	6
3	9
4	12



- c. We graph the (s, P) pairs just as we graph (x, y) pairs. Since lengths are only positive, all possible pairs are in the first quadrant. Notice that the points are aligned.

Example 2

Xavier and Yolanda are playing a game. When Xavier says a number, Yolanda performs an operation with Xavier's number and says the result. The table shows four numbers Xavier said and Yolanda's replies. Describe the rule Yolanda uses, and write the rule as an equation. Then graph the (x, y) pairs from the table and all other pairs of numbers Xavier and Yolanda could say.

x	y
2	4
3	5
4	6
5	7

Solution

Yolanda adds 2 to the number Xavier says. The equation is

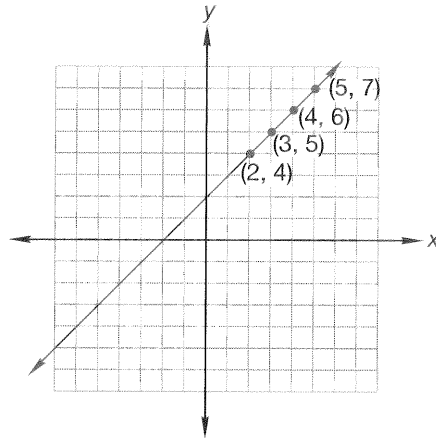
$$y = x + 2.$$

We graph the x, y pairs from the table and see that the points are aligned. We suspect other points along this line also meet the conditions of the problem. We draw a line through and beyond the graphed points and test some x, y pairs on the line to see if they fit the equation $y = x + 2$. Any point we choose on the line satisfies the equation. We conclude that the graph of all possible pairs of numbers Xavier and Yolanda could say using the rule are represented by points on the graphed line.

Thinking Skill

Verify

Select and name coordinates of points on the graphed line that are not in the table. What do the two numbers for each selected point represent? Do the number pairs fit the rule of the game Xavier and Yolanda are playing?



Examples 1 and 2 illustrate special classes of functions. We noted that the points plotted in both examples are aligned. If all the input-output pairs of a function fall on a line, then the function is **linear**. The functions in examples 1 and 2 are linear functions.

However, there is an important difference between the functions in example 1 and example 2. Notice that the ratio of output to input in example 2 is not constant.

Example 1
 $P = 3s$

s	P	$\frac{P}{s}$
1	3	$\frac{3}{1}$
2	6	$\frac{6}{2} = \frac{3}{1}$
3	9	$\frac{9}{3} = \frac{3}{1}$
4	12	$\frac{12}{4} = \frac{3}{1}$

Proportional

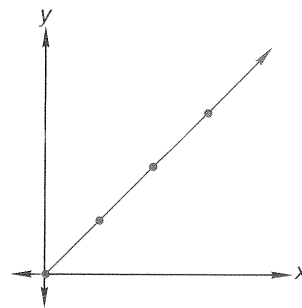
Example 2
 $y = x + 2$

x	y	$\frac{y}{x}$
1	3	$\frac{3}{1}$
2	4	$\frac{4}{2} = \frac{2}{1}$
3	5	$\frac{5}{3}$
4	6	$\frac{6}{4} = \frac{3}{2}$

Not Proportional

This means the relationship of perimeter to side length is **proportional**, but the relationship of Yolanda's number to Xavier's number is **not proportional**. Graphs of proportional functions have these two characteristics.

1. The function is linear.
2. The points are aligned with the origin (0, 0).



Example 3

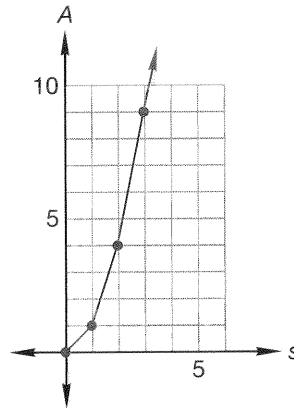
The area of a square is a function of its side length. The graph of an area function is different from the graph of a perimeter function. For the function $A = s^2$, describe the rule with words and make a table that shows at least four pairs of numbers that satisfy the function. Then graph the (s, A) pairs and predict the graph of other points for the function.

Solution

The rule is, “To find A , square s .” We may choose any four numbers for s to put into the function. To keep our calculations simple we choose 0, 1, 2, and 3. Then we find A for each value of s .

$A = s^2$

s	A
0	0
1	1
2	4
3	9



We graph the pairs of numbers from the table with s on the horizontal axis and A on the vertical axis. We see that the points are not aligned. Instead, the points seem to be on a curve that becomes steeper and steeper as the side length increases.

Discuss Is the function of side length to area linear? Is the function proportional?

Example 4

Yanos is playing a numbers game. When Xena says a number, Yanos says a number that is twice Xena's number.

- Write an equation for the game using x for the number Xena says and y for the number Yanos says.
- Make a table that shows some pairs of numbers for the game.
- Graph all the possible pairs of numbers for the game.

Solution

a. $y = 2x$

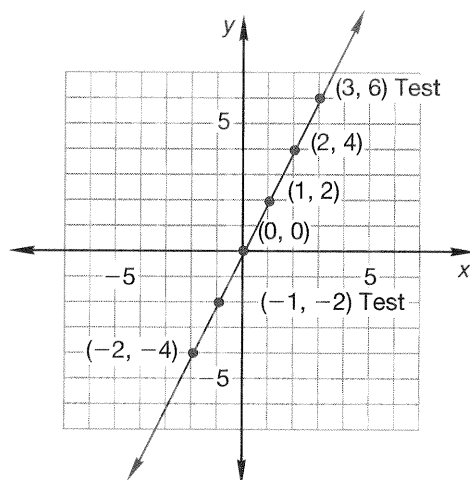
b. We write some numbers Xena could say in the x -column. Then we find each number Yanos would say for each of Xena's numbers and write them in the y -column.

$y = 2x$

x	y
0	0
1	2
2	4
-2	-4

c. The numbers in the function table form ordered pairs of numbers we can graph on a coordinate plane. We graph the ordered pairs and draw a line through the graphed points.

Every point on the line represents an (x, y) pair of numbers that meets the conditions of the game.



To check our work we can test some (x, y) pairs on the line to see if the pairs meet the conditions of the problem. We choose $(3, 6)$ and $(-1, -2)$. If Xena says 3, Yanos doubles the number and says 6. If Xena says -1 , Yanos doubles the number and says -2 . These pairs of numbers satisfy the problem, which verifies our work.

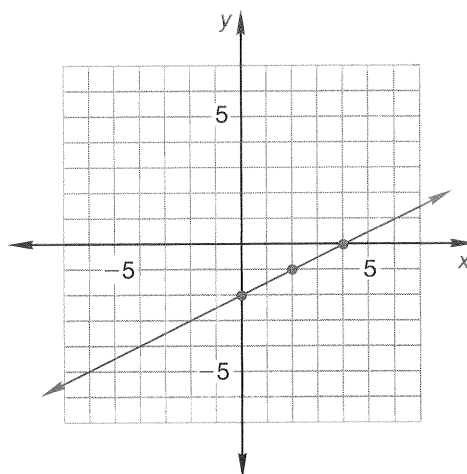
Example 5

Graph the function $y = \frac{1}{2}x - 2$.

Solution

In this lesson we will graph functions by first making a function table of (x, y) pairs that satisfy the equation. We choose values for x and calculate each corresponding value for y . Since x is multiplied by $\frac{1}{2}$, we choose even numbers for x so that y is a whole number. Then we plot the (x, y) ordered pairs and draw a line through them to complete graphing the equation.

x	y
0	-2
2	-1
4	0



In this lesson, we have described functions with words and expressed functions with equations. We have listed pairs of numbers in tables that satisfy selected functions and represented these functions in graphs.

Discuss Describe some of the benefits and limitations of each of these four ways of expressing a function.

Practice Set

- a. **Connect** The relationship between feet and inches is a function. Write an equation that shows how to find the number of inches (n , output) if you know the number feet (i , input). Then make a function table that shows the number of inches in 1, 2, 3, and 4 feet. Is the relationship linear? Is the relationship proportional?
- b. Yolanda played a new number game with Xavier. She used this equation to generate her response to each number Xavier said:

$$y = x - 2$$

Describe with words the rule Yolanda uses. Then make a function table that shows the numbers Yolanda says for four numbers Xavier might say. Use 0 for one of Xavier's numbers and three more numbers of your choosing. Is the function linear? Is it proportional?

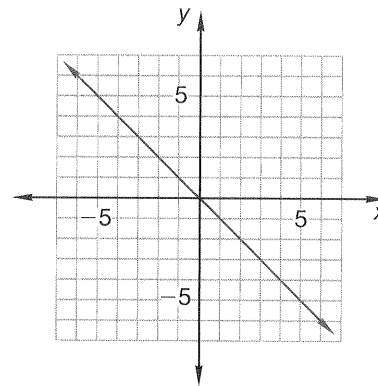
- c. **Formulate** This table shows the capacity in ounces of a given number of pint containers. Describe with words the rule of the function and write an equation that relates pints (p) to ounces (z). Then find the number of ounces in 5 pints.

Pints	Ounces
1	16
2	32
3	48
4	64

- d. State why the relationship between the numbers in this table is not a function.

x	y
0	0
1	1
1	-1
2	2

- e. Yanos and Xena played a numbers game. This graph shows all the possible pairs of numbers they could say following the rule of the game. Make a function table that lists four pairs of numbers they could say. Then describe the rule Yanos followed and write an equation for the rule.

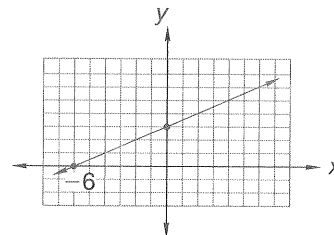


- f. **Formulate** Make up a numbers game Yanos and Xena could play. Describe the rule and write an equation for the rule. Make a function table that has four pairs of numbers they could say. Graph all possible pairs of numbers for the game you create.
- g. **Model** For the function $y = \frac{1}{2}x + 1$, make a function table and use it to graph the equation.

Written Practice

Strengthening Concepts

1. Cathy makes \$18 teaching a half-hour piano lesson. She makes \$32 an hour as a chef at a restaurant. How much does she make in a week if she works as a chef for 2 hours every Friday and teaches piano lessons for one hour on Mondays and 2 hours on Tuesdays?
(3, 4)
- * 2. **Analyze** The cafeteria offered six choices of vegetables and eight choices of fruits. What was the ratio of fruit to vegetable choices?
(29)
3. Socks are sold four pairs per pack. Bobby put three complete packages of socks in the laundry. He later noticed that he had 21 socks. How many socks were misplaced in the laundry?
(3, 4)
- * 4. **Verify** Shown is a graph of the function $y = \frac{1}{2}x + 3$. Does $x = 2$ and $y = 4$ satisfy the function? Demonstrate your answer by substituting the values and simplifying.
(41)



* 5. **Model** Sketch a rectangle and draw its lines of symmetry.
(Inv. 3)

* 6. **Conclude** Which pair of words correctly completes the following sentence? "Similar polygons have corresponding angles that are _____ and corresponding sides lengths that are _____."
(35)

- A similar, congruent B congruent, proportional
C acute, straight D vertexes, lines

* 7. **Generalize** Use $\frac{22}{7}$ for π to find the **a** area and **b** circumference of a circle with a diameter of 14 meters.
(39, 40)

* 8. Find the **a** area and **b** circumference of the circle with radius 1 cm.
(39, 40) Express answers in terms of π .

9. At 7 p.m. the wind-chill temperature was 4°F . By 10 p.m., it had fallen 11 degrees. Write an equation to show the wind-chill temperature at 10 p.m., then solve the equation.
(33)

Solve.

* 10. $-3m = 4.2$
(25, 38)

* 11. $30.7 - x = 20$
(24, 38)

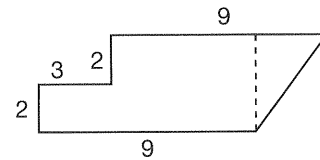
Analyze For problems 12 and 13, refer to the figure at right. (Dimensions are in meters.)

12. Find the area of the figure.
(37)

13. Find the perimeter of the figure.
(8)

14. a. Expand: $-4(2m - 7x + 9)$
(11, 18)

b. Factor: $3x^2 + 3x + 3$



Simplify.

15. $\frac{32xmz}{-4mz}$
(27)

16. $\frac{2}{5} + 4\frac{2}{7} \div 1\frac{11}{49}$
(13, 23)

17. $\frac{2}{3} \cdot 2\frac{1}{4} \cdot 1\frac{4}{5}$
(23)

18. $36 \div [2(1 + 2)^2]$
(21)

19. $\frac{(-3) + (-5)}{-4}$
(31, 36)

20. $\left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^3$
(6, 12)

21. $(-3)^2 + (-3)^1 + (-3)^0$
(27, 36)

22. $(2x^2y)^3$
(27, 36)

23. Write $\frac{11}{20}$ as a **a** decimal and **b** percent.
(12)

24. Three alphabet tiles are face down. The letters on the tiles are S, A, and M. Danielle picks one tile, keeps it, and picks up another.
(32)

a. What is the sample space of the experiment?

b. What is the probability that one of the tiles she picks up will be an A?

Combine like terms to simplify.

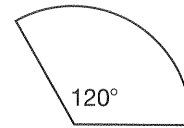
25. $2xy - xy - y + 7y - xy$
(31)

26. $9x^2 - 4x + 7x - 6x^2 + 3$
(31)

Evaluate.

27. For $E = \frac{1}{2}mv^2$, find E when $m = 5$ and $v = 4$.
(14, 15)

28. The radius of the sector shown is 3 ft. State the area of the sector in terms of π .
(11)



29. Florence bought 50 cm of thin leather cord to make jewelry. She used 16 cm of it for a bracelet and 34 cm for a necklace. How many centimeters of leather cord were left?
(31)

- * 30. **Evaluate** Xena and Yanos played a number game. In the table are four numbers Xena said and the numbers Yanos said in reply.
(41)

X	Y
3	0
0	-3
2	-1
5	2

- Use words to describe the rule Yanos followed.
- Write an equation that shows the rule.
- Sketch a graph that shows all the pairs of numbers that fit the rule. Then draw a line through the points to show other x, y pairs of numbers that fit the rule.

• Volume

Power Up

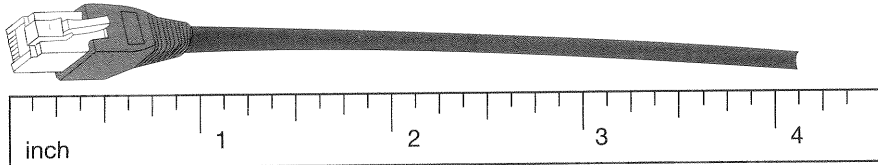
Building Power

facts

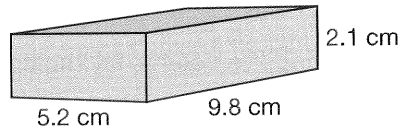
Power Up I

mental math

- a. **Number Sense:** 2×256
- b. **Algebra:** $\frac{x}{4} = 6$
- c. **Measurement:** Find the length of the piece of cable.



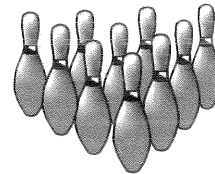
- d. **Percent:** 75% of 40
- e. **Scientific Notation:** Write 6.05×10^5 in standard notation.
- f. **Rate:** Robert biked 12 miles in an hour. On average, how many minutes did it take him to ride each mile?
- g. **Geometry:** Approximate the volume:



- h. **Calculation:** $\sqrt{100}, \times 6, + 4, \sqrt{\quad}, \times 6, + 1, \sqrt{\quad}, \times 9, + 1, \sqrt{\quad}, + 1, \sqrt{\quad}$

problem solving

In each bowling lane, 10 pins are arranged in four rows forming a triangle as shown:
 If the pins from 12 lanes are combined to make one big triangle of pins, how many rows will it have?



New Concept

Increasing Knowledge

In this lesson we will find the volumes of rectangular prisms. In later lessons we will find the volumes and surface areas of other geometric solids.

The **volume** of a solid is the total amount of space occupied or enclosed by the solid. Volume is measured in cubic units such as cubic centimeters (cm^3), cubic inches (in.^3), and cubic feet (ft^3). The following diagram illustrates volume.

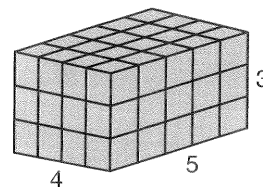
Thinking Skill

Analyze

Why are the formulas $V = Bh$ and $V = lwh$ equivalent formulas for the volume of a rectangular prism?

The bottom layer of this solid has 4 rows of cubes with 5 cubes in each row. Therefore, there are 20 cubes on the bottom layer. There are three layers of cubes, so there are 3×20 , or 60 cubes in all.

Notice that we found the volume (V) by multiplying the area of the base (B) by the height (h). We can use this formula for any prism.



Volume of a Prism

$$V = Bh$$

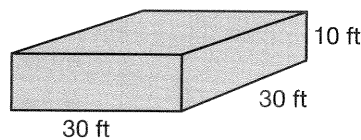
The area of the base of a rectangular prism equals the length (l) times the width (w). Thus, the specific formula for a rectangular prism is:

Volume of a Rectangular Prism

$$V = lwh$$

Example 1

To calculate the heating and cooling requirement for a room, one factor architects consider is the room's volume. A classroom that is 30 feet long, 30 feet wide, and 10 feet high contains how many cubic feet?



Solution

We can find the volume by multiplying the room's length, width, and height.

$$V = lwh$$

$$V = (30 \text{ ft})(30 \text{ ft})(10 \text{ ft})$$

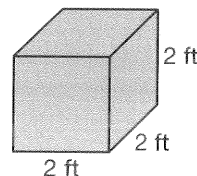
$$V = 9000 \text{ ft}^3$$

Example 2

A sculptor carved a 2-foot cube of ice into a swan. At 57 pounds per cubic foot, what did the 2-foot cube weigh before it was carved?

Solution

A 2-foot cube is a cube with edges 2 feet long. The specific formula for the volume of a cube is $V = s^3$, where s is the edge length. We will use this formula to find the volume of the cube.



Step:

$$V = s^3$$

$$V = (2 \text{ ft})^3$$

$$V = 8 \text{ ft}^3$$

Justification:

Volume of a cube

Substituted

Simplified

The volume of the ice before it was carved was 8 ft^3 . At 57 pounds per ft^3 , the weight of the 2-ft cube was 8×57 , or **456 pounds**.

Analyze Why are the formulas $V = Bh$ and $V = s^3$ equivalent formulas for the volume of a cube?

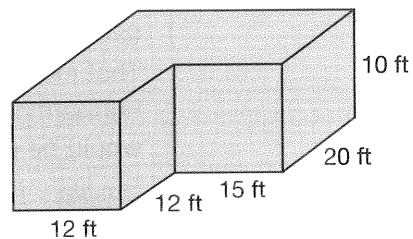
Activity

Volume of a Box

Find a box in the classroom such as a tissue box. Measure the length, width, and height of the box and estimate its volume after rounding the dimensions to the nearest inch.

Example 3

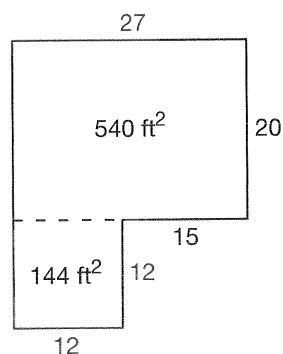
Many buildings (and other objects) are shaped like combinations of geometric solids. Find the volume of a building with this shape. All angles are right angles.



Solution

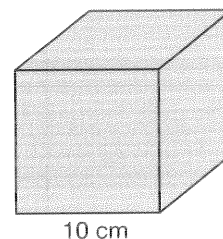
First we find the area of the building's base. We sketch the base and divide it into two rectangles. We add the area of the two rectangles to find the area of the base (684 ft^2). Now we find the volume.

$$\begin{aligned} V &= Bh \\ &= (684 \text{ ft}^2)(10 \text{ ft}) \\ &= 6840 \text{ ft}^3 \end{aligned}$$



Example 4

Find the volume of this cube.



Solution

We can choose from three formulas to calculate the volume of a cube. A cube is a prism, so we may use the formula $V = Bh$. A cube is a rectangular prism, so we may use the formula $V = lwh$. Also, since every edge (s) of a cube is the same length, we may use the formula $V = s^3$.

$$V = s^3$$

$$V = (10 \text{ cm})^3$$

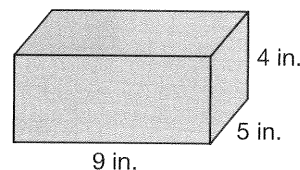
$$V = 1000 \text{ cm}^3$$

The cube in example 4 is a special cube. The metric system relates volume to capacity. The capacity of this cube (1000 cm^3) is equal to one liter. That means a container of this size and shape will hold one liter of liquid. Furthermore, if the liter of liquid is water, then in standard conditions its mass would be one kilogram.

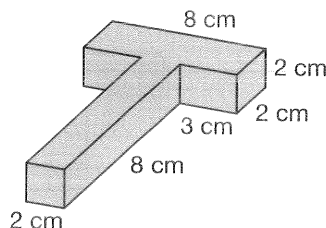
Predict How would the volume of a cube change if you doubled its dimensions?

Practice Set

- a. What is the volume of a tissue box with the dimensions shown?



- b. What is the volume of a cube with edges 2 inches long?
- c. Find the volume of a cube with edges 6 inches long.
- d. How many times greater are the dimensions of the cube in problem c than the cube in problem b? How many times greater is the volume?
- e. What is the volume of this block T?
(All angles are right angles).



- f. **Estimate** Find a box at home that is the shape of a rectangular prism (such as a cereal box) and measure its dimensions. Then sketch the box on your paper and record its length, width, and height. Estimate the capacity of the box in cubic inches by rounding the measurements to the nearest inch before using the volume formula.

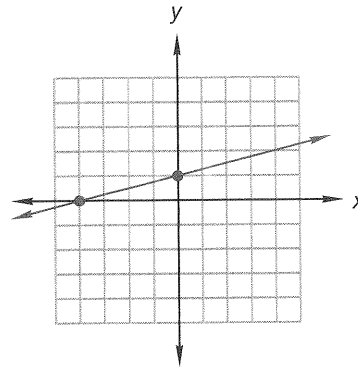
Written Practice

Strengthening Concepts

1. **Evaluate** Jason has two part-time jobs. He earns \$15 an hour working at the help desk at a local library. He also mows lawns earning \$25 per lawn. How much does he earn in a week if he works at the help desk for 2 hours each on Monday and Wednesday and for 4 hours on Friday and he mows 4 lawns on the weekend?

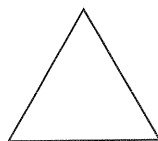
2. There were 35 plastic cups and 15 one-liter bottles of water on a table.
 (29) What was the ratio of plastic cups to bottles of water?
3. The dimensions of a rectangular Olympic-sized swimming pool are
 (42) 2 meters deep, 25 meters wide, and 50 meters long. What is the volume of the pool? If 1 m^3 is equivalent to about 264 gallons, how many gallons of water does the pool hold?

4. **Justify** Shown is a graph of the function
 (41) $y = \frac{1}{4}x + 1$. Does $x = 8$ and $y = 3$ satisfy the function? Demonstrate your answer by substituting the values and simplifying.

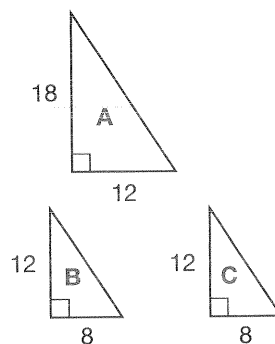


5. a. Using the figure and vanishing point shown below, draw a polyhedron.
 (Inv. 4)
- b. What is the name of the polyhedron you drew?

vanishing point
 ●



6. Lars stitched three right triangular sails for sailboats as shown. Which statement about the sails is true?
 (35)
- A $\triangle A$ and $\triangle B$ are congruent.
- B $\triangle A$ and $\triangle C$ are congruent.
- C $\triangle C$ and $\triangle B$ are similar, but not congruent.
- D $\triangle B$ and $\triangle C$ are congruent and both are similar to $\triangle A$



7. Use a calculator to find the **a** area and **b** circumference of a circle with a
 (4) diameter of 28 feet. Use $\frac{22}{7}$ for π .
8. **Evaluate** On Saturday, Margo bought six packages of rawhide treats
 (3, 4) for her dog. Each package contained 8 treats. By the following Saturday there were 44 treats left. If Margo continues to give her dog treats at this rate, how many weeks will the treats she bought last?

9. Use this table to answer a–c.
(18, 41)

x	y
-2	-6
0	-2
2	2
4	6

- a. Use words to describe the rule shown in the table.
- b. Write an equation that shows the rule.
- c. Sketch a graph that shows the pairs of numbers in the table. Then draw a line through the points to show other (x, y) pairs of numbers that fit the rule.

Simplify.

10. $3 \times 2(6 - 4)^3$
(15, 21)

11. $\frac{5}{8} + 3\frac{2}{3} \div 1\frac{1}{21}$
(13, 22)

12. $\frac{4}{9} \cdot \frac{10}{9} \cdot \frac{3}{8}$
(13, 22)

13. $48 \div [3(4 - 2)^2]$
(15, 21)

14. $\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^4$
(13, 22)

15. $\frac{105xyz}{-5yz}$
(15, 27)

Solve.

16. $-5s = 21.5$
(13, 22)

17. $45.6 - m = 25$
(13, 22)

18. If the thickness of an average sheet of 8.5-inch by 11-inch copy paper is 0.004 inches (or about 0.1 mm) then what is the volume of the sheet of paper in cubic inches?
(42)

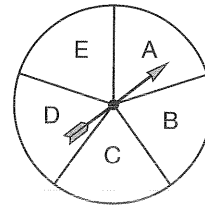
19. a. Expand: $-3(4n - 6w + 5)$
(18, 21)

b. Factor: $8y^2 + 8y + 8$

20. The spinner is spun once. What is the probability the spinner will stop
(32)

a. in sector A?

b. in sector A, B or C?



21. Find the a circumference and b area of a circle with a diameter of 8 meters. Express the answers in terms of π .
(40)

22. In a parking lot, the ratio of cars to trucks is 5 to 1. If there are 20 trucks in the parking lot, how many cars are there? Write a proportion and use it to solve the problem.
(34)

For problems 23–26, find all values of x , if any, that make the equations true.

23. $x^2 + 3 = 7$
(14, 15)

24. $-5|x| = 25$
(14)

25. $3x^2 = 27$
(14, 15)

26. $|x| - 4 = 12$
(14)

27. **Formulate** ⁽³³⁾ A submarine was at a depth of 255 meters below sea level. The submarine rose 65 m. Then it dove 25 m. Write an equation with integers to express the situation. Then find the new depth of the submarine.

Combine like terms to simplify.

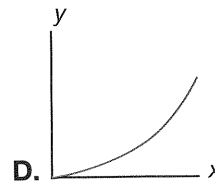
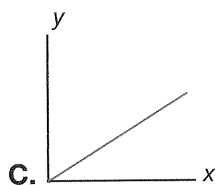
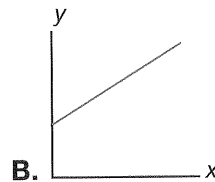
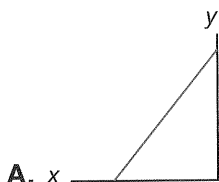
28. ⁽²¹⁾ $3xy - xy - 2y + 5y$

29. ⁽²¹⁾ $5x^2 - 3x + 5x - 4x^2 + 6$

- * 30. ⁽⁴¹⁾ Recall that the graph of a proportional relationship has these characteristics:

1. The graph is a line or aligned points.
2. The graph is aligned with the origin.

Which graph below indicates a proportional relationship? Explain why the other graphs do not show a proportional relationship.



• Surface Area

Power Up

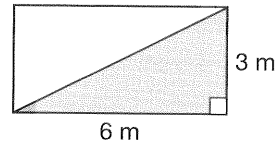
Building Power

facts

mental math

Power Up I

- a. **Number Sense:** $700 \div 200$
- b. **Fractional Parts:** $\frac{1}{3}$ of \$243
- c. **Scientific Notation:** Write 38,100 in scientific notation.
- d. **Proportions:** If Darla can read 6 pages in 10 minutes, how many pages can she read in half an hour?
- e. **Geometry:** Find the area of the rectangle. Then find the area of the shaded triangle.



- f. **Proportions:** $\frac{7}{3} = \frac{x}{9}$
- g. **Powers/Roots:** Estimate $\sqrt{120} - \sqrt{99} + \sqrt{80}$.
- h. **Calculation:** $77 - 7, \div 7, - 7, \times 7, + 7, \div 7 + 7, \times 7$

problem solving

Rolling a number cube has 6 different possible outcomes (from 1 to 6).
 Rolling two number cubes has 11 different possible outcomes (from 2 to 12).
 How many different possible outcomes does rolling 5 number cubes have?

New Concept

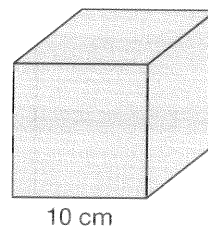
Increasing Knowledge

The surface area of a solid is the combined area of the surfaces of the solid. We distinguish between total surface area and lateral surface area, which is the combined area of surfaces on the sides of a solid and does not include the area(s) of the base(s). We can find the lateral surface area of prisms and cylinders by multiplying the perimeter or circumference of the base by the height.

Example 1

The figure represents a cube with edges 10 cm long.

- a. Find the total surface area of the cube.
- b. Find the lateral surface area of the cube.



Solution

- a. All edges of a cube are the same length. So the length, width, and height of the cube are each 10 cm. Thus, the shape of each face is a square with an area of 100 cm^2 . Since the cube has six congruent faces, the total surface area is

$$6 \times 100 \text{ cm}^2 = \mathbf{600 \text{ cm}^2}$$

- b. The lateral surface area of the cube is the combined area of the four side faces, not including the area of the top or the bottom of the cube. One way to find the lateral surface area of a cube is to multiply the area of one face of the cube by 4.

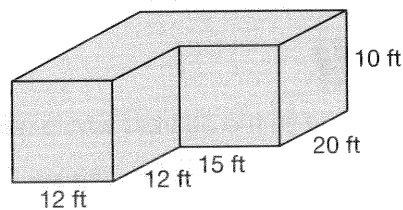
$$\begin{aligned} \text{Lateral surface area of the cube} &= 4 \times 100 \text{ cm}^2 \\ &= \mathbf{400 \text{ cm}^2} \end{aligned}$$

Another way to find the lateral surface area is to multiply the perimeter of the base by the height of the cube.

$$\begin{aligned} \text{Lateral surface area} &= \text{perimeter of base} \cdot \text{height} \\ &= 40 \text{ cm} \cdot 10 \text{ cm} \\ &= \mathbf{400 \text{ cm}^2} \end{aligned}$$

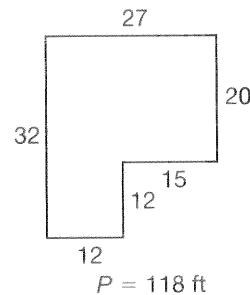
Example 2

Before buying paint for the sides of this building, Malia calculated its lateral surface area. Find its lateral surface area.

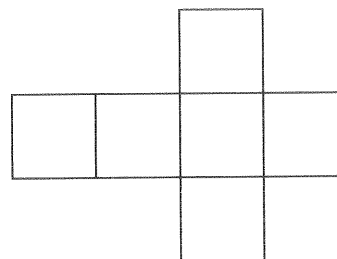
**Solution**

We can find the lateral surface area by multiplying the perimeter of the base by the height of the building.

$$\begin{aligned} \text{Lateral surface area} &= \text{perimeter of base} \cdot \text{height} \\ &= 118 \text{ ft} \cdot 10 \text{ ft} \\ &= \mathbf{1180 \text{ ft}^2} \end{aligned}$$



A net is a two-dimensional image of the surfaces of a solid. A net can help us visualize a solid's surfaces. Here is a net of a cube. We see the six square faces of the cube.



Activity

Surface Area of a Box

Find a box in the classroom that has six complete faces.

1. Identify the faces with terms such as front, back, top, bottom, left side, right side.
2. Which faces are congruent?
3. Sketch a net of the box predicting its appearance if some edges were cut and the box were unfolded.
4. Unfold the box, cutting and taping as necessary, to reveal a net of six rectangles. Find the surface area by adding the areas of the six rectangles.

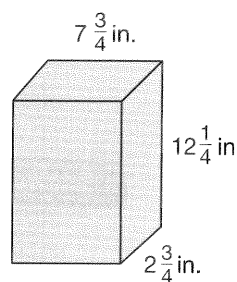
Example 3

Thinking Skill

Connect

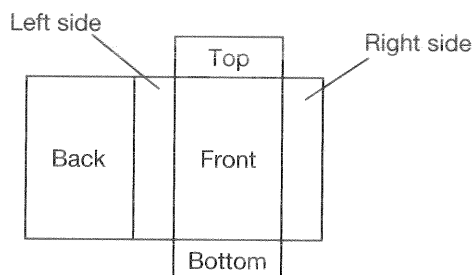
How could we use the lateral surface area of the box to find the total surface area?

The figure shows the dimensions of a cereal box. Estimate the surface area of the box.



Solution

We round to the nearest whole number before performing the calculations. To estimate the surface area we estimate the area of the six surfaces.

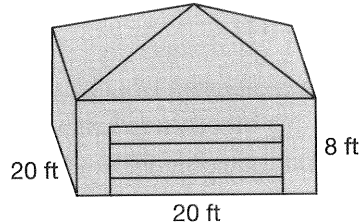
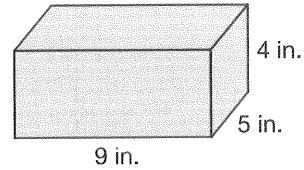


Area of front	$8 \text{ in.} \times 12 \text{ in.} = 96 \text{ in.}^2$
Area of back	$8 \text{ in.} \times 12 \text{ in.} = 96 \text{ in.}^2$
Area of top	$8 \text{ in.} \times 3 \text{ in.} = 24 \text{ in.}^2$
Area of bottom	$8 \text{ in.} \times 3 \text{ in.} = 24 \text{ in.}^2$
Area of left side	$12 \text{ in.} \times 3 \text{ in.} = 36 \text{ in.}^2$
Area of right side	$12 \text{ in.} \times 3 \text{ in.} = 36 \text{ in.}^2$
Total Surface Area	$= 312 \text{ in.}^2$

The surface area of the box is **about 312 in.²**

Practice Set

- a. What is the lateral surface area of a tissue box with the dimensions shown?
- b. What is the total surface area of a cube with edges 2 inches long?
- c. Estimate the surface area of a cube with edges 4.9 cm long.
- d. **Analyze** Kwan is painting a garage. Find the lateral surface area of the building.

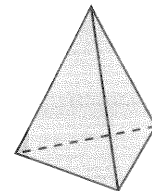


- e. Find a box at home (such as a cereal box) and measure its dimensions. Sketch the box on your paper and record its length, width, and height. Then estimate the number of square inches of cardboard used to construct the box.

Written Practice

Strengthening Concepts

1. Desiree drove north for 30 minutes at 50 miles per hour. Then, she drove south for 60 minutes at 20 miles per hour. How far and in what direction is Desiree from where she started?
(7)
2. In the forest, the ratio of deciduous trees to evergreens is 2 to 7. If there are 400 deciduous trees in the forest, how many trees are in the forest?
(34)
3. Reginald left his house and rode his horse east for 3 hours at 9 miles per hour. How fast and in which direction must he ride to get back to his house in an hour?
(7)
- * 4. **Analyze** What is the volume of a shipping box with dimensions $1\frac{1}{2}$ inches \times 11 inches \times 12 inches?
(42)
5. How many square inches is the surface area of the shipping box described in problem 4?
(43)
- * 6. How many edges, faces, and vertices does a triangular pyramid have?
(Inv. 4)
- * 7. Graph $y = -2x + 3$. Is $(4, -11)$ on the line?
(41)
- * 8. Find **a** the area and **b** circumference of the circle with a radius of 6 in. Express your answer in terms of π .
(39, 40)



- * 9. **Formulate** A hotel shuttles 50 people per bus to an amusement park. The table at the right charts the number of busses x and the corresponding number of people y shuttled to the park. Write an equation that shows the relationship in this table.

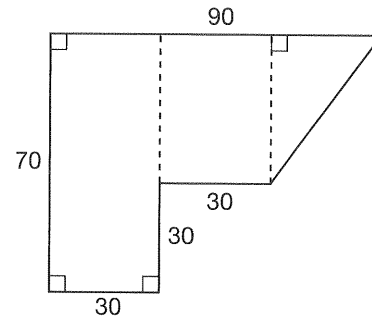
x	y
1	50
2	100
3	150
4	200

For problems 10 and 11, solve using inverse operations:

10. $\frac{x}{1.1} = 11$
(25, 38)

11. $x + 3.2 = 5.14$
(24, 38)

At right is a diagram of a grassy schoolyard. Refer to this diagram to answer problems 12 and 13. Dimensions are in yards.



- * 12. **Evaluate** How many square yards are there to mow?
(37)
13. The athletes do cardiovascular conditioning by running laps around the perimeter of the entire schoolyard. How far do they run in one lap?
(8, Inv. 2)
- * 14. a. Expand: $-5(x^2 - x + 4)$
(21, 36)
- b. Factor: $7x + 7$

Simplify.

15. $\frac{20wx^2}{5x^2}$
(27)

16. $\left(\frac{1}{4}\right)^2 - \left(\frac{1}{2}\right)^4$
(13, 27)

17. $\frac{7}{8} - \frac{12}{13} \div \frac{16}{13}$
(13, 22)

18. $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$
(22)

19. $\frac{1 + 3^2}{(1 + 3)^2}$
(15, 21)

20. $\frac{(-3)(-4)}{(-3) - (-4)}$
(33, 36)

For problems 21–22, find all values of x which make the equations true.

21. $\frac{x^2}{2} = 18$
(14, 15)

22. $\frac{|x|}{5} = 10$
(1, 14)

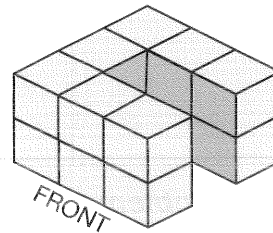
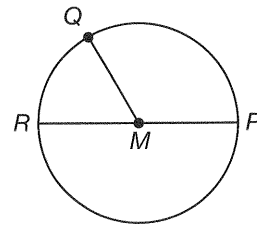
23. Write 0.005 **a** as a percent and **b** as a reduced fraction.
(11, 12)
24. Express $\frac{5}{6}$ **a** as a decimal number, and **b** as a percent.
(30)
25. Write the prime factorization of 9000 with exponents.
(9)

- 26.** Todd wondered if students were likely to favor certain numbers when choosing a number between 1 and 10, so he conducted a survey. He asked 60 students to pick a number from 1 to 10 and tallied the choices.

1	2	3	4	5	6	7	8	9	10

Before the survey, Todd guessed that all numbers were equally likely to be chosen. Based on his hypothesis,

- What is the theoretical probability that a student would choose the number 7?
 - Based on Todd's survey, what is the experimental probability that a student will choose the number 7?
 - Was Todd's hypothesis confirmed in the survey?
- 27.** Combine like terms to simplify: $x^2 + 4x + 4 + 2x^2 - 3x - 4$
(31)
- 28.** Find mgh when $m = 3$, $g = 9.8$, and $h = 10$.
(14)
- 29.** Segment PR is a diameter of circle M and measures 20 in. Central angle QMR measures 60° .
(18, 40)
- Find $m \angle QMP$.
 - Find the area of the semicircle RMP (Use $\pi \approx 3.14$).
- * **30.** *Model* For **a–c**, refer to this figure constructed of 1-cm cubes.
(Inv. 4, 42, 43)



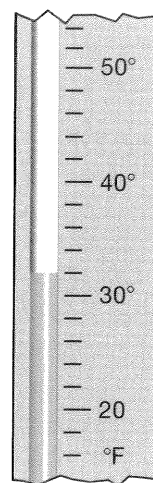
- On grid paper draw the front view, top view, and right-side view of this figure.
- What is the total surface area of this figure?
- What is the volume of this figure?

- Solving Proportions Using Cross Products
- Slope of a Line

Power Up*Building Power***facts****mental math**

Power Up I

- Number Sense:** $6\frac{1}{2} \times 20$
- Fractional Parts:** David tipped $\frac{1}{5}$ of the \$35 meal bill. What was his tip?
- Measurement:** Find the temperature indicated on this thermometer.
- Rate:** Charlene drove 55 miles per hour for 4 hours. Ralph drove 50 miles per hour for 5 hours. How far did each drive?
- Geometry:** Two sides of a triangle are 16 m and 20 m. The third side is between what two lengths?
- Scientific Notation:** Write 2.38×10^7 in standard notation.
- Estimation:** Approximate the total for this shopping bill: 3 items at \$1.99 each, 4 items at \$2.49 each, and 2 items at \$6.99 each.
- Calculation:** $5 + 2, \times 6, \div 7, \times 8, + 1, \div 7, \div 7, \times 12, \div 3, \times 5, + 1, \div 7$

**problem solving**

- Find the square root of the sum of the first 2 positive odd numbers.
- Find the square root of the sum of the first 3 positive odd numbers.
- Find the square root of the sum of the first 4 positive odd numbers.
- Describe the pattern.
- Using the pattern you noticed, what do you think is the square root of the sum of the first 17 positive odd numbers?

New Concepts*Increasing Knowledge***solving proportions using cross products**

We have solved proportions by finding equivalent ratios just as we found equivalent fractions. To solve some proportions it is helpful to use another method. In this lesson we will practice using cross products to solve proportions.

Math Language

Recall that a **proportion** shows that two ratios are equal.

A **cross product** is the result of multiplying the denominator of one fraction and the numerator of another fraction. A characteristic of equal ratios is that their cross products are equal.

$$8 \cdot 3 = 24 \qquad 4 \cdot 6 = 24$$

If we know that two ratios are equal, we can use cross products to help us find an unknown term in one of the ratios.

$$4n \qquad 6 \cdot 6$$

We do not know n , but we know that the cross product $4n$ equals the cross product $6 \cdot 6$.

$$4n = 6 \cdot 6$$

We can solve this equation to find n .

Step:	Justification:
$4n = 6 \cdot 6$	Equal ratios have equal cross products
$\frac{4n}{4} = \frac{6 \cdot 6}{4}$	Divide both sides by 4.
$n = 9$	Simplified

Example 1

Solve using cross products: $\frac{m}{10} = \frac{15}{25}$

Solution

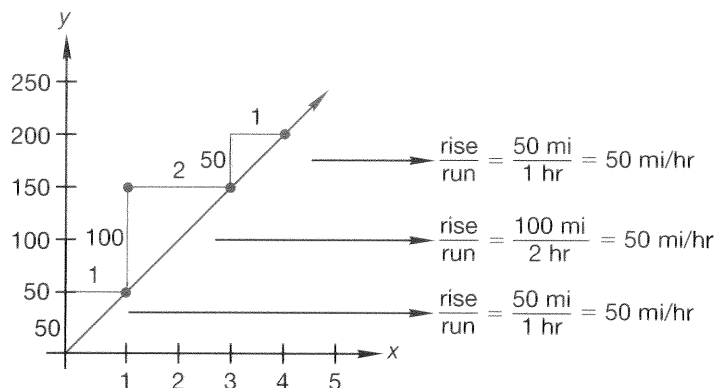
The ratios are equal. We find the cross products and solve for m .

Step:	Justification:
$\frac{m}{10} = \frac{15}{25}$	Given proportion
$25m = 10 \cdot 15$	Equal ratios have equal cross products.
$\frac{25m}{25} = \frac{10 \cdot 15}{25}$	Divide both sides by 25.
$m = 6$	Simplified

Justify Explain how we can check the solution.

Solution

We find that the average rate of change is **50 miles per hour** for each of the three intervals.



Notice that if a function is linear (the points are aligned as on the graph above), then the slope between any two points is constant.

Practice Set

Analyze Use cross products to solve the proportions in **a–d**.

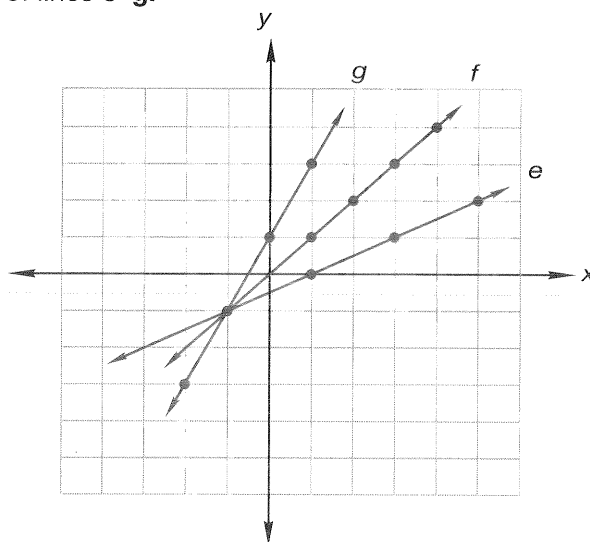
a. $\frac{8}{12} = \frac{12}{w}$

b. $\frac{3}{12} = \frac{x}{1.6}$

c. $\frac{y}{18} = \frac{16}{24}$

d. $\frac{0.8}{z} = \frac{5}{1.5}$

Find the slope of lines **e–g**.



h. Generalize Which statement best describes the relationship between the slope and the steepness of the line?

- A The greater the uphill steepness, the less the slope.
- B The greater the uphill steepness, the greater the slope.
- C The less the uphill steepness, the greater the slope.

i. Represent Graph the line that passes through the origin and $(2, -4)$. Then find the slope of the line.

j. What is the slope of a line passing through $(1, 1)$ and $(5, -1)$?

1. ⁽³⁴⁾ A marine biologist reported the results of a survey that found 3 out of 5 lobsters are undersized and if caught, must be put back in the water. If a lobsterman traps 260 lobsters, about how many should he expect to toss back? Write a proportion and solve.
2. ⁽²²⁾ During their third season, a baseball team played 36 games and won $\frac{5}{9}$ of them. How many games did they win?
A 20 **B** 25 **C** 30 **D** 45
- * 3. ⁽³⁹⁾ **Connect** Tomás plans to build a circular pen for his dog. The radius of the pen will be 17.5 feet. Approximately, how many feet of fencing will Tomás use for the pen?
- * 4. ⁽⁴³⁾ **Analyze** The dimensions of a pasta box are 14 in. by 1.5 in. by 3 in. What is the surface area of the box?
- * 5. ⁽⁴²⁾ Geri wants to fill a rectangular planter box with soil. If the dimensions of the box are $2\frac{1}{2}$ ft by $1\frac{3}{4}$ ft by 4 ft long. How many cubic feet of soil will Geri need?
- * 6. ⁽⁴¹⁾ Anna makes leather coin purses and donates them to the local children's hospital fundraiser fair. The coin purses sell for \$4 each. Make a function table that relates the number of coin purses sold to the amount of money collected. Write an equation for the rule. Is the function linear? Is it proportional?
- * 7. ⁽⁴⁰⁾ An artist makes rugs by painting intricate designs on canvas. He has been commissioned by a local store to create a 12-foot diameter circular rug. He prices his rugs by their area. About what is the area of this 12-foot rug?
- * 8. ⁽⁴¹⁾ **Justify** Consider the function $y = \frac{2}{3}x - 1$.
 a. Does (9, -5) satisfy this function? Why or why not?

 b. If the value of x is 15, what is y? Write your answer as an ordered pair.

Simplify.

9. ^(13, 22) $6\frac{3}{4} - \frac{3}{5} \div \frac{1}{5}$

10. ^(15, 25) $(2.5)^2 - \left(\frac{3}{2}\right)^2$

11. ⁽³³⁾ $(-10) - (-4)$

12. ⁽³⁶⁾ $\frac{(-4)(-8) \div (-2)}{-2}$

* 13. ^(27, 36) $(3x^2y)(-2xy)^3$

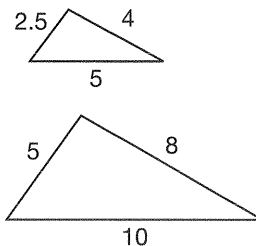
* 14. ^(21, 36) $5(40) \div [-5(1 - 3)^3]$

15. ^(13, 27) $\left[\frac{3}{5} - \left(-\frac{1}{2}\right)^2 + \frac{9}{10}\right]^0$

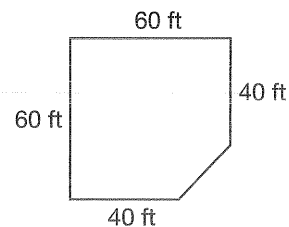
16. ⁽²⁷⁾ $\frac{10^8}{10^3}$

17. Francisco and Rafael are baseball players. Francisco has had 31 hits in ⁽³²⁾ 75 at-bats, while Rafael has had 40 hits in 83 at-bats. Between the two, who is more likely to get a hit in his next at-bat? Explain your answer.
18. A 1-inch garden hose had a small leak. Ali patched the leak by wrapping ⁽³⁹⁾ plastic tape around the hose 6 times. About how many inches of tape did Ali use?
A 6 in. **B** 12 in. **C** 18 in. **D** 21 in.
19. There was very little rain in Austin one summer. In June, the depth ⁽³¹⁾ of Lake Travis fell 3 inches from its usual level. In July, it went down another 5 inches. In August it finally rained, adding 2 inches to the lake's depth. How many inches below the usual level was the lake at that point?
20. It is estimated that at the turn of the 20th century, one farmer in the U.S. ⁽³⁴⁾ could feed 25 people, whereas today, that ratio is about 1 to 130. How many farmers would it take today to feed 2600 people?
21. Pedro collects baseball caps. Three-fourths of the caps are red, and he ⁽³⁸⁾ has 12 red caps. How many caps does Pedro have? Write an equation and solve.

22. The larger triangle is a dilation of the smaller ⁽³⁵⁾ triangle. What is the scale factor?



23. Sergio's front yard has the shape shown. He ⁽³⁷⁾ is going to sod his lawn. Before he buys the sod, Sergio needs to know the area. What is the area of the front yard?

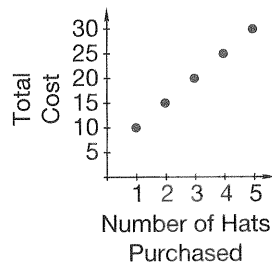


24. a. Write 4% as a reduced fraction and as a decimal. ⁽¹¹⁾
 b. Which form would you find most convenient to answer the following question? "If 4% of a class of 25 students were absent, how many were absent?"
25. Solve by inspection. $5x + 3 = 28$ ⁽¹⁴⁾
26. Find the mean time of Bert's last five downhill skiing runs: 98 sec, ⁽⁷⁾ 90 sec, 102 sec, 97 sec, and 113 sec.
27. Find the area and perimeter of the quadrilateral with vertices ^(8, Inv. 3) $(-4, -1)$, $(-4, 7)$, $(4, 7)$ and $(4, -1)$. What kind of quadrilateral is it?

28. ⁽¹⁴⁾ The science club charges \$1.00 to join and 50¢ dues per week so that they can go on a field trip at the end of the year. The equation for dues (d) collected and the number of weeks (w) is $d = 0.50w + 1$. Katja has been a member for 18 weeks. How much has she paid?

29. ⁽⁴⁴⁾ A line passes through points (4, 2) and (0, -6). Sketch the line and find its slope.

* 30. ⁽⁴¹⁾ **Evaluate** At Hilbert's hat store, Hilbert has a strategy for pricing hats. The rates are shown on the graph below.



Explain Hilbert's pricing strategy. Is the relation proportional? Why or why not? If it is a proportional relationship, write an equation that describes the graph and state the constant of proportionality.

Early Finishers

Real-World Application

A local rental company rents cars for \$45.50 per day plus \$0.10 per mile. Azam rented a car for one day to take a drive along the coast. Write a function that represents the cost of his trip (use c to represent cost and m to represent distance in miles) and create a function table showing the cost of a 50-mile trip, a 125-mile trip, and a 160-mile trip.

• Ratio Problems Involving Totals

Power Up

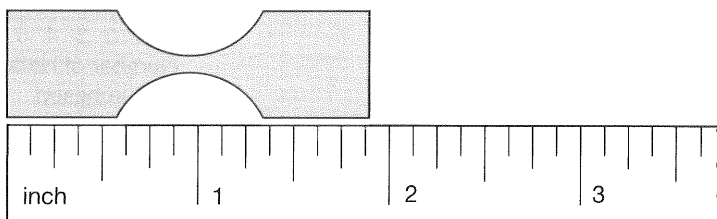
Building Power

facts

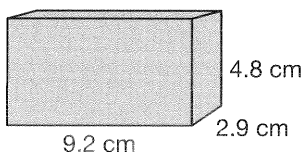
Power Up I

mental math

- a. **Number Sense:** $8\frac{1}{2} \times 4$
- b. **Probability:** What is the probability of rolling a number less than 1 on a number cube?
- c. **Algebra:** $12x = 6$
- d. **Measurement:** Find the length of the object in inches:



- e. **Scientific Notation:** Write 40,800 in scientific notation.
- f. **Rate:** Nathaniel drives north at a rate of 60 miles per hour. Sally drives south at a rate of 60 miles per hour. If they started at the same place and time, how far apart are they after one hour?
- g. **Geometry:** Approximate the volume of this box:



- h. **Calculation:** $10 \times 10, + 44, \sqrt{\quad}, - 3, \sqrt{\quad}, - 3, \sqrt{\quad}$

problem solving

Three friends ordered 3 different sandwiches and 3 different drinks. Annie ordered tuna, but didn't order water. Bernice ordered lemonade. Calvin did not order grilled cheese. Who ordered the cranberry juice? Who ordered the chicken sandwich?

New Concept

Increasing Knowledge

Some ratio problems require us to consider the total to solve the problem. For these problems we add a third row for the total to our ratio table.

Example 1

Acrobats and clowns converged on the center ring in the ratio of 3 to 5. If a total of 24 acrobats and clowns performed in the center ring, how many were clowns?

Solution

We are given the ratio of acrobats to clowns. We can add the ratio numbers to find a ratio number for the total. We are given the actual total.

	Ratio	Actual Count
Acrobats	3	a
Clowns	5	c
Total	8	24

There are three rows in the table. We can use the numbers in two rows to write a proportion. We use the row for the number we want to find (clowns), and we use the row in which we know both numbers (total).

	Ratio	Actual Count
Acrobats	3	a
Clowns	5	c
Total	8	24

$$\rightarrow \frac{5}{8} = \frac{c}{24}$$

$$\rightarrow c = 15$$

We solve the proportion and find that there were **15 clowns**.

Connect What is the relationship between the ratios and the actual counts in example 1? How could you use this information to solve the proportion a different way?

Thinking Skill**Formulate**

Write and solve a proportion to find the total number of acrobats.

Example 2

A bus company has small and large buses in the ratio of 2 to 7. If the company has 84 large buses, how many buses does it have?

Solution

The question involves the total number of buses, so we use a three-row table. To write the proportion, we use numbers from the row with two known numbers and from the row with the unknown we want to find.

	Ratio	Actual Count
Small Buses	2	s
Large Buses	7	84
Total	9	t

$$\rightarrow \frac{7}{9} = \frac{84}{t}$$

$$\rightarrow 7t = 9 \cdot 84$$

$$t = 108$$

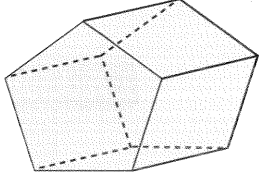
The answer is reasonable because the ratio of 2 to 7 means that there are fewer small buses than large buses. Therefore, the total number of buses is just a little more than the number of large buses.

Practice Set

- a. The ratio of boys to girls at the assembly was 5 to 4. If there were 180 students at the assembly, how many girls were there? Explain why your answer is reasonable.
- b. The coin jar was filled with pennies and nickels in the ratio of 7 to 2. If there were 28 nickels in the jar, how many coins were there?
- c. The ratio of football players to soccer players at the park was 5 to 7. If the total number of players was 48, how many were football players?

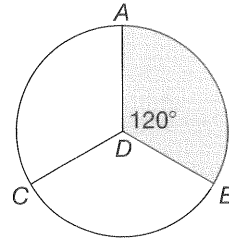
Written Practice

Strengthening Concepts

- * 1. **Analyze** The ratio of house finches to goldfinches is 7 to 3. If there are 80 in all, how many goldfinches are there?
(45)
 2. If it was 113.5 degrees in Amarillo and 95.7 degrees in Phoenix, how much hotter was it in Amarillo than it was in Phoenix?
(24)
 - * 3. The ratio of hours that the power is on to the hours that the power is off is 5 to 7. If there are 720 hours in the month, how many hours is the power on?
(45)
 - * 4. Two guests out of every 25 guests at an amusement park bought popcorn. If there were 5175 guests at the amusement park, how many bought popcorn?
(45)
 5. **a.** A pentagonal prism has how many faces, edges, and vertices?
(Inv. 4)
 - b.** Sketch a net of the prism.
- 
- * 6. Find the volume of a speaker cabinet with dimensions 8 in. by 8 in. by 11 in.
(42)
 - * 7. **Model** Graph the equation $y = -x$. Is (5, 5) on the line? What is the slope of the line?
(41, 44)

- * 8. **Evaluate** Radius DB measures 6,
(40) $m\angle ADB = 120^\circ$.

- Find the area of the circle. (Leave in terms of π .)
- What fraction of the area of the circle does sector ADB cover?
- Find the area of sector ADB . (Leave in terms of π .)

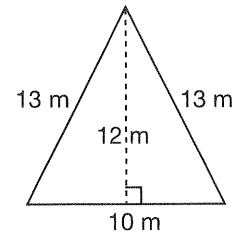


- * 9. Write an equation for the function shown in the table:
(41)

x	y
-1	.5
0	1.5
1	2.5
2	3.5

10. **Classify** a. Classify this triangle by
(20) sides.

- What is its area?
- What is its perimeter?



Solve for x .

11. $\frac{2}{3}x = 30$
(13, 22)

12. $x - \frac{2}{3} = \frac{1}{9}$
(13)

For problems 13–14, find all values of x which make the equations true.

13. $x^2 + 1 = 145$
(38)

14. $7 - |x| = 1$
(38)

Simplify.

15. $\frac{mn}{2m^2}$
(15)

16. $\frac{4}{5} - \frac{1}{5} \div \frac{1}{4}$
(13, 22)

17. $\frac{2}{7} \cdot \frac{3}{4} + \frac{11}{14}$
(13, 22)

18. $\left(\frac{3}{5}\right)^2 + \frac{3}{5}$
(13, 15)

19. Simplify and compare: $\frac{-5 + \sqrt{25-16}}{2} \bigcirc \frac{-5 - \sqrt{25-16}}{2}$
(21, 31)

* 20. Simplify and compare: 0.5% of 1000 \bigcirc 101% of 5
(11, 25)

21. a. Write 45% as a decimal and as a reduced fraction.
(11)

- b. Which of the three forms would be convenient for computing a 45% discount of a \$40 shirt?

22. A chalk artist draws the earth as a circle with a diameter of
(89, 40) 14 m. Find the a area and b circumference of the circle. Express the measures in terms of π .

- 23.** Consider the drawing from problem 22.
(39, 40)
- The number of square meters of a drawing gives the artist an indication of how much chalk is needed. Estimate the number of square meters the drawing will cover.
 - The artist will rope off the drawing to protect the work in progress. Estimate the length of rope needed to rope off the art.

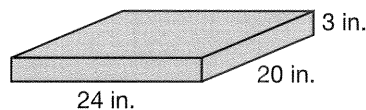
24. For $E = \frac{1}{2}mv^2$, find E when $m = 6$ and $v = 2$.
(14)

25. Which shows how to rewrite $7 \cdot 3 + 7 \cdot 5$?
(21)

- A** $7 + 3 \times 7 + 5$ **B** $7 \times 7 + 3 \times 5$
C $7(3 + 5)$ **D** $8(7 + 7)$

26. Frank earned \$80.00 this week working at a supermarket. He owed his father \$12.00 for a book and \$6.00 for drawing paper that he bought for school last week. How much money does Frank still have from his weekly pay?
(31)

- * **27.** **Analyze** Maria wants to make a cover for the cushion on her chair. She needs to know the total surface area before she buys material. Find the total surface area of the cushion.
(43)



- * **28.** Venus and Earth are often called twin planets because they are almost the same size. Earth has a circumference of 40,070 km. Venus has a diameter of 12,100 km. Which planet is larger? Hint: find the circumference of Venus.
(39)
- * **29.** Lisa stood next to a Ponderosa pine tree that is 5 times her height. If the tree is 6.85 meters high, how tall is Lisa? Write an equation and solve.
(38)

- * **30.** Leora's lasagna recipe required 3 cups of spaghetti sauce to make 4 servings. Leora is serving lasagna to a large group of people. Leora makes a table that shows the number of cups of sauce she needs for multiples of four servings. Copy and complete the table. What is the constant of proportionality? Use the information in the table to find the minimum number of cups of sauce she will need to serve 18 people.
(34)

Number (n) of Servings	Number of Cups of Sauce (s)	Ratio $\frac{s}{n}$
4	3	$\frac{3}{4}$
8	6	
12		
16		
20		